Design of a large transformer

Large transformers are typically designed according to the specifications and needs of the end user (customer). The specifications needed are

- Rated power (S_{N})
- Rated voltages (U_{N1}, U_{N2})
- Rated frequency (f_{N})
- Relative short-circuit impedance (z_{k})
- Connection of the windings
- Transformer standard followed and
- Service type and cooling method.

Design of a large transformer

Economical aspects require knowledge of

- Cost of the core material including the costs of raw material and manufacturing h_{fe},
- Cost of the winding material including the costs of raw material and manufacturing h_{cu},

Tot. manufacturing cost → $$H_{\mathrm{m}} = m_{\mathrm{fe}} h_{\mathrm{fe}} + m_{\mathrm{cu}} h_{\mathrm{cu}}$$

- Capitalisation factor for no-load losses (K_0),
- Capitalisation factor for load losses (K_{k}),
- Capitalisation factor for the production costs (K_{m}). If the transformer has no re-circulation value, K_{m} is one.

Total cost of ownership → $$H = K_{\mathrm{m}} H_{\mathrm{m}} + K_0 P_{\mathrm{fe}} + K_{\mathrm{k}} P_{\mathrm{cu}}$$

Design of a large transformer

1. The free variables (design parameters) must be chosen. Four variables are needed to define the basic design of a transformer, and typically, the following ones are chosen

- Flux density of the limb;

  initial value: $\hat{b}_{\mathrm{limb}} = 1.7 \ldots 1.9\ \mathrm{T}$
- Current density in the primary winding;

  initial value: $J_1 = 2.2 \ldots 3.8\ \mathrm{A/mm^2}$
- Current density in the secondary winding;

  initial value: $J_2 = 2.0 \ldots 2.8\ \mathrm{A/mm^2}$
- Cross-sectional area of a limb from an empirical equation; initial value:

$$A_{\mathrm{fe,limb}}\left[\mathrm{m}^2\right] = (0.005\mathrm{K}\ 0.010) \times \sqrt{\frac{S_{\mathrm{N}}\left[\mathrm{kVA}\right]}{f\left[\mathrm{Hz}\right]}}$$

Design of a large transformer

2. Cross-sectional areas needed for the conductors of the primary and secondary windings

$$A_{\mathrm{j1}} = \frac{I_{\mathrm{N1}}}{J_1} = \frac{S_{\mathrm{N}}}{3\ U_{\mathrm{N1}} J_1} \qquad A_{\mathrm{j2}} = \frac{I_{\mathrm{N2}}}{J_2} = \frac{S_{\mathrm{N}}}{3 U_{\mathrm{N2}} J_2}$$

3. Numbers of turns

$$N_1 = \frac{\hat{e}_1}{\omega \hat{\Phi}} = \frac{\sqrt{2}\, U_{\mathrm{N1}}}{\omega \hat{b}_{\mathrm{limb}} A_{\mathrm{fe,limb}}}$$

$$N_2 = \frac{\hat{e}_2}{\omega \hat{\Phi}} = \frac{\sqrt{2}\, U_{\mathrm{N2}}}{\omega \hat{b}_{\mathrm{limb}} A_{\mathrm{fe,limb}}} = \frac{U_{\mathrm{N2}}}{U_{\mathrm{N1}}} N_1$$

Design of a large transformer

4. Cross-sectional area needed for the windings

$$A = A_{j1} + A_{j2}$$

5. The height and width of the cross-sectional winding area affect the short-circuit impedance of the transformer. They are fixed so that the specified impedance is obtained.

