Resistive losses of a winding

The DC-resistance of a winding can be calculated from the dimensions of the conductor

$$R_{\mathrm{coil}} = \frac{N\, l_{\mathrm{turn}}}{\gamma_{\mathrm{c}} A_{\mathrm{c}}}$$

where γ_{c} is the electrical conductivity,
N is the number of turns,
l_{turn} is the average length of a turn and
A_{c} is the cross-sectional area of the conductor.

The conductivity of copper is $\gamma_{20} = 58.5 \cdot 10^6\ 1/\Omega\mathrm{m}$ at 20 ºC. It depends significantly on the temperature.

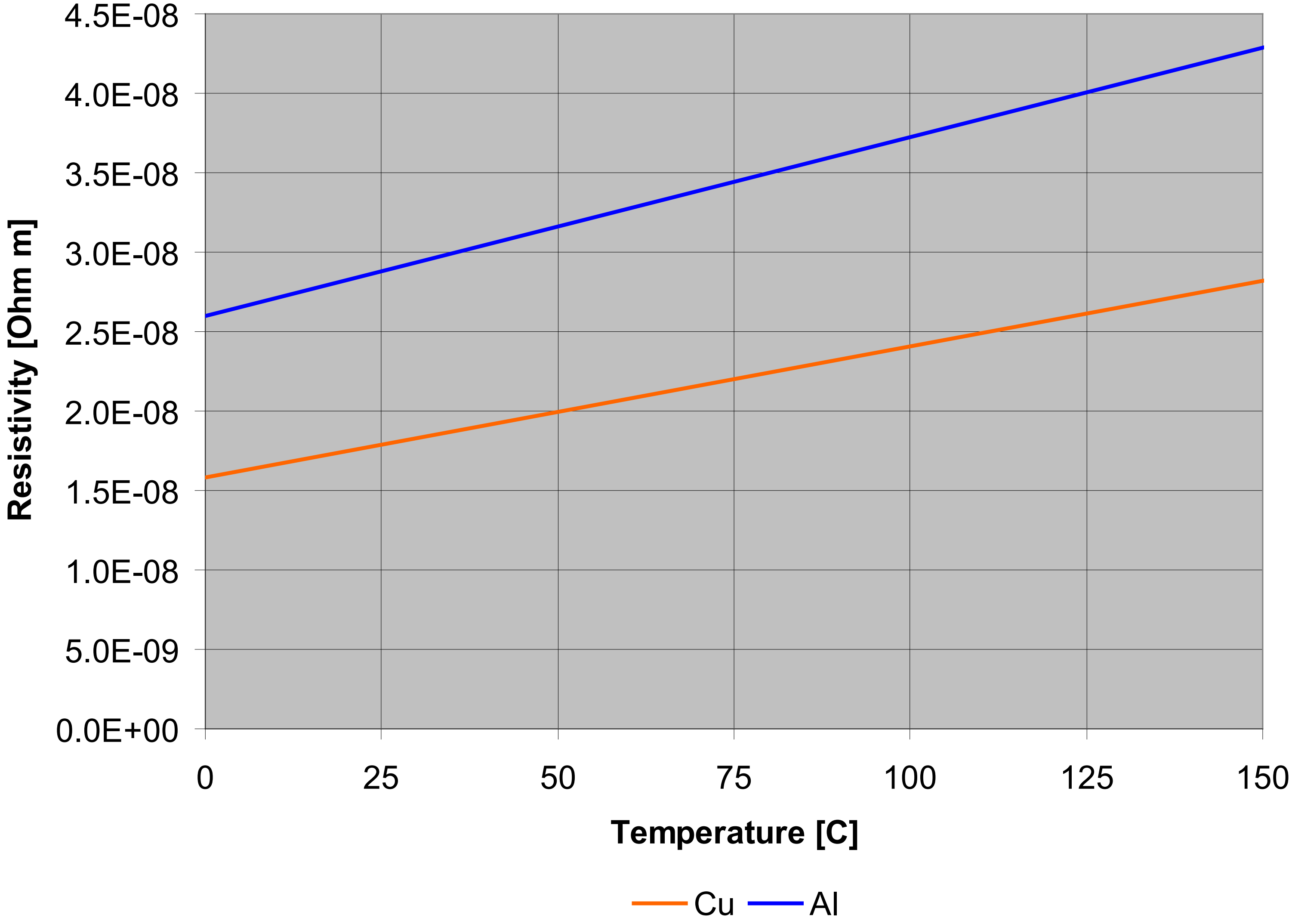
Thermal dependence of resistance

The electrical conductivity of copper at other temperatures can be calculated from equation

