



Aalto University  
School of Electrical  
Engineering

# ELEC-E8126 Robotic Manipulation

## Introduction

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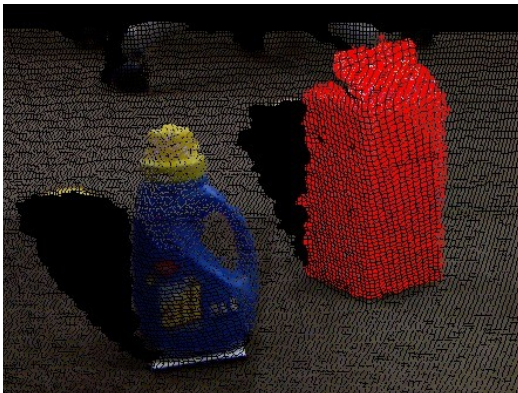
16.1.2023

# Today

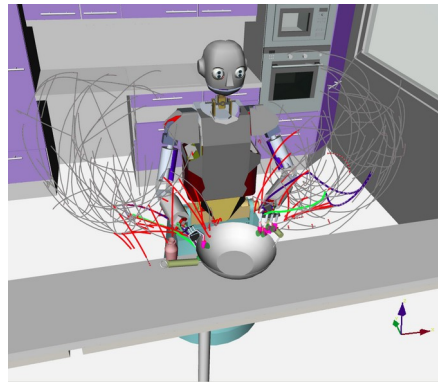
- Course arrangements (see another slide set)
- Quick overview of course contents
- Re-cap of many things

