Aalto University
School of Electrical
Engineering

ELEC-E8126: Robotic Manipulation
Control in contact

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Learning goals

- Understand basic approaches of force and impedance control.
- Understand how control can be partitioned with multiple objectives.

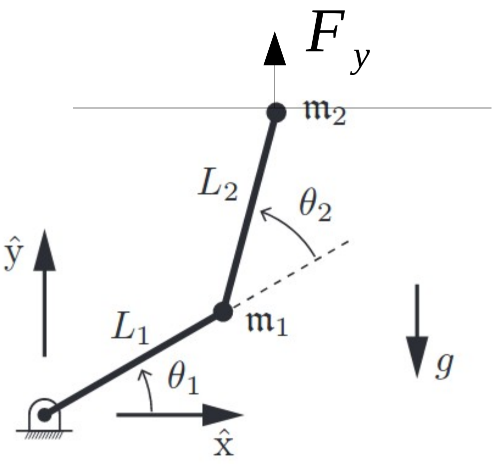
Contact

- Contact requires control of interaction forces.

- Approaches:
  - Control interaction forces to desired values → force control
  - Control interaction behavior → impedance control

Force control

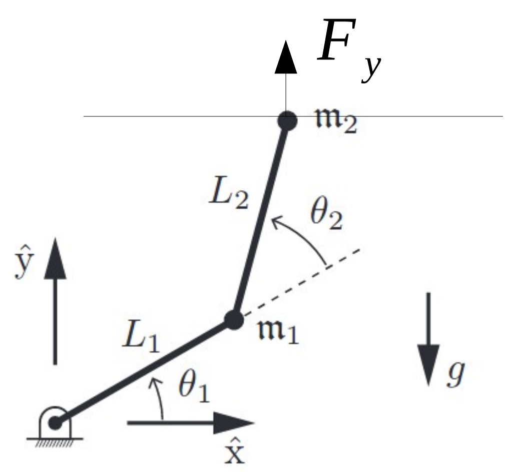
- Try to achieve vertical contact force F_{yd}



- Propose a feedforward controller!

Force control

- Try to achieve vertical contact force F_{yd}



- Propose a feedforward controller!

$$\boldsymbol{\tau} = J_y^T(\boldsymbol{\theta}) F_{yd} + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

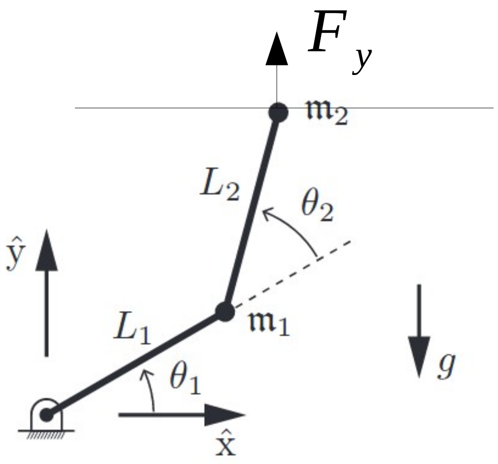
What's this?
What is its meaning?

Dynamics (gravity) compensation

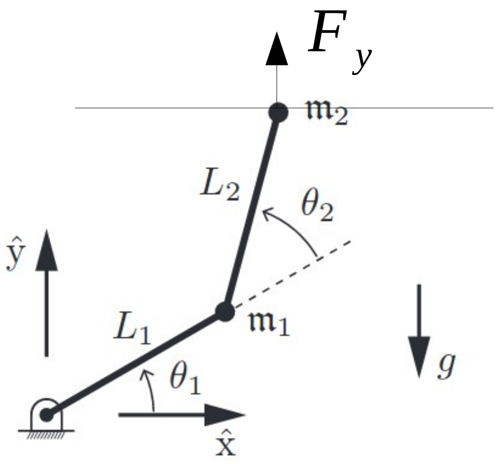
What happens if there are disturbances such as error in estimated mass (gravity term)?