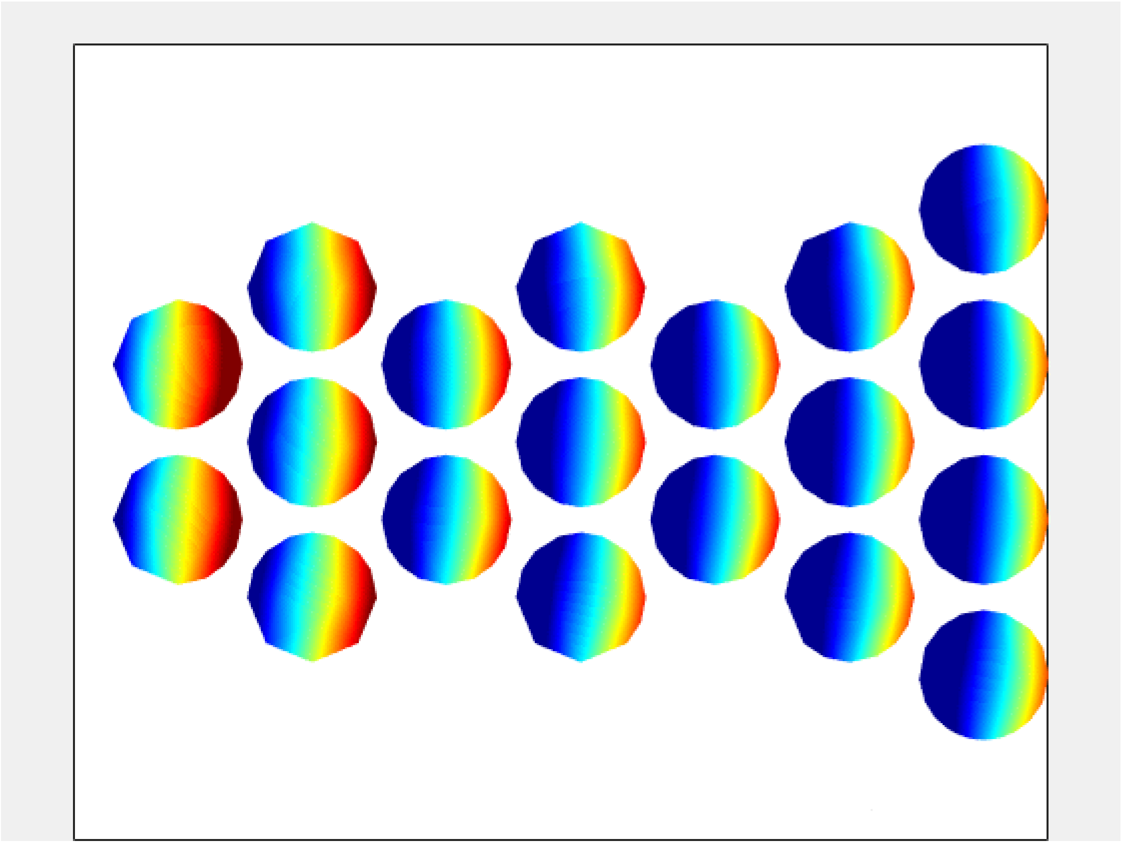
$$\gamma_{\mathrm{cu}}(T) = \frac{235\ ^{\mathrm{o}}\mathrm{C} + 20\ ^{\mathrm{o}}\mathrm{C}}{235\ ^{\mathrm{o}}\mathrm{C} + T}\gamma_{20}$$

where the temperature T has to be expressed in ºC.