# Typical (advanced) manipulation pipeline

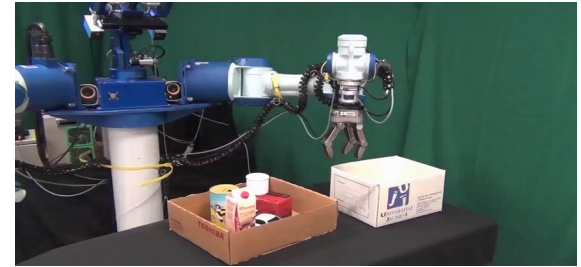
Perception



Planning

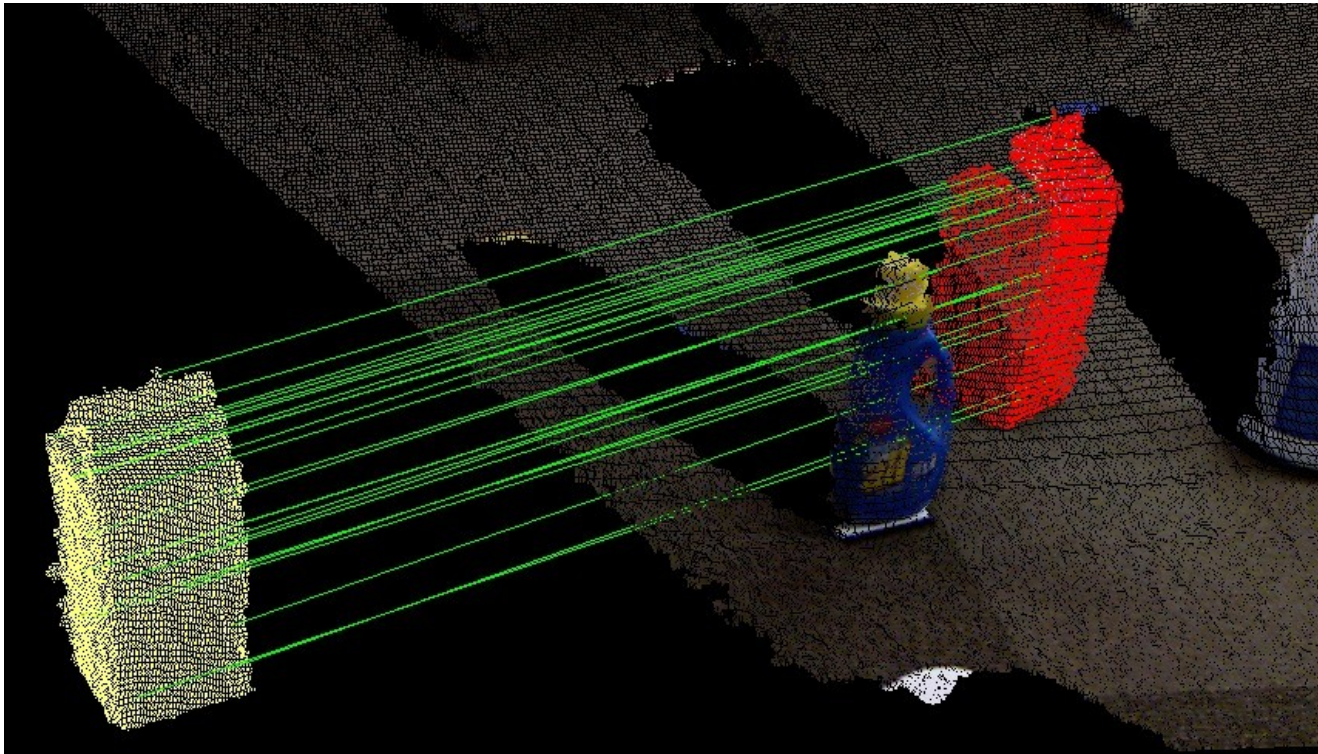


Control / Execution



# Perception

- Primarily: Detection of target objects and obstacles



# Planning problems in manipulation

- How a robot can re-arrange objects surrounding it in order to reach a particular goal? E.g. complete an assembly.
  - Mixture of mechanics and planning (synthesis)
- Hierarchy of techniques: (for finding a sequence of actions)
  - Kinematic manipulation: Based on kinematics. E.g. how to move joints to move from a start to end position without collisions. *Lecture 2.*
  - Static manipulation: Based on statics and kinematics. E.g. how to place an object at rest on a table.
  - Quasi-static manipulation: Kinematics, statics, dynamics without inertia. E.g. grasping stably. *Lectures 5-6.*
  - Dynamic manipulation: Kinematics, statics, dynamics. E.g. throwing an object.

# Control problems in manipulation

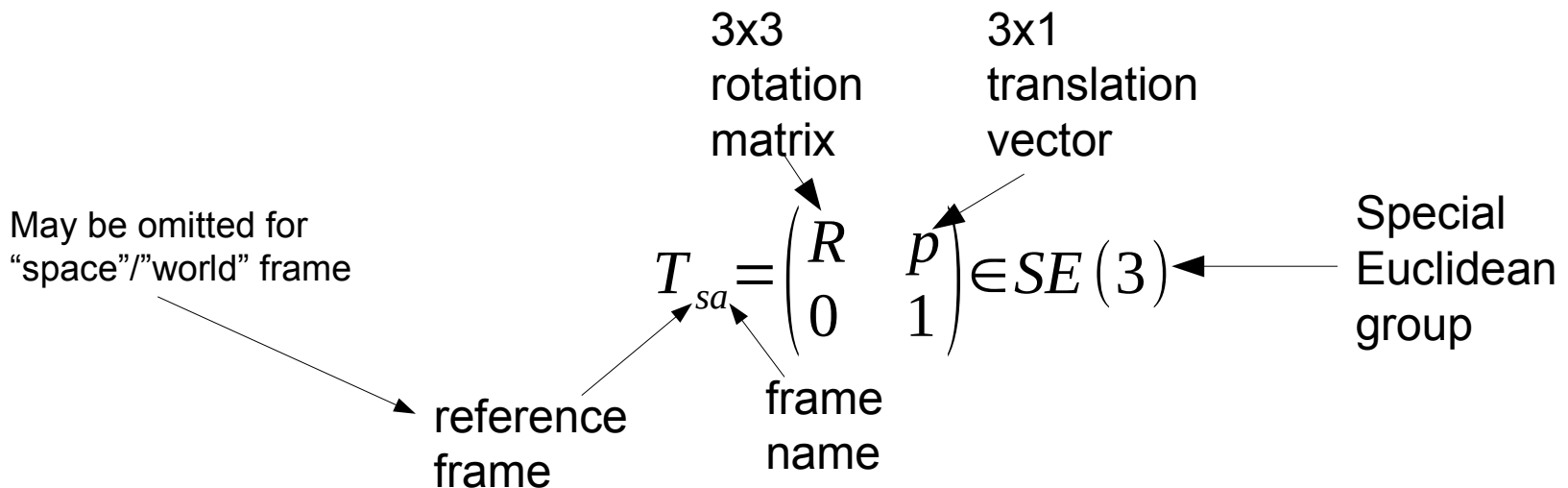
- How to move along a trajectory?
- How to perform several simultaneous tasks? *Lecture 3*
  - E.g. avoid obstacles while moving
- How to perform in-contact motions? *Lecture 3*
- How to perform coordinated motions with several manipulators? *Lecture 8*

