Advanced microeconomics 3: game theory
Spring 2023

Problem set 3 (Due 13.2.2023)

1. Consider the simple card game discussed in the lecture notes: Players 1 and 2 put one dollar each in a pot. Then, player 1 draws a card from a stack, observes privately the card, and decieds wheter to "raise" or "fold". In case of "fold", game ends and player 1 gets the money if the card is red, while player 2 gets the money if black. In case of "raise", player 1 adds another dollar in the pot, and player 2 must decide whether to "meet" or "pass". In case of "pass", game ends and player 1 takes the money in the pot. In case of "meet", player 2 adds another dollar in the pot, and player 1 shows the card. Player 1 takes the money if the card is red, while player 2 takes the money if black.

   (a) Formulate the card game as an extensive form game.

   (b) Represent the game in strategic form and find the unique mixed strategy Nash equilibrium of the game.

   (c) Write the corresponding equilibrium using behavior strategies.

   (d) Derive a belief system (probabilities for nodes within each information set) that is consistent with the equilibrium strategies (i.e., derived using Bayesian rule).

   (e) Check that the equilibrium strategies are sequentially rational given the belief system that you derived in d).

2. Players 1 and 2 want to divide a dollar and they have two periods to reach an agreement. Players are risk-neutral, and if the agreement is not reached by the end of period 2, the dollar will be destroyed. Nature chooses player 1 to make a proposal on a division of the dollar in period $t \in \{1, 2\}$ with probability π, and with complementary probability it is player 2, who gets to make a proposal in period t. That is, in period 1 the player recognized as the proposer suggests a division of the dollar $(x^1, 1 - x^1)$ and the other player can either accept or refuse

this proposal. If the offer is accepted, the game ends with payoffs $(x^1, 1-x^1)$. If the offer is refused, the game moves to period 2, where Nature chooses a proposer again and the recognized player proposes a division $(x^2, 1-x^2)$. If the offer is accepted, the game ends with payoffs $(\delta x^2, \delta(1-x^2))$. If the offer is rejected, the game ends with payoffs $(0,0)$. Find the unique SPE of the game. Give an example of a Nash equilibrium that is not sub-game perfect.

3. Consider the following strategic form game:

	a	b	c
A	$0,0$	$3,4$	$6,0$
B	$4,3$	$0,0$	$0,0$
C	$0,6$	$0,0$	$5,5$

   (a) Find all Nash equilibria of the static game.

   (b) Suppose that the above described stage-game is repeated twice, so that before playing the second stage the players observe each others' action choices for thse first stage. A player's payoff is the sum of the stage-game payoffs. Find all sub-game perfect Nash equilibria of the game.

   (c) Suppose that the players discount their stage-two payoffs relative to stage-one payoffs with discount factor $\delta < 1$. For which values of δ does an equilibrium exists where (C, c) is played?

4. Consider a two-stage game with observed actions, where in the first stage players choose simultaneously U1 or D1 (player 1) and L1 or R1 (player 2), and in the second stage players choose simultaneously U2 or D2 (player 1) and L2 or R2 (player 2). The payoffs of the stage games are shown in the tables below:

First stage:

	$L1$	$R1$
$U1$	$2,2$	$-1,3$
$D1$	$3,-1$	$0,0$

Second stage:

	$L2$	$R2$
$U2$	$6,4$	$3,3$
$D2$	$3,3$	$4,6$

The players maximize the sum of their stage-game payoffs.

(a) Find the Nash equilibria of the two static stage-games.

(b) Find the subgame-perfect equilibria of the two-stage game.

(c) Suppose that the players can jointly observe the outcome y_1 of a public randomizing device before choosing their first-stage actions, where y_1 is drawn from uniform distribution on the unit interval. Find the set of subgame-perfect equilibria, and compare the set of possible payoffs against the possible payoffs in b).

(d) Suppose that the players jointly observe y_1 at the beginning of stage 1 and y_2 at the beginning of stage 2, where y_1 and y_2 are independent draws from a uniform distribution on a unit interval. Again, find the sub-game perfect equilibriua and possible payoffs.

5. (Folk Theorem) Consider an infinitely repeated game with a stage game given in the following matrix:

	L	R
U	$5, 0$	$0, 1$
M	$3, 0$	$3, 3$
D	$0, -1$	$0, -1$

Players have a common discount factor.

(a) Find the minmax payoffs for each of the players.

(b) Characterize the set of feasible payoff vectors of the stage game (Assume that a public randomization device is available).

(c) What is the set of normalized payoff vectors for the repeated game, such that each element in the set is a subgame perfect equilibrium payoff vector for some value of the discount factor?

(d) Can you construct some subgame perfect equilibrium strategies leading to the constant play of (U, L) in the equilibrium path?

(e) Let's change the game so that payoffs for (D, L) and (D, R) are $(0, 0)$. Can there now be an equilibrium with a constant play of (U, L)?