Thermal dependence of resistivity



Skin effect and AC resistance

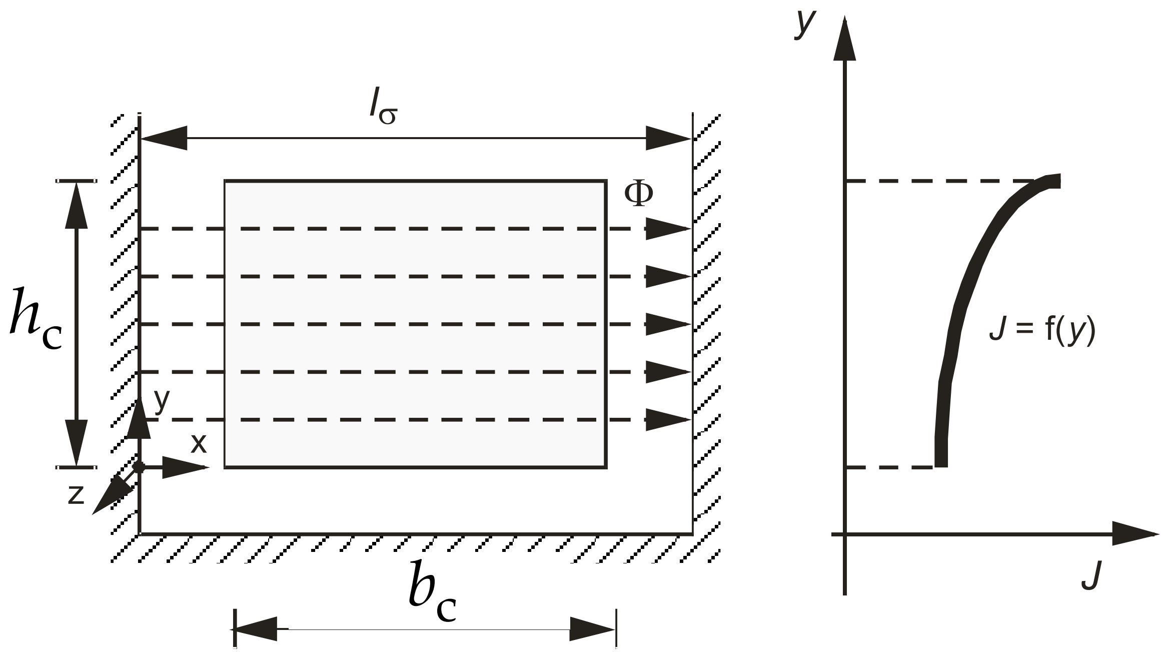


http://www.anttilehikoinen.fi/wp-content/uploads/2016/10/2016-10-20_eddies.gif

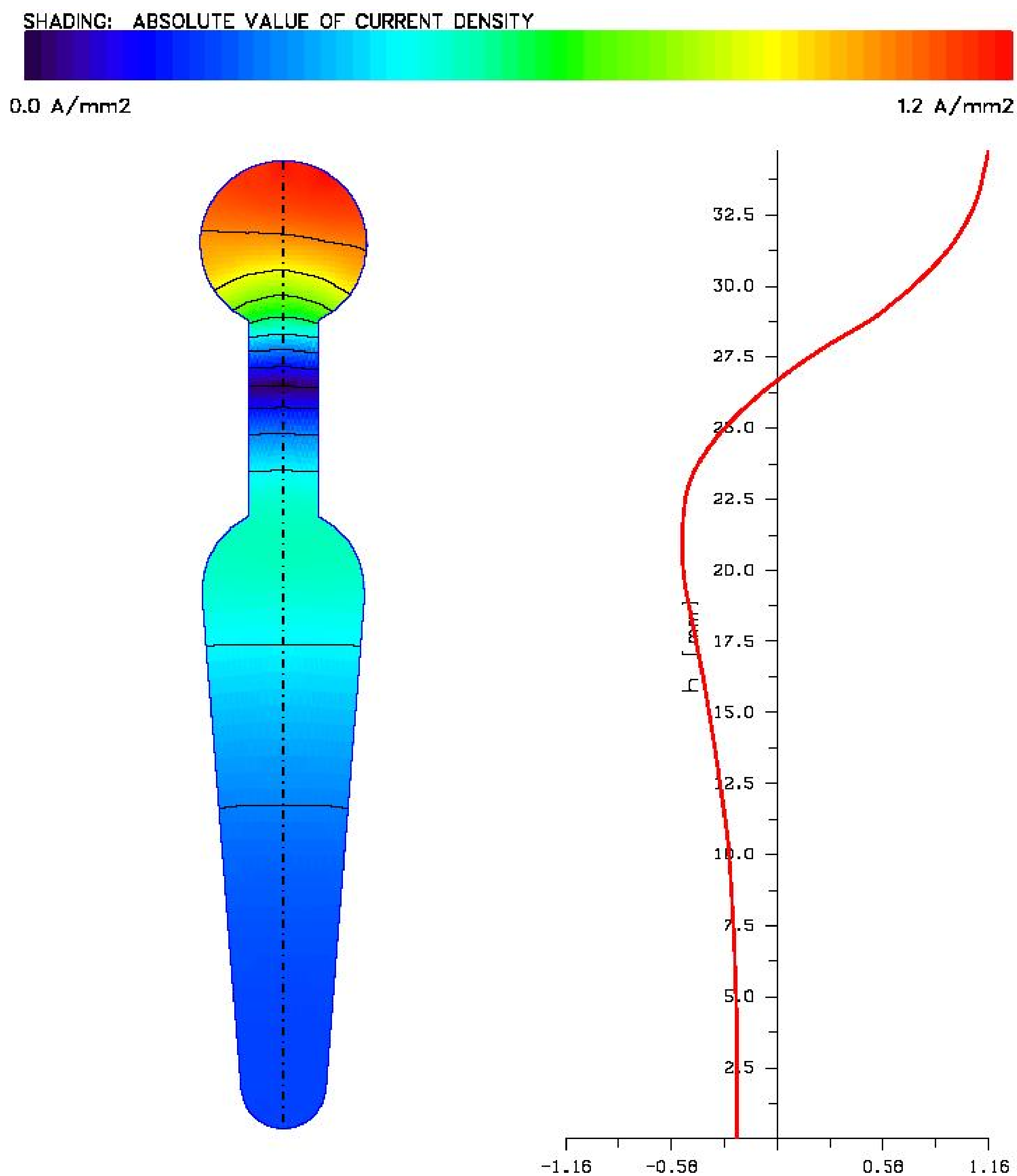
AC resistance and DC resistance

Skin effect modifies the current distribution in a conductor and affects its effective resistance R_{ac},

