Advanced microeconoics 3: game theory
Spring 2023

Problem set 1 (Due 23.1.2023)

1. Consider the battle of the sexes game:

	Ballet	*Football*
Ballet	$2,1$	$0,0$
Football	$0,0$	$1,2$

Denote by $u_i(\sigma)$ the (expected) payoff of i given mixed strategy profile $\sigma = (\sigma_1, \sigma_2)$.

(a) Define Σ_i, the set of mixed strategies for each player.

(b) Denote by $B_i(\sigma_j) \subset \Sigma_i$ the set of best-responses of player i to player j's strategy σ_j. In other words,

$$B_i(\sigma_j) := \left\{\sigma_i \in \Sigma_i \,\middle|\, u_i(\sigma_i, \sigma_j) \geq u_i\left(\sigma_i', \sigma_j\right) \text{ for all } \sigma_i' \in \Sigma_i.\right\}$$

Derive $B_1(\sigma_2)$ for all $\sigma_2 \in \Sigma_2$. Draw a figure of the best-response correspondences of the two players (Note: with two pure strategies you can express a mixed strategy as a single number).

(c) Find all Nash equilibria of the game.

You can also sketch a similar figure as in b) for other classical 2x2 games listed in the lecture slides (e.g. prisoner's dilemma, stag hunt, hawk-dove, matching pennies)

2. Consider strategy $\sigma_1 = \frac{1}{2}T \circ \frac{1}{2}B$ for player one in the following game:

	L	R
T	$5,2$	$0,1$
C	$2,6$	$4,3$
B	$0,1$	$5,0$

(a) Is there a strategy (mixed or pure) for player 2 such that σ_1 is a best-response to that strategy? Either find such a strategy or show that such a strategy does not exist.

(b) Is there a strategy for player 1 that strictly dominates σ_1? Either find such a strategy or show that such a strategy does not exist.

3. Consider the following normal form game:

	b_1	b_2	b_3	b_4
a_1	$0, 7$	$2, 5$	$7, 0$	$0, 1$
a_2	$5, 2$	$3, 3$	$5, 2$	$0, 1$
a_3	$7, 0$	$2, 5$	$0, 7$	$0, 1$
a_4	$0, 0$	$0, -2$	$0, 0$	$10, -1$

(a) Find for each player the set of strategies that survive iterated deletion of strictly dominated strategies.

(b) Which strategies are rationalizable?

(c) Find the set of Nash equilibria of the game.

4. (Guess the average). Consider the n-player game where all the players announce simultaneously a number in the set $\{1, ..., K\}$ and a price of \$1 is split equally among all the players having the guess closest to $\frac{2}{3}$ of the average of the announced numbers. Find the strategies that are rationalizable (i.e. survive iterated elimination of strictly dominated strategies) and find all Nash equilibria of the game.

5. Consider a simple model of R&D race. Two firms choose simultaneously how much money to invest and the winner is the firm who invests more. The winner gets a prize worth 1 million Euros. If both firms invest the same amount, then each firm wins with probability 1/2. The loser gets no prize. Formulate this situation as a strategic form game and analyze it (hint: look for a symmetric Nash equilibrium in mixed strategies i.e. a probability density function for investment level that keeps players indifferent between different amounts).