Advanced microeconoics 3: game theory
Spring 2023

Problem set 2 (Due 30.1.2023)

1. Consider the following two-player game:

	L	R
U	$5,1$	$0,0$
D	$4,4$	$1,5$

   (a) Find all Nash equilibria of the game. What is the best payoff that the players can get in a symmetric equilibrium?

   (b) Suppose that before choosing their actions, the players first toss a coin. After publicly observing the outcome of the coin toss, they choose simultaneously their action. Draw the extensive form game and define available strategies for the players. Find a Nash equilibrium that gives both players a higher payoff than the symmetric equilibrium in a).

   (c) Suppose that there is a mediator that can make a recommendation separately for each player. Suppose that the mediator makes recommendation (U, L), (D, L) or (D, R), each with probability $1/3$. Each player only observes her own action choice recommendation (so that, e.g., player one upon seeing recommendation D does not know whether the recommended profile is (D, L) or (D, R)). Does any of the players have an incentive to deviate from the recommended action? What is the expected payoff under this scheme?

   The solution concept that this excercise demonstrates is called correlated equilibrium (see Myerson, Osborne-Rubinstein, or Fudenberg-Tirole for full discussion of the concept).

2. Show that a finite two-player zero-sum game has a Nash-equilibrium in pure strategies if and only if

$$\max_{s_1 \in S_1} \min_{s_2 \in S_2} u(s_1, s_2) = \min_{s_2 \in S_2} \max_{s_1 \in S_1} u(s_1, s_2),$$

where S_i, $i = 1, 2$, is the set of pure strategies available for i and $u(s_1, s_2)$ is the payoff of player 1 (since this is a zero-sum game, the payoff of player 2 is $-u(s_1, s_2)$).

3. Consider the following game:

	L	R
T	$0, 2$	$2, 2$
C	$0, 0$	$1, 1$
B	$2, 4$	$0, 0$

   (a) Find all Nash equilibria of the game.

   (b) Verify that all equilibria are trembling hand perfect (by constructing an appropriate sequence of "trembles")

   (c) Change the game to:

	L	R
T	$0, 2$	$2, 2$
C	$0, 0$	$1, 0$
B	$2, 4$	$0, 0$

   Are all equilibria still trembling hand perfect?

4. Two firms compete in the market, and there are three possible output levels available for both players: low (L), medium (M), and high (H). The profits for the diffent combinations of outputs are given below:

	L	M	H
L	$18, 18$	$15, 20$	$9, 18$
M	$20, 15$	$16, 16$	$8, 12$
H	$18, 9$	$12, 8$	$0, 0$

   (a) (Cournot model) Suppose that the players chooose their outputs simultaneously. Draw the extensive form game. Find the Nash equilibria.

(b) (Stackelberg model) Suppose that player 1 moves first and player 2 moves after observing the output of firm 1. Draw the extensive form game. Find the sub-game perfect equilibria of the game.

(c) Consider the normal form representation (or strategic form) of the extensive form game derived in (b). Give an example of a pure strategy profile. How many pure strategies are available to each of the players? Give an example of a Nash equilibrium that is not sub-game perfect.

5. (War of attrition) Two players are fighting for a prize whose current value at any time $t = 0, 1, 2, ...$ is $v > 1$. Fighting costs 1 unit per period. The game ends as soon as one of the players stops fighting. If one player stops fighting in period t, he gets no prize and incurs no more costs, while his opponent wins the prize without incurring a fighting cost. If both players stop fighting at the same period, then neither of them gets the prize. The players discount their costs and payoffs with discount factor δ per period.

   This is a multi-stage game with observed actions, where the action set for each player in period t is $A_i(t) = \{0, 1\}$, where 0 means continue fighting and 1 means stop. A pure strategy s_i is a mapping $s_i : \{0, 1, ...\} \to A_i(t)$ such that $s_i(t)$ descibes the action that a player takes in period t if no player has stopped the game in periods $0, ..., t-1$. A behavior strategy $b_i(t)$ defines a probability of stopping in period t if no player has yet stopped.

   (a) Consider a strategy profile $s_1(t) = 1$ for all t and $s_2(t) = 0$ for all t. Is this a Nash equilibrium?

   (b) Find a stationary symmetric Nash equilibrium, where both players stop with the same constant probability in each period.

   (c) Are the strategy profiles considered above sub-game perfect equilibria?

   (d) Can you think of other strategy profiles that would constitute a sub-game perfect equilibrium?