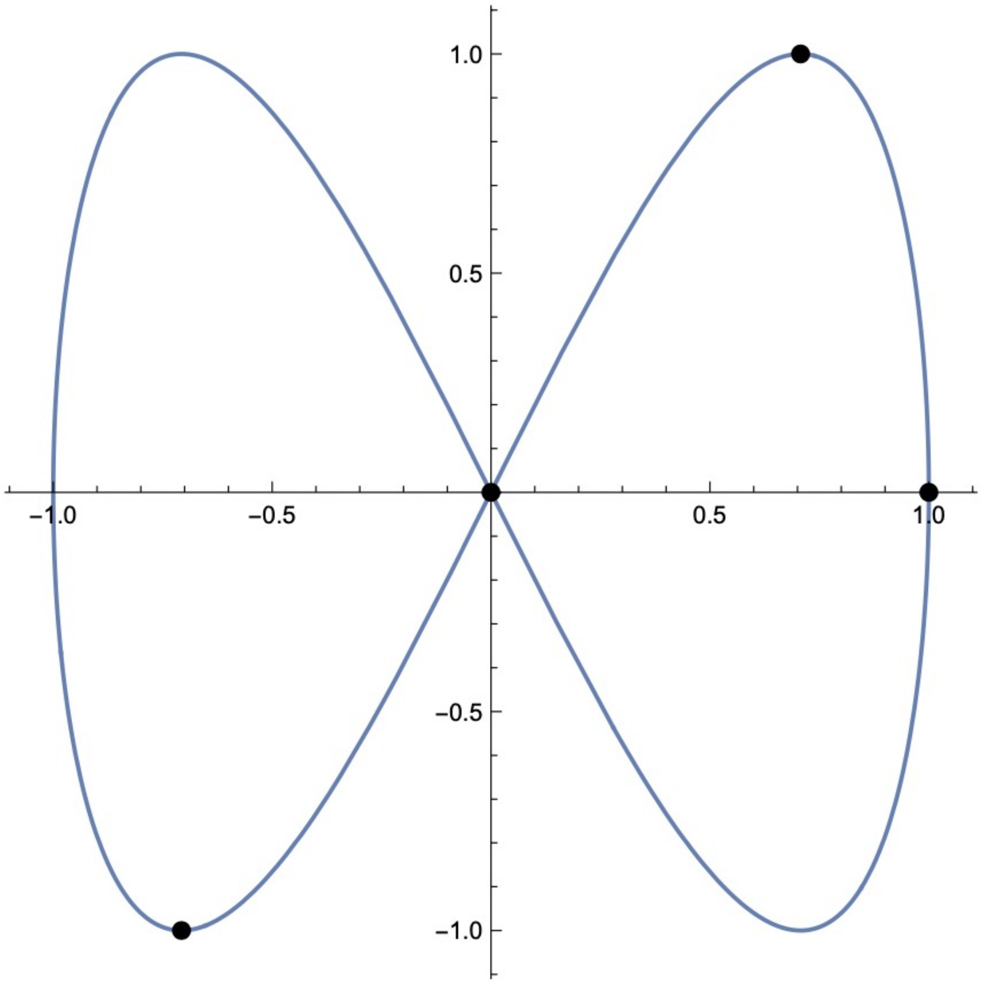
K1 a) Kyseessä on ns. rusettikäyrä.

t	x	y
0	1	0
$\pi/4$	$1/\sqrt{2}$	1
$\pi/2$	0	0
$3\pi/4$	$-1/\sqrt{2}$	-1



Kiertosuunta on vastapäivään $x > 0$ ja myötäpäivään $x < 0$.

b) $\underline{r}(t) = \cos t\,\underline{i} + \sin 2t\,\underline{j}$

$\underline{r}'(t) = -\sin t\,\underline{i} + 2\cos 2t\,\underline{j}$

Leikkauskohta $(0,0)$ saavutetaan parametrin arvoilla $t = \pi/2$ ja $t = 3\pi/2$

Tangenttivektorit ovat siis

$$\underline{t}_1 = \underline{r}'(\pi/2) = -\underline{i} - 2\underline{j}$$

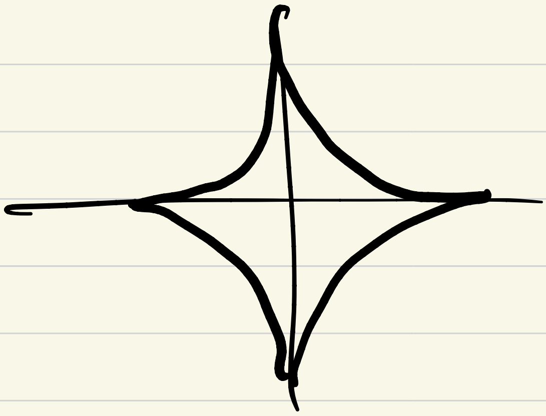
$$\underline{t}_2 = \underline{r}'(3\pi/2) = \underline{i} - 2\underline{j}$$

$$\underline{t}_1, \underline{t}_2 \neq \underline{0} \Rightarrow \cos\measuredangle(\underline{t}_1, \underline{t}_2) = \frac{\underline{t}_1 \cdot \underline{t}_2}{\|\underline{t}_1\|\,\|\underline{t}_2\|}$$

$$\Rightarrow \arccos \frac{3}{\sqrt{5}\sqrt{5}} = \arccos \frac{3}{5}$$

K2

Asteroidi :



$x(t) = a\cos^3 t$
$y(t) = a\sin^3 t$

Symmetria: $$4\int_0^{\pi/2} \sqrt{x'(t)^2 + y'(t)^2}\, dt = I_s$$

$x'(t) = -3a\cos^2 t \sin t$
$y'(t) = 3a\cos t \sin^2 t$

$$I_s = 4 \cdot 3 \cdot a \int_0^{\pi/2} \sqrt{\cos^4 t \sin^2 t + \cos^2 t \sin^4 t}\, dt$$

$$= 12a \int_0^{\pi/2} \cos t \sin t\, dt = 12a \Big/_0^{\pi/2} -\frac{1}{2}\cos^2 t$$

$$= 6a$$