$$P = R_{\mathrm{ac}} I^2 = \frac{1}{\gamma} \int_{V_c} J^2 dV$$

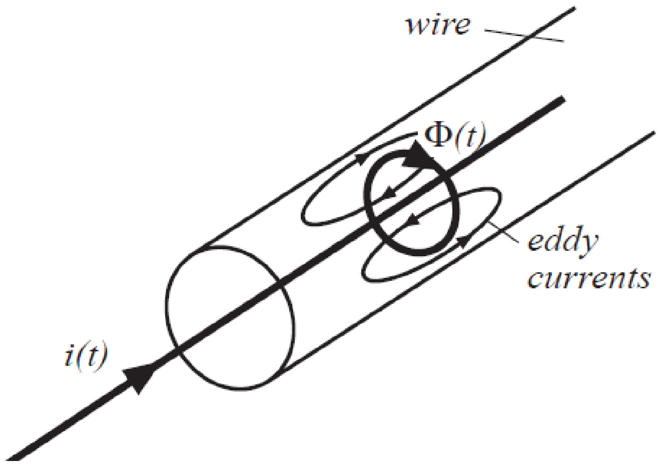


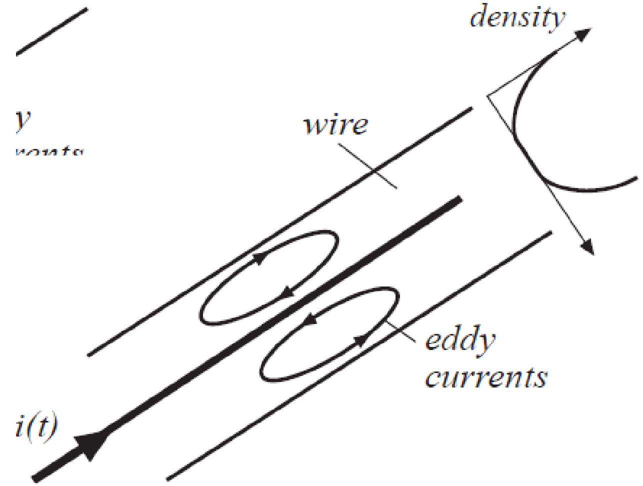
Skin effect in a rotor bar



Skin Effect

$$J = J_o e^{-x/\delta}$$

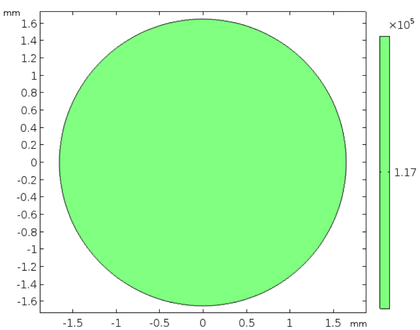




- Eddy currents are induced inside the main conductor due to main AC current. This causes current redistribution.

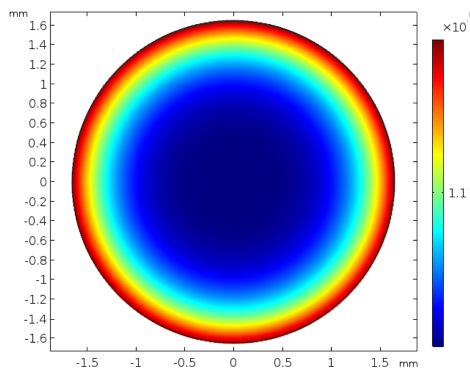
Current Density Distribution For 3.3mm diameter wire 1m thickness

0Hz



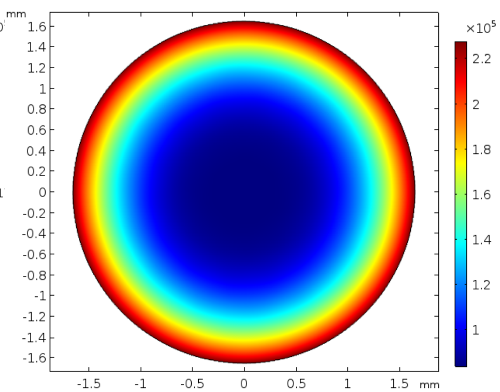
$$R_{dc} = 1.9\text{m}\Omega$$
$$\delta = \text{Inf}$$

50Hz



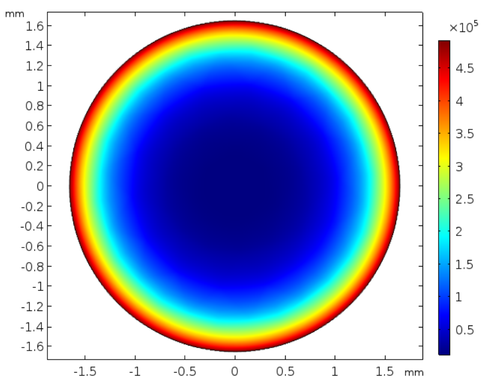
$$R_{ac} = 1.9\text{m}\Omega$$
$$\delta = 9.2\text{mm}$$