Force control

- Try to achieve vertical contact force F_{yd}
- Now assume force can be measured, giving error $F_{ye} = F_y - F_{yd}$



Force control



- Try to achieve vertical contact force F_{yd}
- Now assume force can be measured, giving error $F_{ye} = F_y - F_{yd}$

- PI-controller with feedforward can then be constructed:

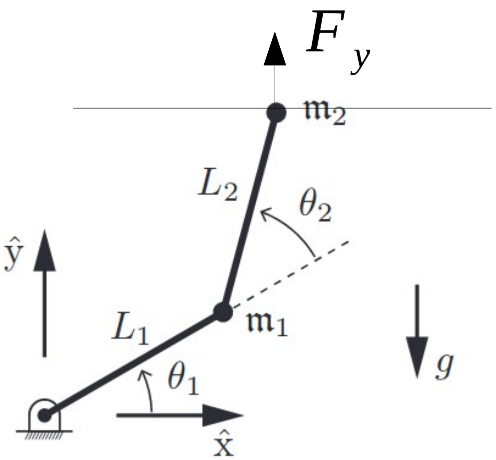
$$\boldsymbol{\tau} = J_y^T(\boldsymbol{\theta})\left(F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye}\right) + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

Dynamics compensation

What happens if there are disturbances such as error in estimated mass (gravity term)?

Recap: Position control

- Propose a position controller to remain in horizontal position x_d



Recap: Position control

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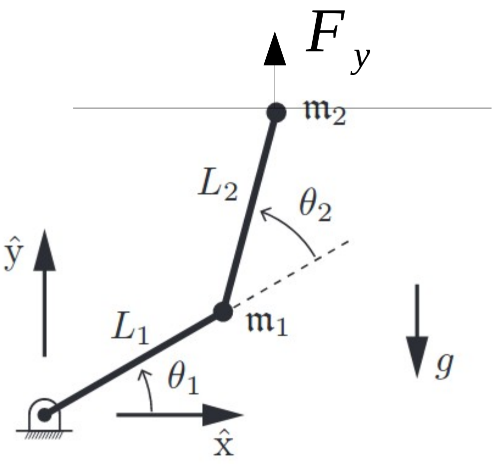

- Computed torque controller:

$$\boldsymbol{\tau} = J_{x}^{T}(\boldsymbol{\theta})\left(M_C(\theta)\left(\ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e\right)\right) + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

Recap: Position control

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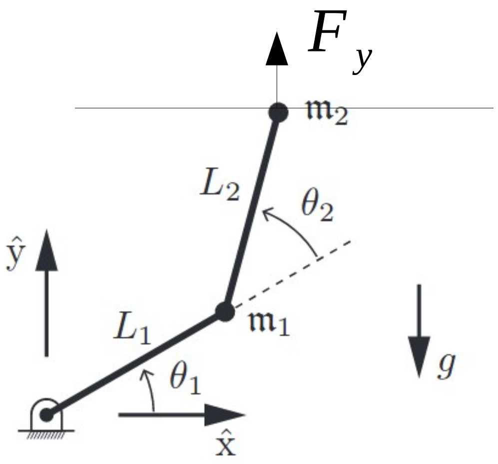


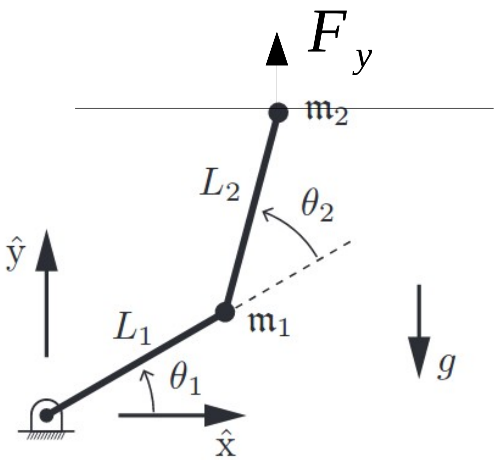
- Computed torque controller:

$$\boldsymbol{\tau} = J_x^T(\boldsymbol{\theta})\left(M_C(\theta)\left(\ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e\right)\right) + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

How to combine with force controller?

$$\boldsymbol{\tau} = J_y^T(\boldsymbol{\theta})\left(F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye}\right) + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

Hybrid control (example)



- Hybrid just combination of the two

$$\boldsymbol{\tau} = J_x^T(\boldsymbol{\theta})\left(M_C(\theta)\left(\ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e\right)\right) + J_y^T(\boldsymbol{\theta})\left(F_{yd} + K_{fp} F_{ye} + K_{fi}\int F_{ye}\right) + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

- Note: $J = \begin{bmatrix} J_x \\ J_y \end{bmatrix}$ (ignoring J_θ)

- Can we write this now somehow in form $\boldsymbol{\tau} = J^T(\ldots)$?