6. The cross-section of the yoke and lengths of the limb and yoke are defined.

7. Weight of the core and conductor materials, and further, the material and production costs are calculated

$$H_{\mathrm{m}} = m_{\mathrm{fe}} h_{\mathrm{fe}} + m_{\mathrm{cu}} h_{\mathrm{cu}}$$

Design of a large transformer

8. The core and copper losses of the transformer as well as the capitalisation cost are calculated

$$H = K_{\mathrm{m}} H_{\mathrm{m}} + K_0 P_{\mathrm{fe}} + K_{\mathrm{k}} P_{\mathrm{cu}}$$

9. The cost H is a function of the four free variables

$$H = H\left(\hat{b}_{\mathrm{limb}}, A_{\mathrm{fe,limb}}, J_1, J_2\right)$$

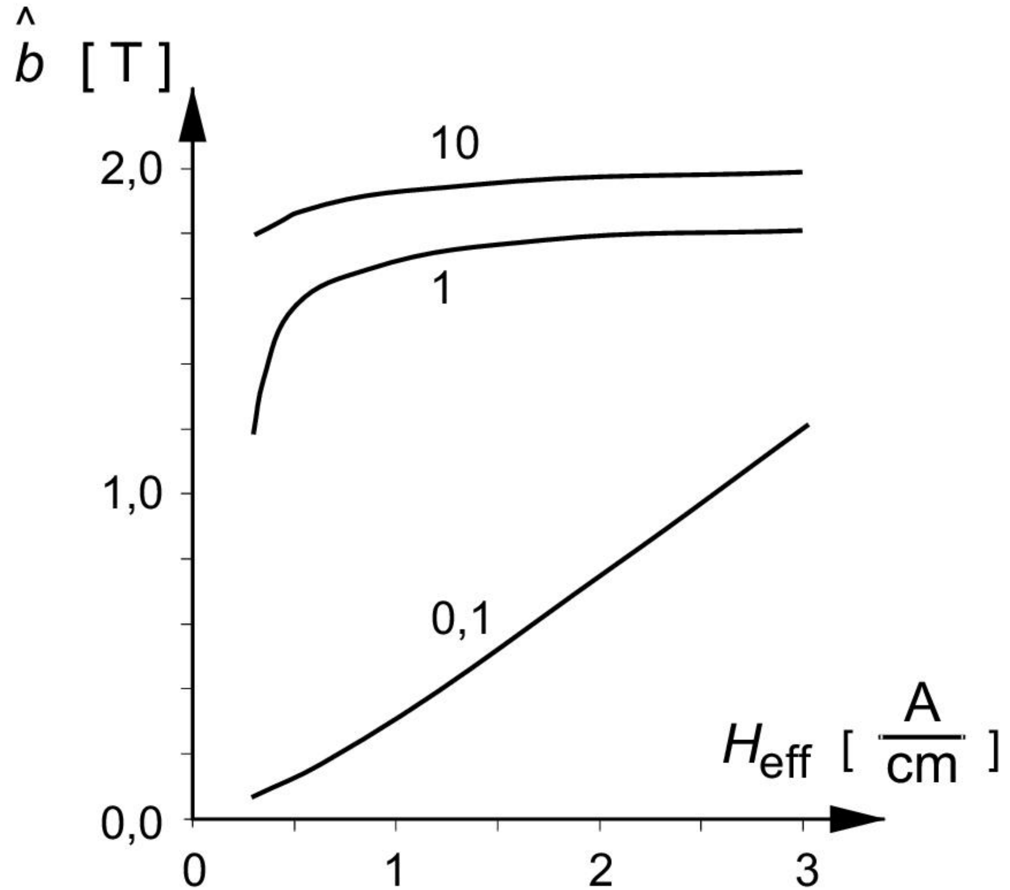
The minimum of this function is searched while keeping the constraints of step 1 in mind. Also, the temperature should stay within the temperature class of the windings.

