Class exercises for Week 3. To be done in class. These exercises do not need to be returned, and they are not marked.

1. Suppose that at the point $(1, 2)$ the directional derivative of the function $z = f(x, y)$ in the direction $3\mathbf{i} + 4\mathbf{j}$ is equal to 2 and the directional derivative of $z = f(x, y)$ in the direction $5\mathbf{i} + 12\mathbf{j}$ is equal to 1.

   (a) Determine the gradient of f at the point $(1, 2)$.

   (b) What is the maximum rate of change of the function f at the point $(1, 2)$? And in what direction does this maximum occur?

2. Consider the function

$$f(x, y) = \sin(xe^y) - x + 3.$$

   (a) Compute all the 1st and 2nd order partial derivatives of $f(x, y)$.

   (b) A given fact is that $f(0.2, 0.1) = 3.01924$ accurate to 5 decimal places. Taking $(0, 0)$ as the reference point, use **linear approximation** to find an approximation of $f(0.2, 0.1)$.

   (c) By referring to the idea of a tangent plane, explain why you obtained the answer you did in part (b).

   (d) Use a **2nd order Taylor polynomial** to find an approximation of $f(0.2, 0.1)$. Is this approximation better or worse than the linear approximation?

   (e) Find a critical point of $f(x, y)$ and determine if it is a local min, local max or saddle. You do not need to find all of them.

3. Let $f(x, y) = y - x^2$.

   (a) Make a sketch of the level curves $f(x, y) = 1$ and $f(x, y) = 2$ on the same axes (that is, make a contour plot with these two level curves).

   (b) On the above plot, draw a point P and a vector $\mathbf{u}$ (with initial point at P) such that the directional derivative $D_{\mathbf{u}}(P)$ is negative.

   (c) At the point $(1, 3)$, find the direction (vector) in which $f(x, y)$ is increasing the fastest. Sketch the point and the vector on the above plot.