We try to generalize the approach to any number of position and force controlled degrees of freedom.

Hybrid control (example)

$$\boldsymbol{\tau} = J_x^T(\boldsymbol{\theta})\left(M_C(\theta)\left(\ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e\right)\right) + J_y^T(\boldsymbol{\theta})\left(F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye}\right) + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

$$J = \begin{bmatrix} J_x \\ J_y \end{bmatrix}$$

$$\boldsymbol{\tau} = J^T(\boldsymbol{\theta})\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} M_C(\theta)\left(\ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e\right) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye}\right)\right) + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

“select” a column of the preceding matrix J^T

Hybrid control (example)

$$\boldsymbol{\tau} = J^T(\boldsymbol{\theta})\left(\begin{bmatrix}1\\0\end{bmatrix} M_C(\theta)\left(\ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e\right) + \begin{bmatrix}0\\1\end{bmatrix}\left(F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye}\right)\right) + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

Position and force measurements usually measure all degrees of freedom. Can we do a “selection” here as well and use feedback variables

$$\boldsymbol{x} = \begin{bmatrix}x\\y\end{bmatrix}, \boldsymbol{F} = \begin{bmatrix}F_x\\F_y\end{bmatrix}?$$

Hybrid control (example)

$$\boldsymbol{\tau} = J^T(\boldsymbol{\theta})\left(\begin{bmatrix}1\\0\end{bmatrix} M_C(\theta)\left(\ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e\right) + \begin{bmatrix}0\\1\end{bmatrix}\left(F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye}\right)\right) + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

Orthogonal directions where controllers act

$$\boldsymbol{\tau} = J^T(\boldsymbol{\theta})\left(\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} M_C(\boldsymbol{\theta})\left(\ddot{\boldsymbol{x}}_{\boldsymbol{d}} + K_p \boldsymbol{x}_{\boldsymbol{e}} + K_i \int \boldsymbol{x}_{\boldsymbol{e}} + K_d \dot{\boldsymbol{x}}_{\boldsymbol{e}}\right) + \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\left(\boldsymbol{F}_{\boldsymbol{d}} + K_{fp} \boldsymbol{F}_{\boldsymbol{ye}} + K_{fi} \int \boldsymbol{F}_{\boldsymbol{ye}}\right)\right) + \hat{\boldsymbol{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

$$\boldsymbol{x} = \begin{bmatrix}x\\y\end{bmatrix} \qquad \boldsymbol{F} = \begin{bmatrix}F_x\\F_y\end{bmatrix}$$

Note: all dimensions in both

Hybrid control

screw: position + orientation

- General formulation

$$\tau = J_b^{\mathrm{T}}(\theta)\Bigg(\underbrace{\boxed{P(\theta)}\left(\tilde{\Lambda}(\theta)\left(\frac{d}{dt}([\mathrm{Ad}_{X^{-1}X_d}]\mathcal{V}_d) + K_p X_e + K_i \int X_e(\mathrm{t})dt + K_d \mathcal{V}_e\right)\right)}_{\text{motion control}}$$
$$+ \underbrace{\boxed{(I - P(\theta))}\left(\mathcal{F}_d + K_{fp}\mathcal{F}_e + K_{fi}\int \mathcal{F}_e(\mathrm{t})dt\right)}_{\text{force control}}$$
$$+ \underbrace{\tilde{\eta}(\theta, \mathcal{V}_b)}_{\text{Coriolis and gravity}}\Bigg). \quad (11.61)$$

selection matrices

wrench: linear force + torque

Simultaneous tasks
Examples of alternative formulations

- Torque control, two Cartesian space controllers

$$\boldsymbol{\tau} = J^T(\boldsymbol{\theta})\left(P C_1(\boldsymbol{\theta}) + (I - P) C_2(\boldsymbol{\theta})\right) \qquad (+dyn)$$

- Velocity control for prioritized tasks

$$\dot{\boldsymbol{\theta}} = J_1^+ \dot{\boldsymbol{x}}_1 + N_1\left(J_2^+ \dot{\boldsymbol{x}}_2 + N_2(\ldots)\right)$$