Magnetisation current

The flux density of the yoke is obtained from the flux density of the limb and their known cross-sections

$$\hat{b}_{\text{yoke}} = \frac{A_{\text{fe,limb}}}{A_{\text{fe,yoke}}} \hat{b}_{\text{limb}}$$

The magnetic field strengths in the limb and yoke are obtained from the magnetisation curve of the core material

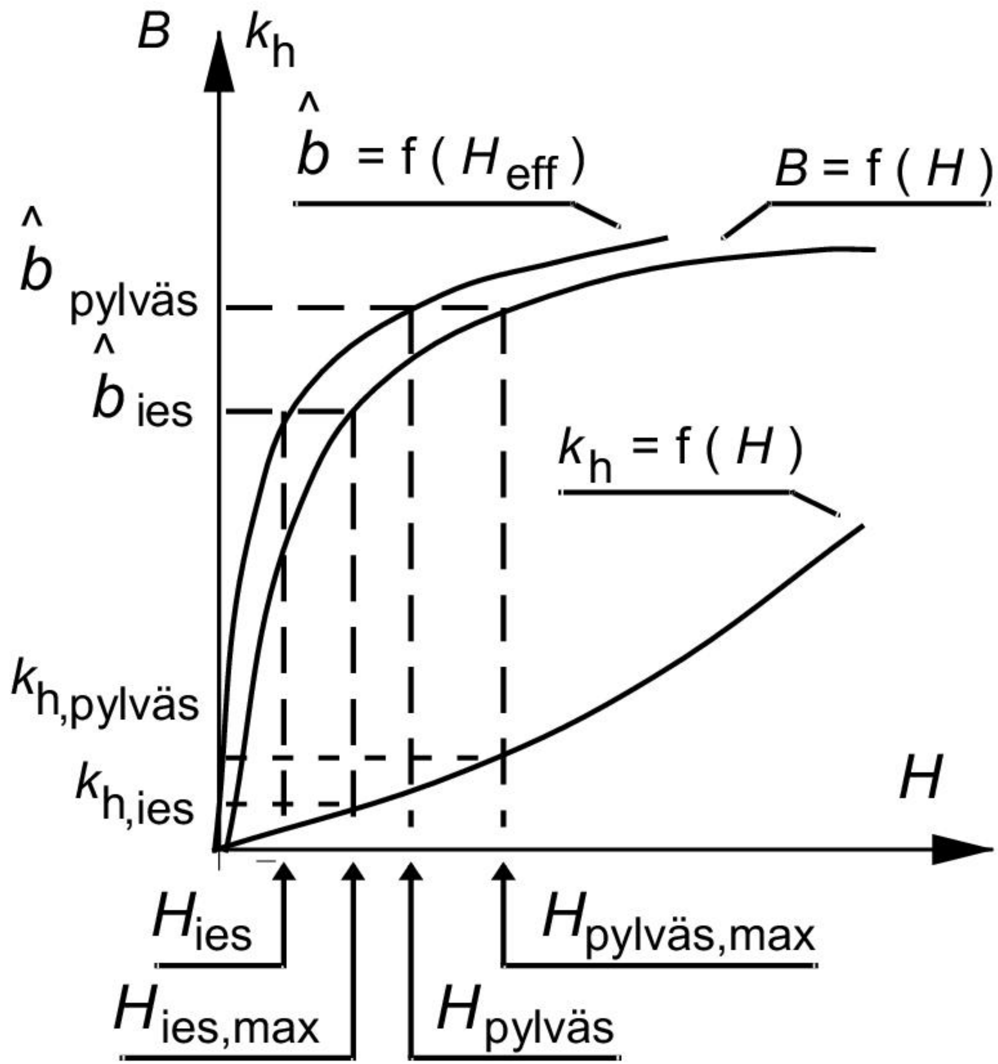


Magnetisation current

The magnetisation curve of the core material can also be presented in the form

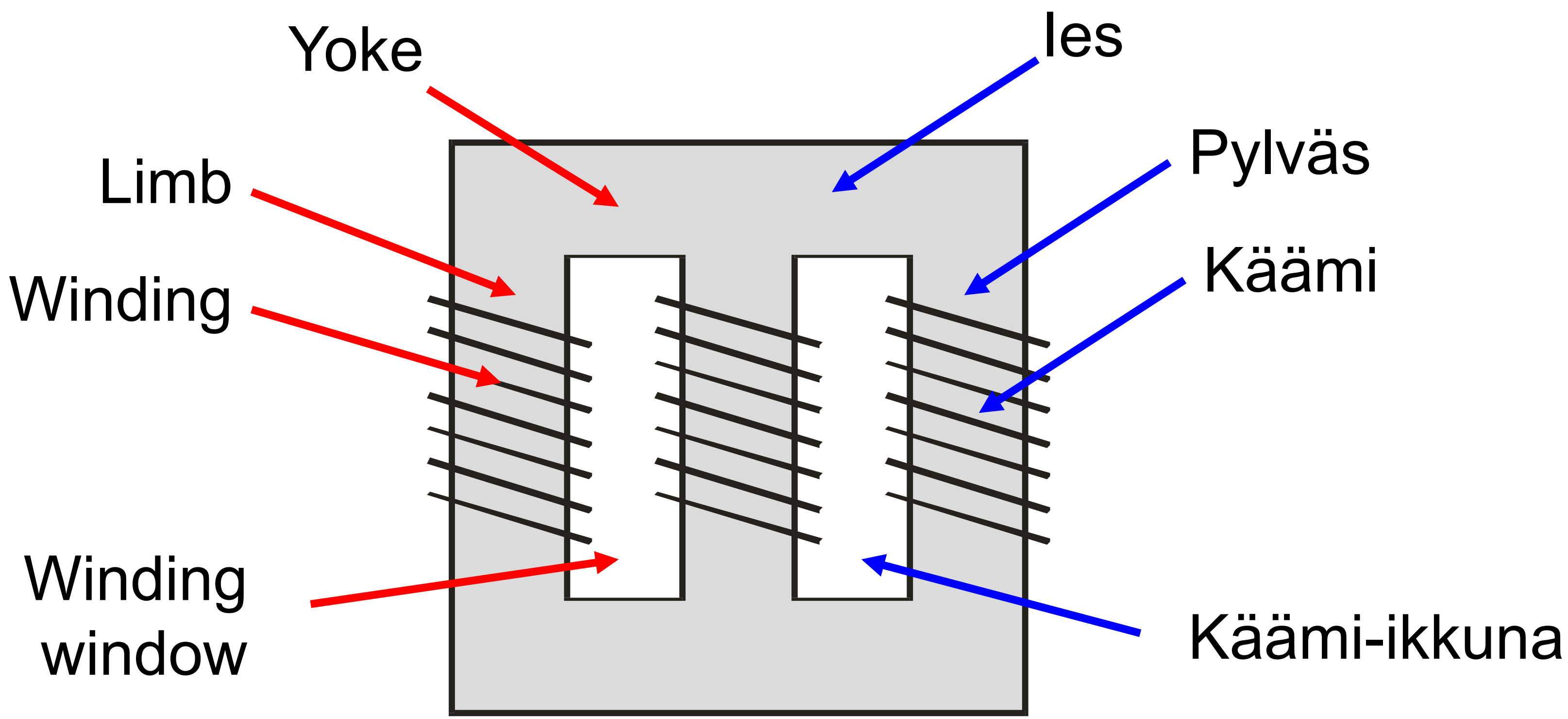
$$\hat{b} = \mathrm{f}\left(H_{\mathrm{eff}}\right)$$

In this case, the peak value of flux density is obtained directly as a function of the effective field strength.



pylväs = limb, ies = yoke

Transformer designs – Basic terms



Magnetisation current

The total magnetomotive force in a three-phase transformer is

$$\sum F_{\mathrm{m}} = 3 l_{\mathrm{limb}} H_{\mathrm{limb}} + 2 l_{\mathrm{yoke}} H_{\mathrm{yoke}} + 3 V_{\mathrm{m\delta}}$$

l_{limb} is the magnetic length of a limb,
l_{yoke} is the magnetic length of a yoke and
$V_{\mathrm{m\delta}}$ is the magnetomotive force needed to drive the flux over the sheet-sheet contacts of one limb.