# Towards state-of-the-art

- Manipulation with multiple manipulators simultaneously *Lecture 9*
- Modeling and learning manipulation skills *Lecture 10*

# Re-cap: Coordinate frames and transforms

- Coordinate frame  $\{a\}$  can be represented as a 4x4 matrix consisting of translation and rotation



- May also be used to change reference frame of a position vector or frame.  $T_{sb} = T_{sa} T_{ab}$      $v_b = T_{ba} v_a$



# Exponential coordinates for rotation

- Any rotation can be obtained from  $I$  by rotating it by some  $\theta$  about axis  $\hat{\omega}$  (axis-angle representation)
- Can be combined to  $\hat{\omega}\theta \in \mathbb{R}^3$  called exponential coordinates for rotation
- What's the relationship between exponential coordinates and rotation matrix?

# Exponential coordinates cont'd

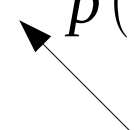
- Velocity of a point in rotation
- Angular velocity
- $$\dot{p} = \hat{\omega} \times p$$
- $$\dot{p} = [\hat{\omega}] p$$
- $$[x] = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$
- Solution?
- $$\dot{x} = a x$$
- $$x(t) = e^{at} x(0)$$

# Exponential coordinates cont'd

- Solution to previous

$$\dot{p} = [\hat{\omega}] p$$
$$p(t) = e^{[\hat{\omega}]t} p(0)$$

Rotation  
matrix



$$[\hat{\omega}] \theta = [\hat{\omega} \theta] = \log R \quad R = e^{[\hat{\omega}] \theta}$$

- Rodrigues' formula

$$R(\hat{\omega}, \theta) = e^{[\hat{\omega}] \theta} = I + \sin \theta [\hat{\omega}] + (1 - \cos \theta) [\hat{\omega}]^2$$

# Spatial velocity

- Similar to angular velocity, we can define spatial velocity as *twist*

$$V = \begin{pmatrix} \omega \\ v \end{pmatrix} \in \mathbb{R}^6$$

← translational velocity

- Let's define skew-operator for twist as

$$[V] = \begin{pmatrix} [\omega] & v \\ 0 & 0 \end{pmatrix} \in se(3)$$

- Transform between frames

$$V_a = \underbrace{\begin{pmatrix} R_{ab} & 0 \\ [p_{ab}] R_{ab} & R_{ab} \end{pmatrix}}_{Ad_{T_{ab}}} V_b = [Ad_{T_{ab}}] V_b$$

# Exponential coordinates of rigid-body motion

- To define unique twist, let us define *screw axis*  $S$

$$S = \begin{pmatrix} \omega \\ v \end{pmatrix} \in \mathbb{R}^6$$

such that  $\|\omega\|=1$  or  $\|v\|=1, \|\omega\|=0$

- Analogous to rotations, we can then define exponential coordinates for rigid-body motions

$$[S]\theta = \log T \in se(3) \quad T = e^{[S]\theta} \in SE(3)$$

# Wrenches

- We can define a similar representation for linear forces acting on the rigid body.
- The 6-dimensional combination of moment and force is called a **wrench**

$$F = \begin{pmatrix} m \\ f \end{pmatrix} \in \mathbb{R}^6$$

- Coordinate frame for wrenches can be changed using the adjoint

$$F_b = Ad_{T_{ab}}^T F_a$$

# Re-cap: Forward kinematics

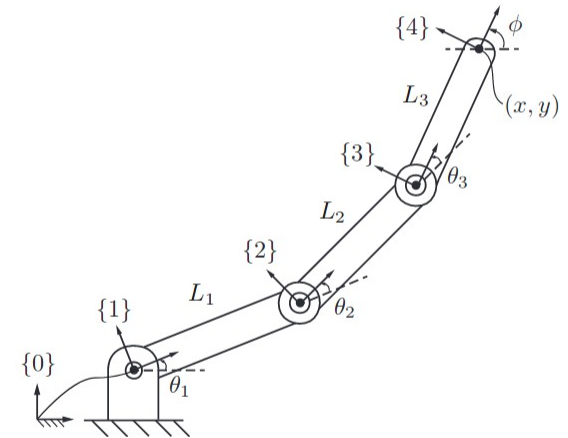
- Forward kinematics is mapping from joint values to end-effector pose
- Forward kinematics of serial chain can be obtained from product of transformation matrices