Jacobian of 1st task

Null space of 1st task

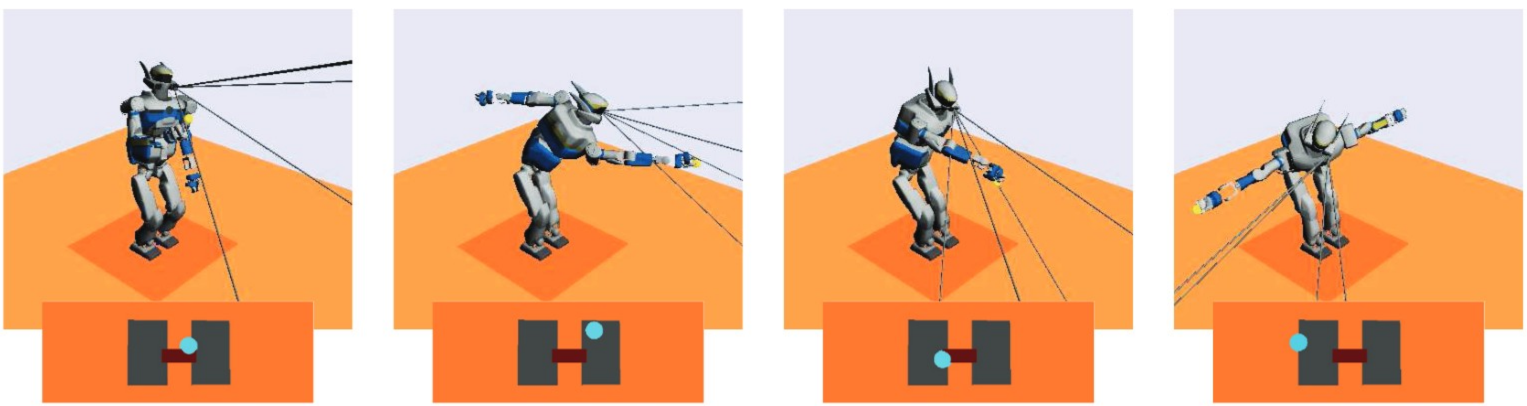
$$N_i = I - J_i^+ J_i$$

What would happen without *N* terms?

Task formalism for control

- Stack of tasks (e.g. Mansard 2009) provides hierarchical approach of execution of multiple simultaneous tasks.
- Tasks prioritized.

- Example: simultaneous balancing, reaching, field of view



Escande et al., IJRR 2014

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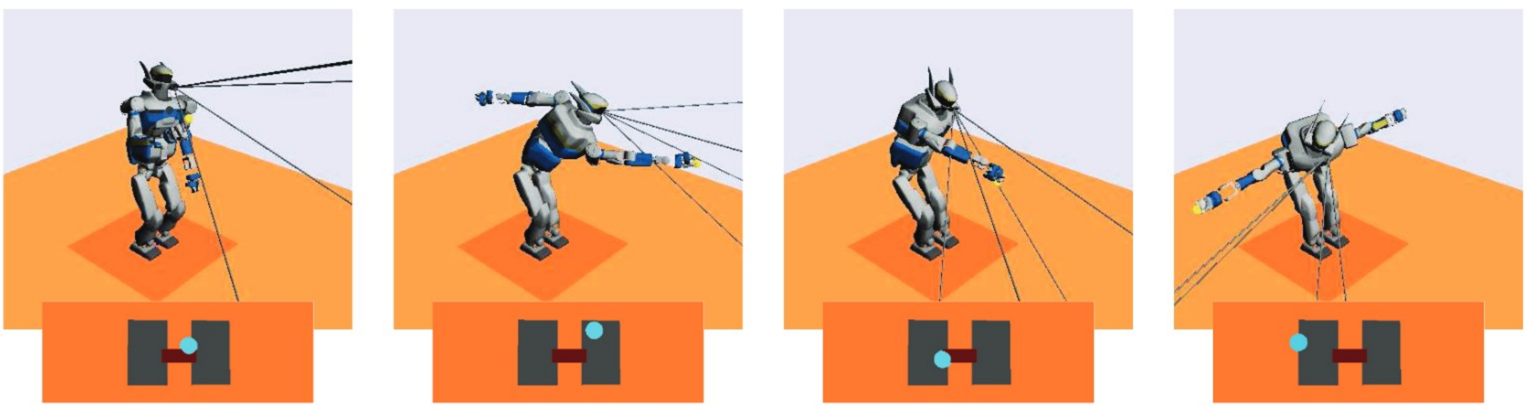

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Back to in-contact control!