The magnetomotive force over a contact is obtained from equation

$$V_{\mathrm{m\delta}} = \frac{2\delta}{\mu_0} \frac{\hat{b}_{\delta}}{\sqrt{2}}$$

Magnetisation current

The magnetisation current is

$$I_{\mathrm{m,phase}} = \frac{1}{3N_1}\sum F_{\mathrm{m}} = \frac{1}{N_1}\left(l_{\mathrm{limb}} H_{\mathrm{limb}} + \frac{2}{3} l_{\mathrm{yoke}} H_{\mathrm{yoke}} + V_{\mathrm{m}\delta} \right)$$

No-load current and losses

The no-load losses are composed of two components, i.e. resistive losses of the primary winding and the core losses. For a three-phase transformer

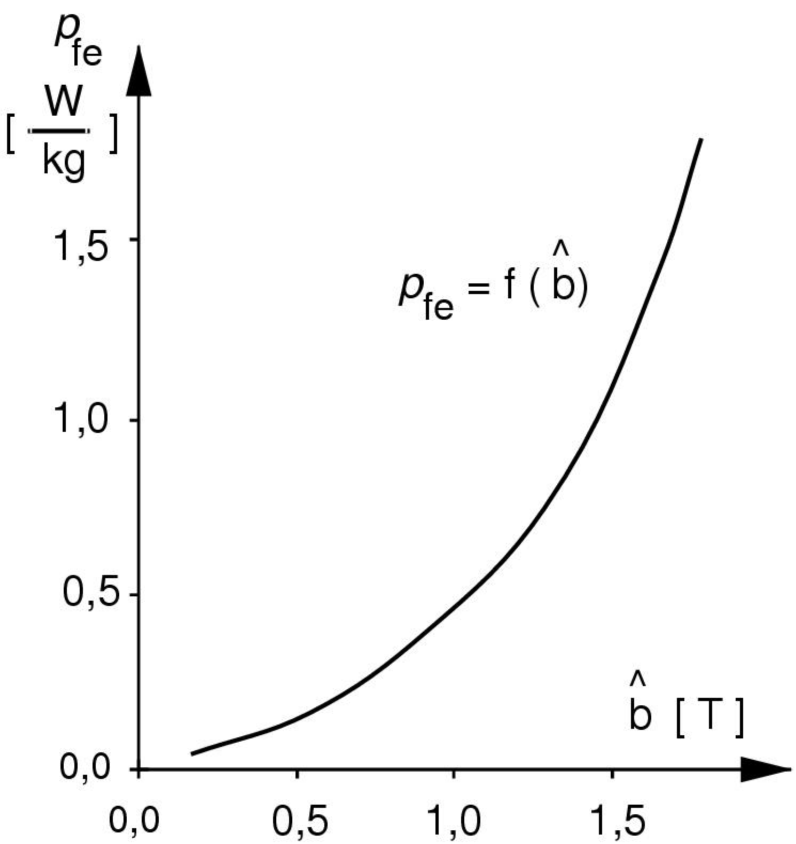
$$P_0 = P_{\mathrm{fe}} + 3R_{\mathrm{cu1}}I_0^2$$

where R_{cu1} is the phase resistance of primary winding. The ratio of the no-load current and rated current strongly depends on the size of the transformer

