Industrial Organization

Question: Estimation of auction models (by Janne Tukiainen)

Return answers to janne.tukiainen@vatt.fi by Mon Feb 13, 2023, Midnight Helsinki time as a single PDF file (including the code).

The exercise concerns how to calculate necessary FOC for some standard auction models and how to estimate valuations both parametrically using FOC under various assumptions.

When handing in your solutions to this question, please supplement your answers with estimation code **commented in detail**. If you think there may be mistakes in the questions, please inform: janne.tukiainen@vatt.fi.

1 Solving the models

Assume IPVP, no reserve prices or entry costs and N symmetric potential bidders. Assume $F_V(v) = \frac{v}{\theta}$ for all $v \in [0, \theta]$. Thus, $f_V(v) = \frac{1}{\theta}$ for all $v \in [0, \theta]$.

a) Find the Bayes-Nash equilibrium bid function $\sigma(v)$ for at a FPSB auction. How does this differ from second price auction bid function.

b) Find the probability density and the cumulative distribution function of $Y = V_{(2:N)}$.

c) Find the mean and variance of Y.

d) Find the probability density and the cumulative distribution function of $Z = V_{(1:N)}$.

e) Find the mean and variance of Z.

f) Find the probability density and the cumulative distribution function of the winning bid in FPSB auction $W = \sigma(V_{(1:N)})$.

g) Find the mean and variance of W.

2 Estimation

Assume IPVP, no reserve prices or entry costs and N symmetric potential bidders. Assume $F_V(v) = \frac{v}{\theta}$ for all $v \in [0, \theta]$. Thus, $f_V(v) = \frac{1}{\theta}$ for all $v \in [0, \theta]$. In each part, discuss and interpret your findings.

a) Simulate the data. Assume $\theta = 1$. Generate data on valuations for 100 auctions where $N = 3$ (bidder id's 1-3) and another 100 auctions where $N = 5$ (bidder id's 1-5) . Treat these as a single data set with auction id 1-200. Use the results from (1) to calculate the FPSB auction equilibrium bids and the SPSB auction equilibrium bids for all bidders in all these auctions. Add the bid variables to the data set.

b) Estimate the value function nonparametrically using bid data under the SPSB. Estimate the value function also parametrically under correct parametrization. Estimate the value function parametrically also assuming Weibull. Compare graphically. Then, plot the confidence intervals (analytical using the Hessian or you can conduct sampling inference).