10kHz



$$R_{ac} = 3\text{m}\Omega$$
$$\delta = 0.6\text{mm}$$

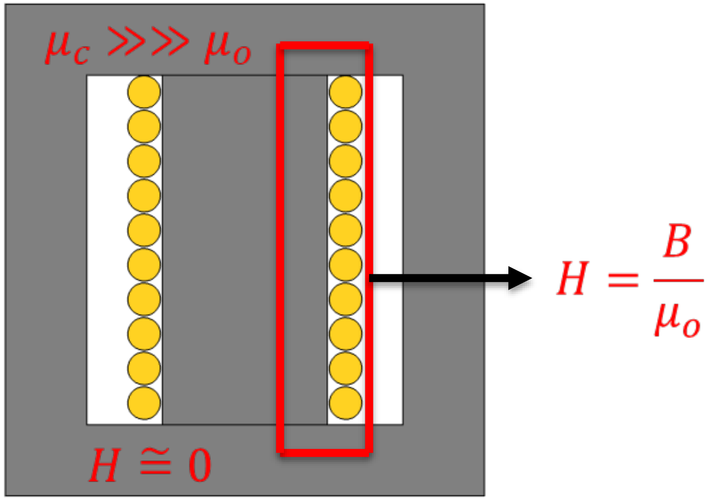
50kHz



$$R_{ac} = 6\text{m}\Omega$$
$$\delta = 0.29\text{mm}$$

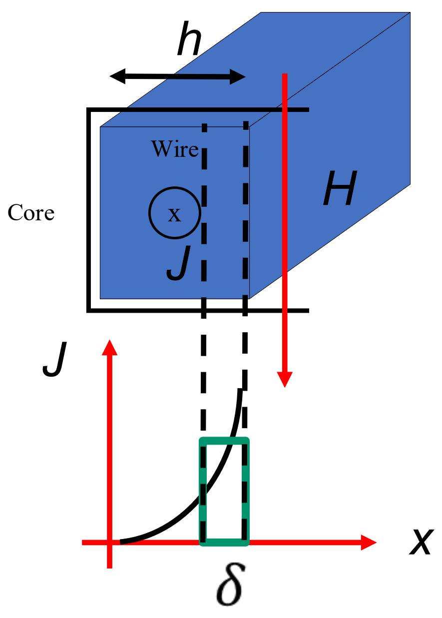
Fields in Winding in a Core

- Applying Ampere's law on a closed loop.
- $\oint H.dl = \oiint J.dS$
- Because of symmetry of winding



Effective AC Resistance

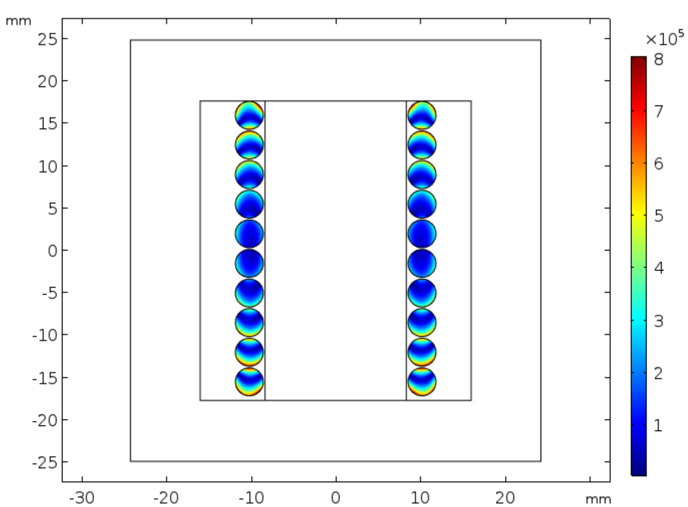
- Due to varying magnetic fields, eddy currents are induced. This reduces the currents near the core area. And increases the currents near the edge to the air. This factor is dependent on the skin depth.
- The skin depth $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$.
- For $\mathrm{h} \gg \delta$,
- The effective area changes by a factor of $\frac{\delta}{h}$
- $R_{\mathrm{ac}} = R_{\mathrm{dc}}\frac{h}{\delta}$, $R_{\mathrm{dc}} = \frac{\rho l_{\mathrm{wire}}}{A_{\mathrm{wire}}}$
- For optimum resistance $h = \delta$. In this case $R_{\mathrm{ac}} = R_{\mathrm{dc}}$.

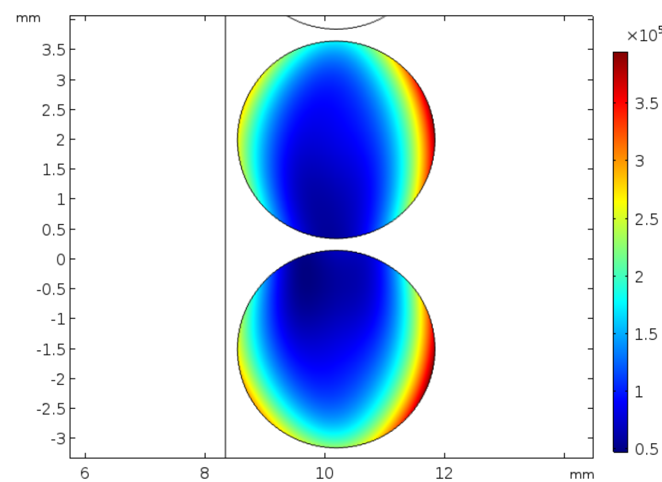


10kHz EI core FE results for the Current Density Distribution for 3.3mm round conductors

- Skin Depth = 0.65mm
- $R_{\mathrm{dc}} = 40\mathrm{m}\Omega,\ R_{\mathrm{ac}} = 190\mathrm{m}\Omega$
- The AC resistance computed from