Making robot compliant to external forces

- Instead of particular contact force, make robot mimic desired impedance characteristics (mass, spring, damper) when responding to external forces.
  - Robot acts as a virtual tool, e.g. with interacting human.
- Two approaches:
  - Sense robot (endpoint) motion and command torques → impedance control
  - Sense interaction forces and command positions → admittance control

Impedance control

Measure position difference!

- Desired behavior: mass-spring-damper with respect to a reference trajectory

diff. to desired

$$\boldsymbol{F}_{ext} = M\ddot{\boldsymbol{x}}_e + B\dot{\boldsymbol{x}}_e + K\boldsymbol{x}_e$$

- Ideal control law

$$\boldsymbol{\tau} = J^T(\boldsymbol{\theta})\left(\underbrace{M_C(\boldsymbol{x})\ddot{\mathbf{x}} + \boldsymbol{h}_C(\boldsymbol{x}, \dot{\boldsymbol{x}})}_{\text{dynamics compensation}} - \underbrace{\left(M\ddot{\boldsymbol{x}}_e + B\dot{\boldsymbol{x}}_e + K\boldsymbol{x}_e\right)}_{\text{desired behavior}}\right)$$

Negate true dynamics

Replace with desired dynamics

Impedance control in practice

- Typical control law

May contain only gravity

$$\boldsymbol{\tau} = J^T(\boldsymbol{\theta})\left(-K(\boldsymbol{x}-\boldsymbol{x}_d) - B(\dot{\boldsymbol{x}}-\dot{\boldsymbol{x}}_d)\right) + \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

What's the inertia that's felt?

Compare to ideal

$$\boldsymbol{\tau} = J^T(\boldsymbol{\theta})\left(M_C(\boldsymbol{x})\ddot{\boldsymbol{x}} + \boldsymbol{h}_C(\boldsymbol{x}, \dot{\boldsymbol{x}}) - \left(M\ddot{\boldsymbol{x}}_e + B\dot{\boldsymbol{x}}_e + K\boldsymbol{x}_e\right)\right)$$

difficult to measure (noisy)

When force measurement and position (not torque) input are available

Admittance control

- Measure external force $\boldsymbol{F}_{ext}$, respond according to desired impedance behavior

$$\boldsymbol{F}_{ext} = M\ddot{\boldsymbol{x}} + B\dot{\boldsymbol{x}} + K\boldsymbol{x}$$

- Desired acceleration then

$$\ddot{\boldsymbol{x}}_d = M^{-1}\left(\boldsymbol{F}_{ext} - K\boldsymbol{x} - B\dot{\boldsymbol{x}}\right)$$

- Desired accelerations in joint space

$$\ddot{\boldsymbol{\theta}}_d = J^{+}(\boldsymbol{\theta})\left(\ddot{\boldsymbol{x}}_d - \dot{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}\right)$$

$$\dot{\boldsymbol{x}} = J(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}$$

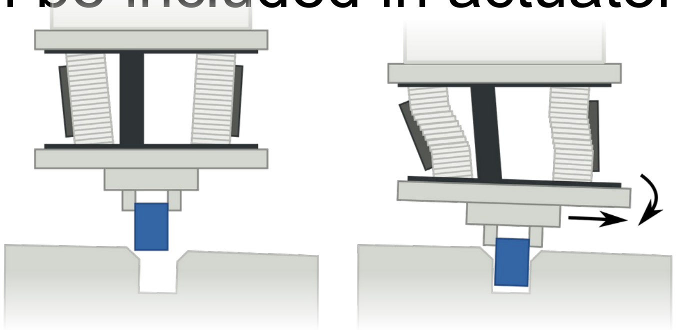
Actuator effects

- Actuators do not produce torque exactly and may have significant internal dynamics.
  - Gearing introduces backlash.
  - Strain gauges may be used for torque measurement to close loop in torque.

Also variable impedance possible.

- Passive compliance can be included in actuator.

Torsional spring of series elastic actuator



Remote center of compliance device

Why include passive compliance?
What are the effects?

Summary

- Force control is used when desired forces can be specified.

- Impedance control used for designed contact interaction characteristics, typical for physically interacting with humans.

Next time: Grasping and statics

- Readings:
  - Lynch & Park, Chapter 12.-12.1.3