$$T_{04} = T_{01} T_{12} T_{23} T_{34}$$

- Forward kinematics can also be expressed as product of exponentials

$$T(\theta) = e^{[S_1]\theta_1} \dots e^{[S_N]\theta_N} M$$

End-effector pose at zero position



# Re-cap: Velocity kinematics

- Jacobian: mapping from joint velocities to Cartesian velocities (expressed e.g. as twists)

$$V = J(\theta) \dot{\theta}$$

- Using screw representation of kinematics,  $i$ :th column of Jacobian in space frame is

$$J_{si}(\theta) = [Ad_{e^{[S_1]\theta_1} \dots e^{[S_{i-1}]\theta_{i-1}}}] S_i$$

- Kinematic singularity: Jacobian is not full rank
  - Can you name examples?



# Re-cap: Forward kinematics

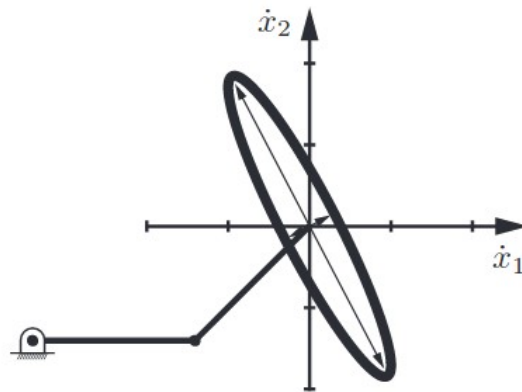
- Fwd kinematics
  - Serial chain, product of exponentials
- Jacobian & body-Jacobian
  - Null-space
  - Singularities
- Inverse kinematics
  - Analytical or numerical

# Manipulability and force ellipsoids

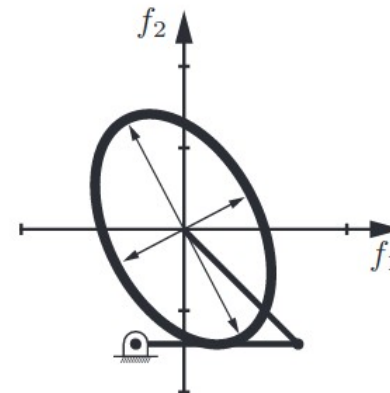
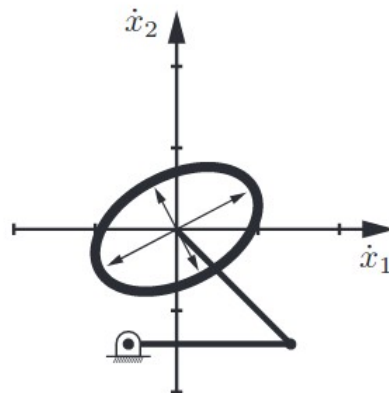
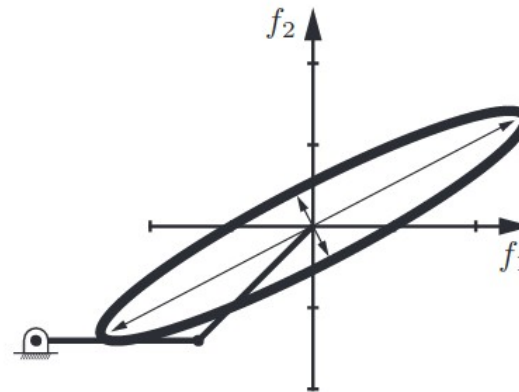
- Manipulability ellipsoid: how easily the robot can move in different directions, corresponds to eigenvalue decomposition of  $J J^T$  ← PCA
- Force ellipsoid: how easily the robot can produce forces in different directions, corresponds to eigenvalue decomposition of  $(J J^T)^{-1}$
- What happens to these at a singularity?

# Manipulability and force ellipsoids

Manipulability



Force



# For next time

- To complement this lecture, read L&P chapter 5-5.1.4 (also ch. 3 is useful)
- Next time we'll talk about motion planning (ch. 10)

# Extra: Series representation of solution of differential equations

$$\begin{aligned}\dot{x}(t) &= a x(t) \\ x(t) &= e^{at} x(0)\end{aligned}$$

$$e^{at} = 1 + at + \frac{(at)^2}{2!} + \frac{(at)^3}{3!} + \dots$$

$$\begin{aligned}\dot{x}(t) &= A x(t) \\ x(t) &= e^{At} x(0)\end{aligned}$$

$$e^{At} = 1 + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \dots$$