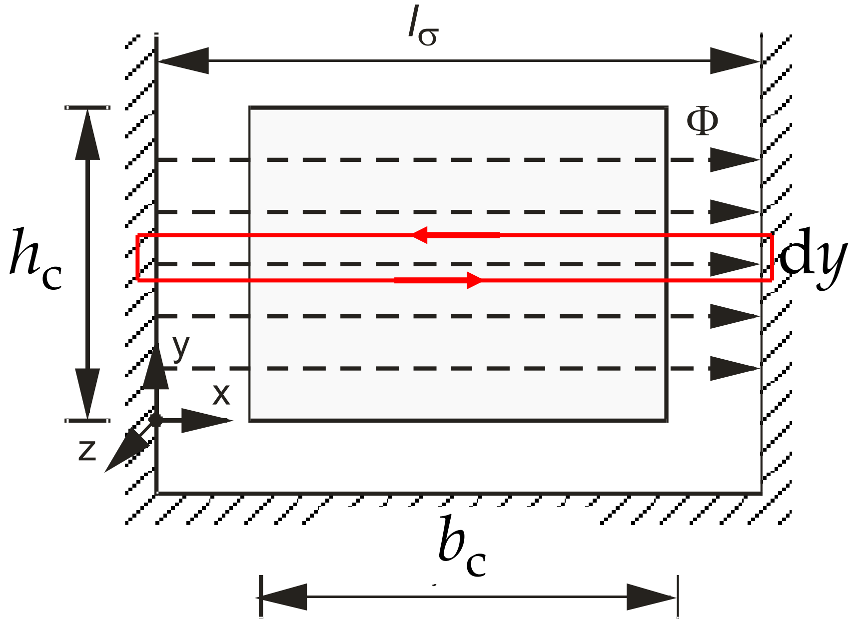
$$R_{\mathrm{ac}} = R_{\mathrm{dc}}\frac{3.3}{0.65} = 200\mathrm{m}\Omega$$





Skin effect – Conductor in a slot

Line integral around a surface element $\mathrm{d}A = l_\sigma\, \mathrm{d}y$



$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{s} = H\, l_\sigma - \left(H + \frac{\partial H}{\partial y} \mathrm{d}y \right) l_\sigma$$

$$= I_{\mathrm{tot}} = J b_c \mathrm{d}y$$

$$\Rightarrow \quad \boxed{-\frac{\partial H}{\partial y} = \frac{b_c\, J}{l_\sigma}}$$

From Maxwell's equation

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} \quad \Rightarrow \quad \boxed{\frac{\partial E}{\partial y} = -\mu_0 \frac{\partial H}{\partial t}}$$

Skin effect – Conductor in a slot

The inverse of $\alpha(1/m)$ is called penetration depth and $\alpha(1/m)$ can be computed as

$$\alpha = \sqrt{\omega\,\mu_0\,\gamma\,\frac{b_\mathrm{c}}{2\,l_\sigma}}$$

It defines a dimensionless number ξ

$$\xi = \alpha h_c$$

The resistive loss

$$P_\mathrm{AC} = \frac{b_\mathrm{c}\,l_\mathrm{c}}{2\,\gamma}\int_0^{h_\mathrm{c}} \hat{j}^2\,\mathrm{d}y$$

Where l_c is the axial length of the slot.

Skin effect – Conductor in a slot

Equation for the dc loss

$$P_{\mathrm{DC}} = \frac{l_{\mathrm{c}}}{\gamma\, b_{\mathrm{c}} h_{\mathrm{c}}} \frac{\hat{i}^2}{2}$$

Resistance ratio (ratio of the ac and dc resistive loss when the total current is kept constant)

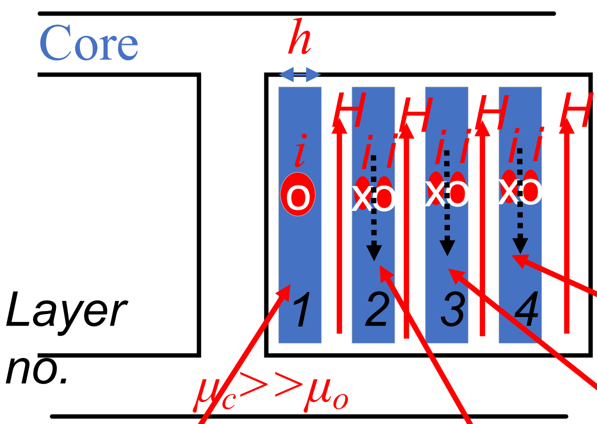
$$k_R = \frac{R_{\mathrm{AC}}}{R_{\mathrm{DC}}} = \frac{P_{\mathrm{AC}}}{P_{\mathrm{DC}}} = \frac{b_{\mathrm{c}}^2 h_{\mathrm{c}}}{\hat{i}^2} \int_0^{h_{\mathrm{c}}} \hat{j}^2 \, \mathrm{d}y$$