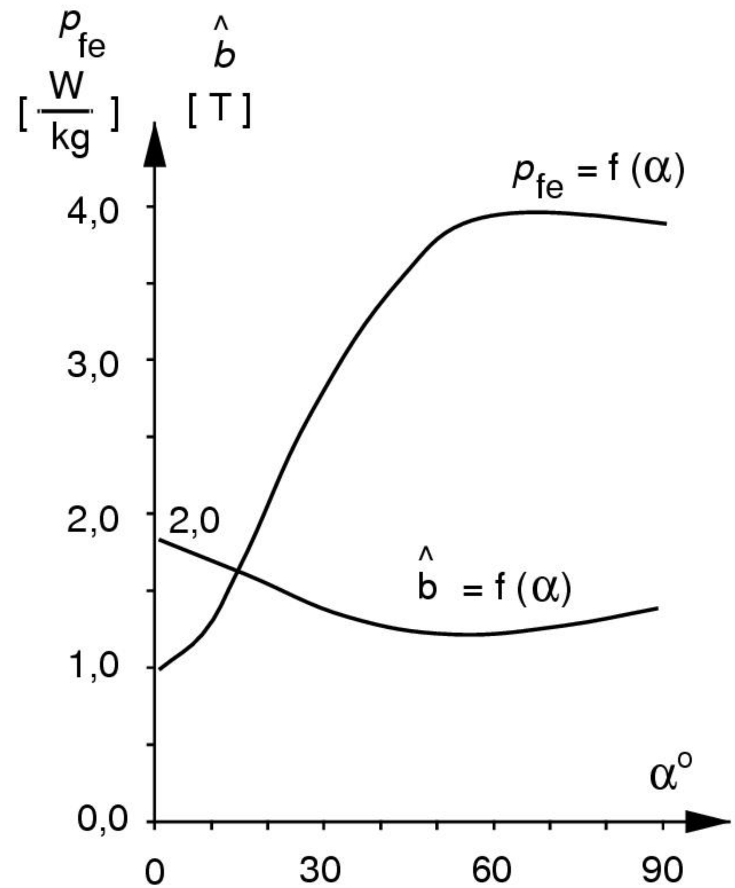
S_N/kVA	0,0025	0,100	2,0	30	100	200	1000
I_0/I_N	0,740	0,420	0,240	0,037	0,020	0,016	0,012

The producers of electrical steels give the loss coefficient p_{fe0} for their electrical steel sheets. These are typically given per mass unit (kg) at specified peak flux densities (1.0 or 1.5 T) and frequencies.

No-load losses



Losses per unit mass for an electrical steel sheet as a function of the peak flux density. Frequency is constant.



Dependence of the losses and peak flux density on the angle α from the direction of the rolling direction. The field strength H is kept constant.

No-load losses

The leftmost figure on the previous slide implies that the losses can be expressed as a polynomial or exponent function on the peak value of flux density

$$p_{\mathrm{fe}} = \left(\frac{\hat{b}}{\hat{b}_0} \right)^{\beta_0} p_{\mathrm{fe},0}$$

The exponent β_0 is about 2.0 for non-oriented steel sheets. For oriented sheets, the exponent varies more. Close to the peak value 1.5 T, the exponent is about 2.8.

The core losses of a transformer are

$$P_{\mathrm{fe}} = p_{\mathrm{fe,limb}} m_{\mathrm{limb}} + p_{\mathrm{fe,yoke}} m_{\mathrm{yoke}} + P_{\mathrm{L,fe}}$$

where $p_{\mathrm{fe,L}}$ are the additional losses of the sheets.

