Class exercises for Week 4. To be done in class. These exercises do not need to be returned, and they are not marked.

1. A flat circular plate has the shape of the region $x^2+y^2 \leq 1$. The plate (including the boundary $x^2+y^2=1$) is heated so that the temperature $T(x,y)$ at any point (x,y) is given by

$$T(x,y) = x^4 - 4x^2 + 2y^2.$$

   Locate the hottest and coldest points of the plate and determine the temperature at each of these points.

2. Use the method of Lagrange multipliers to find the absolute extrema of $f(x,y) = 4xy$ on the region $4x^2+y^2 \leq 8$.

3. Let $a_1, a_2, \ldots, a_n$ be positive numbers

   (a) Find the maximum value of the expression $a_1x_1 + a_2x_2 + \cdots + a_nx_n$ if the variables $x_1, x_2, \ldots, x_n$ are restricted so that the sum of their squares is one.

   (b) What is the minimum value of $a_1x_1 + a_2x_2 + \cdots + a_nx_n$ in this case?

   (c) Give an interpretation of your results using the dot product in the case $n = 2$.

4. Find approximate solutions of the system of equations

$$\begin{cases} x^2+y^2-1 &= 0 \\ y - e^x &= 0 \end{cases}.$$

   For that, we can program the Newton's Method on a computer (e.g. using Maple, Python, Matlab) to generate the required approximations. In the exercise class you could try to program this or just discuss how you would implement practically this using Newton's method.