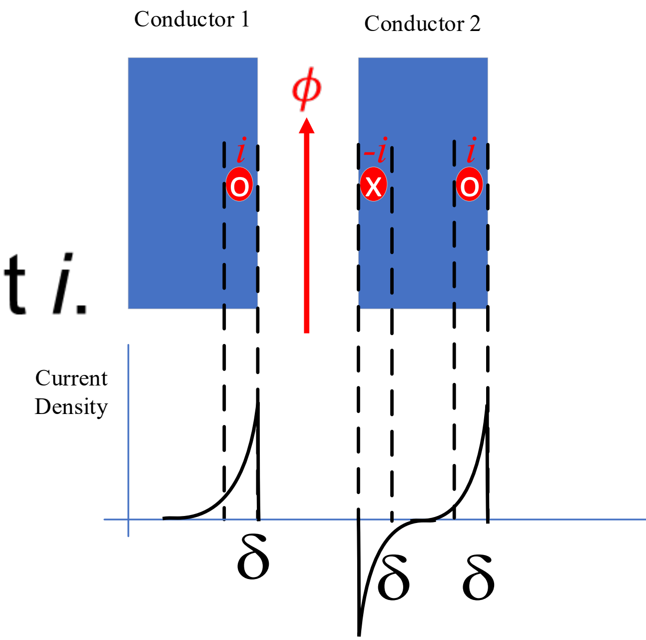
Eddy factor

$$k_{\mathrm{eddy}} = \frac{R_{\mathrm{AC}} - R_{\mathrm{DC}}}{R_{\mathrm{DC}}} = k_R - 1$$

Proximity Effect

- $h \gg \delta$
- Only first layer has a current *i*.





$$P_{1ac} = I_{rms}^2 R_{dc} \frac{h}{\delta}$$

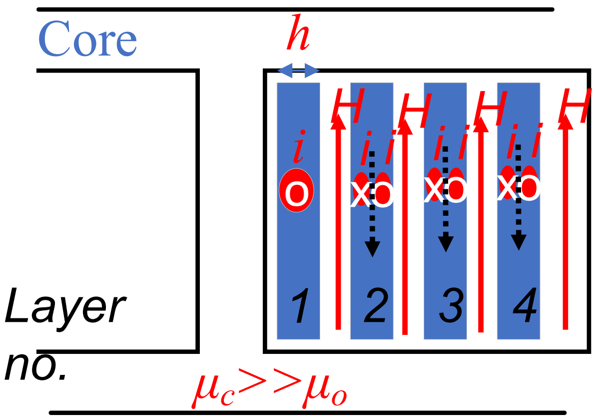
$$P_{2ac} = 2I_{rms}^2 R_{dc} \frac{h}{\delta} = 2P_{1ac}$$

$$P_{3ac} = 2I_{rms}^2 R_{dc} \frac{h}{\delta} = 2P_{1ac}$$

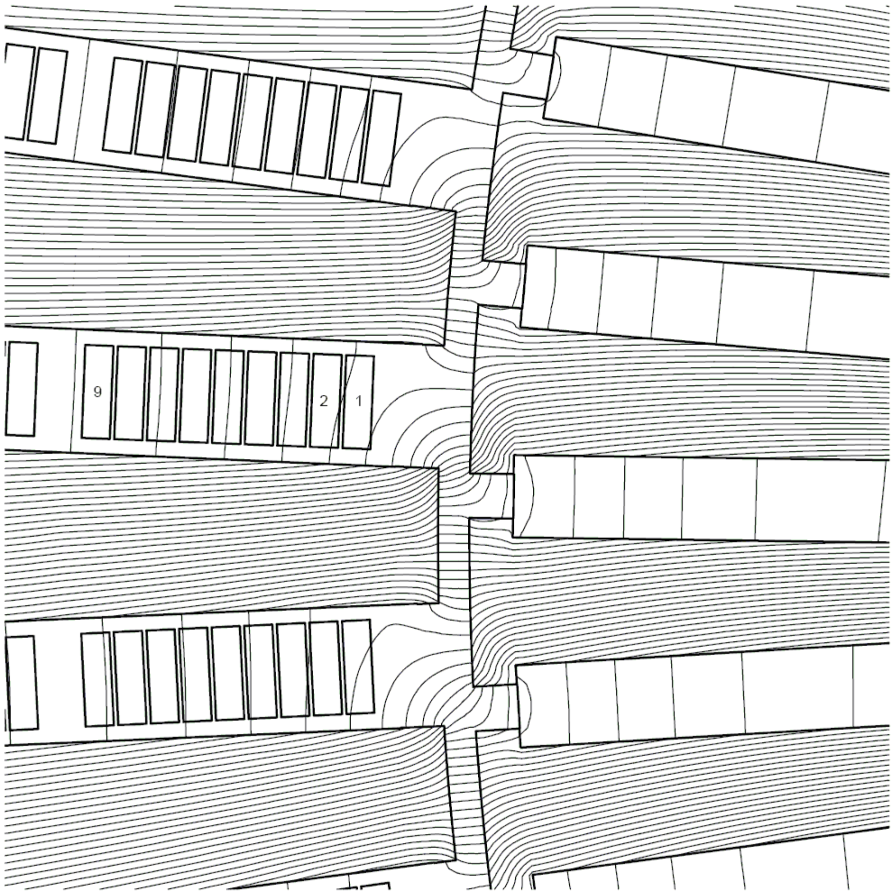
$$P_{4ac} = 2I_{rms}^2 R_{dc} \frac{h}{\delta} = 2P_{1ac}$$