No-load losses

The additional losses are produced because

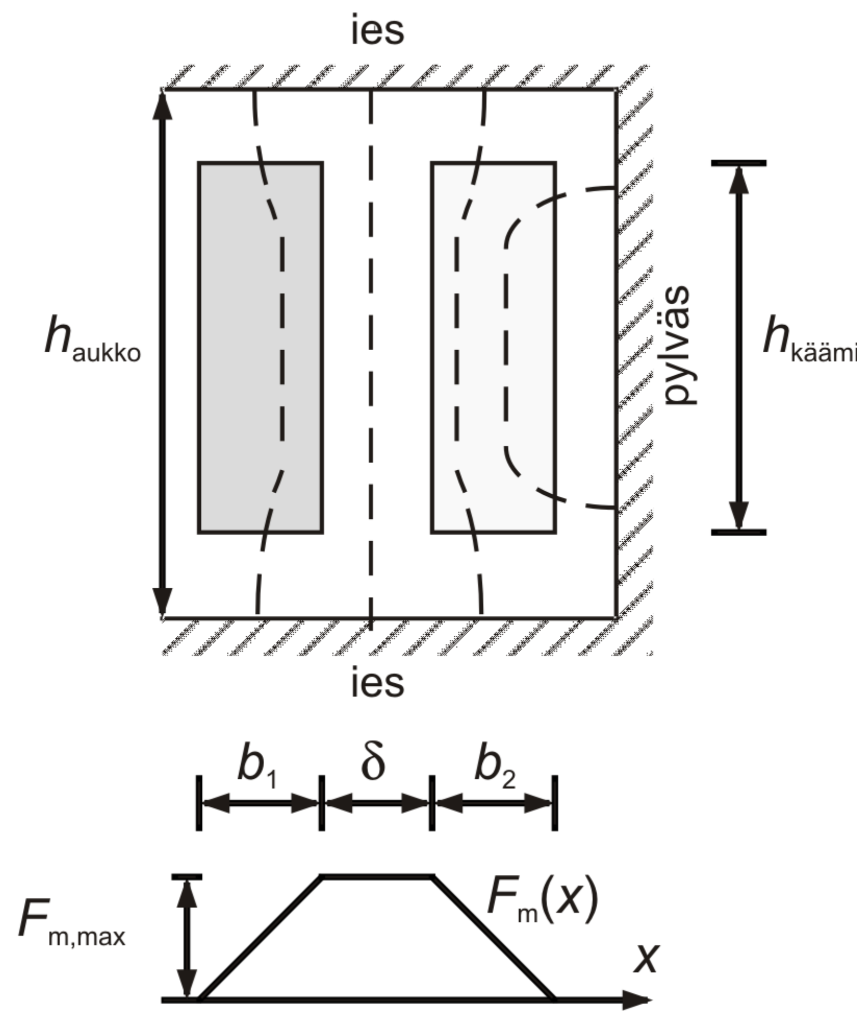
- The direction of the flux differs from the rolling direction, for instance, at the contact region of a limb and yoke,
- The flux is somewhat unevenly distributed in the core, i.e. slightly larger close to the winding window,
- Punching and mechanical treatment of the sheets causes degradation in the characteristics of the sheet. A heat treatment after machining improves the characteristics to some extent.

Leakage reactance

Leakage reactance is needed for the equivalent circuit of a transformer. It can be obtained from the energy of the leakage flux

$$\mathrm{d}W = \frac{1}{2}\mu_0 \int_V H_V^2 \,\mathrm{d}V$$

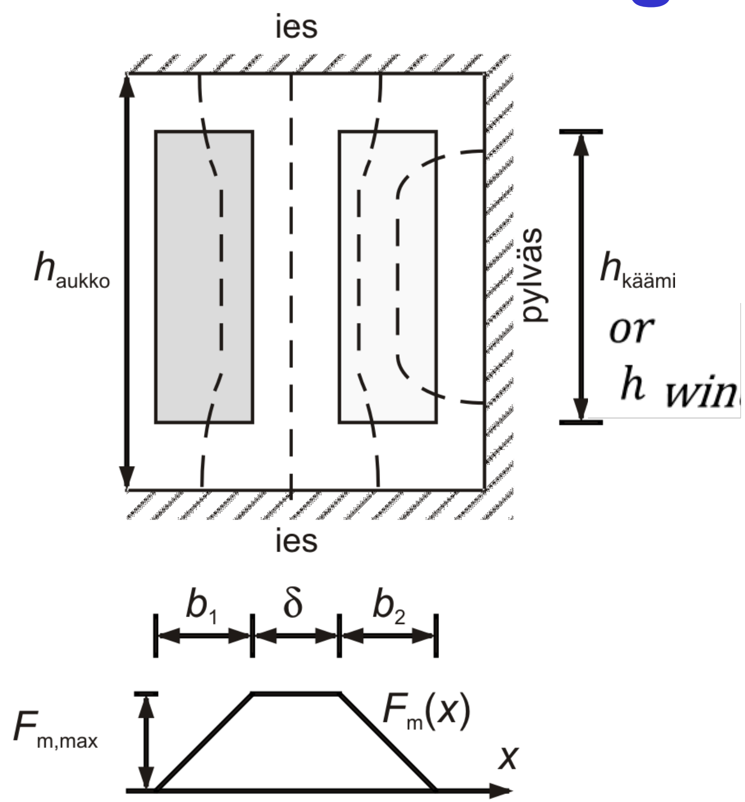
μ_0 is the permeability of the volume element (conductor or air). HV is magnetic field strength in the volume element.



Leakage reactance of a concentric winding

Distribution of the magnetomotive force driving the leakage flux

$$\begin{cases} F_{\mathrm{m}}(x) = \dfrac{x}{b_1} F_{\mathrm{m,max}}, & 0 \le x \le b_1 \\ F_{\mathrm{m}}(x) = F_{\mathrm{m,max}} & 0 \le x \le \delta \\ F_{\mathrm{m}}(x) = \dfrac{x}{b_2} F_{\mathrm{m,max}} & 0 \le x \le b_2 \end{cases}$$



Relation between the mmf and field strength

$$H_V(x) = \frac{F_{\mathrm{m}}(x)}{l_{\mathrm{m}}} = \frac{F_{\mathrm{m}}(x)}{\sqrt{\lambda_{\mathrm{x}}}\, h_{\mathrm{wind}}},$$

$$l_{\mathrm{m}} = \sqrt{\lambda_{\mathrm{x}}}\, h_{\mathrm{wind}}$$

where l_{m} is an effective length of a flux line and λ_x is a correction factor.

Leakage reactance of a concentric winding

Integration over the winding window

$$W = \frac{\mu_0}{2} \pi D_{\text{käämi}}\; h_{\text{käämi}} \int_{x=0}^{x=b_1+\delta+b_2} \left(\frac{F_{\mathrm{m}}(x)}{\sqrt{\lambda_{\mathrm{x}}}\, h_{\text{käämi}}} \right)^2 \mathrm{d}x$$

$$= \frac{\mu_0}{2} \frac{\pi D_{\text{käämi}}}{\lambda_{\mathrm{x}} h_{\text{käämi}}} \left[\int_0^{b_1} \left(\frac{x}{b_1} F_{\mathrm{m,max}} \right)^2 \mathrm{d}x + \int_0^{\delta} F_{\mathrm{m,max}}^2 \mathrm{d}x + \int_0^{b_2} \left(\frac{x}{b_2} F_{\mathrm{m,max}} \right)^2 \mathrm{d}x \right]$$

$$= \frac{\mu_0}{2} \frac{\pi D_{\text{käämi}}}{\lambda_{\mathrm{x}} h_{\text{käämi}}} N^2 I^2 \left[\delta + \frac{b_1 + b_2}{3} \right]$$

Energy from the inductance and current

$$W = \frac{1}{2} L_\sigma I^2 \quad \Rightarrow \quad X_\sigma = \omega L_\sigma = \omega \mu_0 \frac{\pi D_{\text{käämi}}}{\lambda_{\mathrm{x}} h_{\text{käämi}}} N^2 \left[\delta + \frac{b_1 + b_2}{3} \right]$$

Leakage reactance of a concentric winding

Correction factor according to Rogowski

$$\lambda_x = \frac{\kappa}{\kappa - 1 + e^{-\kappa}} \approx \frac{\kappa}{\kappa - 1}$$

where

$$\kappa = \frac{\pi h_{\text{käämi}}}{b_1 + \delta + b_2}$$

These equations are valid when

$$\begin{cases} \delta < \frac{1}{2}\left(b_1 + \delta + b_2\right) \\ h_{\text{käämi}} > \frac{1}{2}\left(b_1 + \delta + b_2\right) \end{cases}$$