Total Losses in This Case

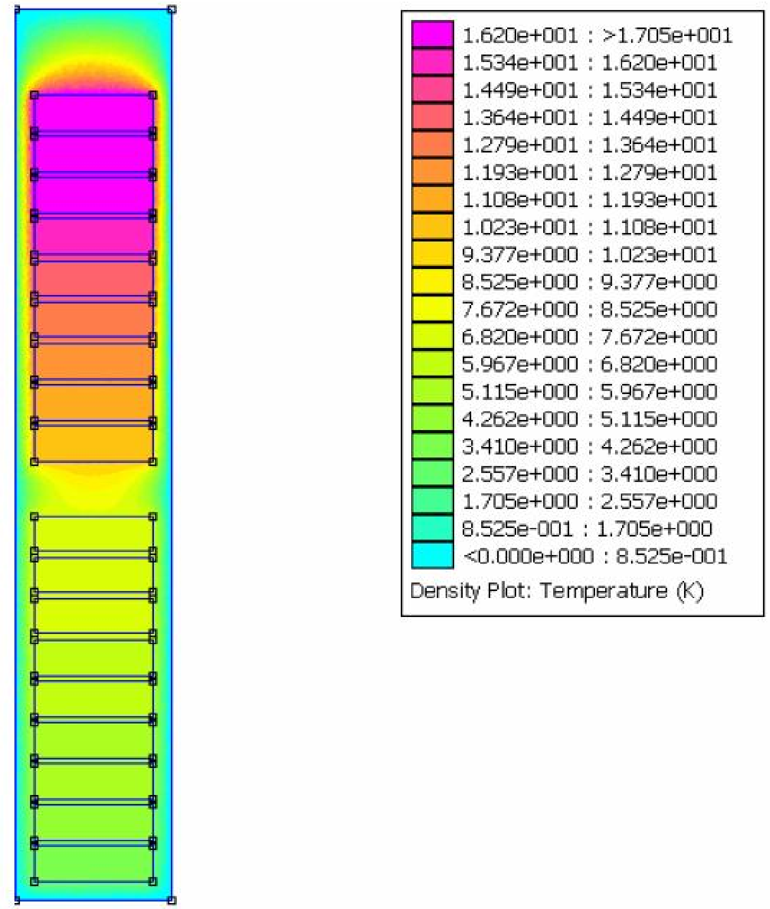
- $P_{tot} = 7P_{1ac}$ -------------→ 7 is due to the proximity effect.
- This occurs due to proximity and skin effect.
- This is the worst case scenario because $\mathrm{h} \gg \delta$.
- Proximity effect is the



Numerical modelling of eddy currents



Air gap



Magnetic flux at the air-gap region at one instant of time

Temperature rise associated with the resistive losses

Skin effect in many conductor windings

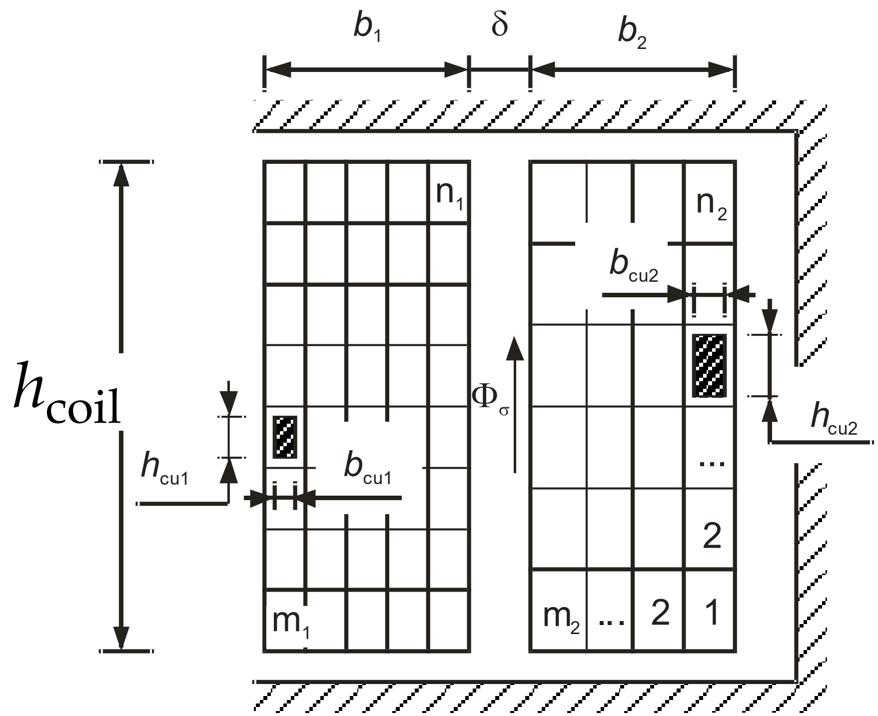
Definitions

$$l_\sigma = h_{\text{coil}} + 2\delta_{\text{ins}}$$

$$\alpha = \sqrt{\omega \, \mu_0 \, \gamma \, \frac{n_1 \, h_{\text{cu1}}}{2 \, l_\sigma}}$$

The normalised conductor heights for the two windings are

$$\xi_1 = \alpha \, b_{\text{cu}}$$



Contd.

This gives a resistance ratio

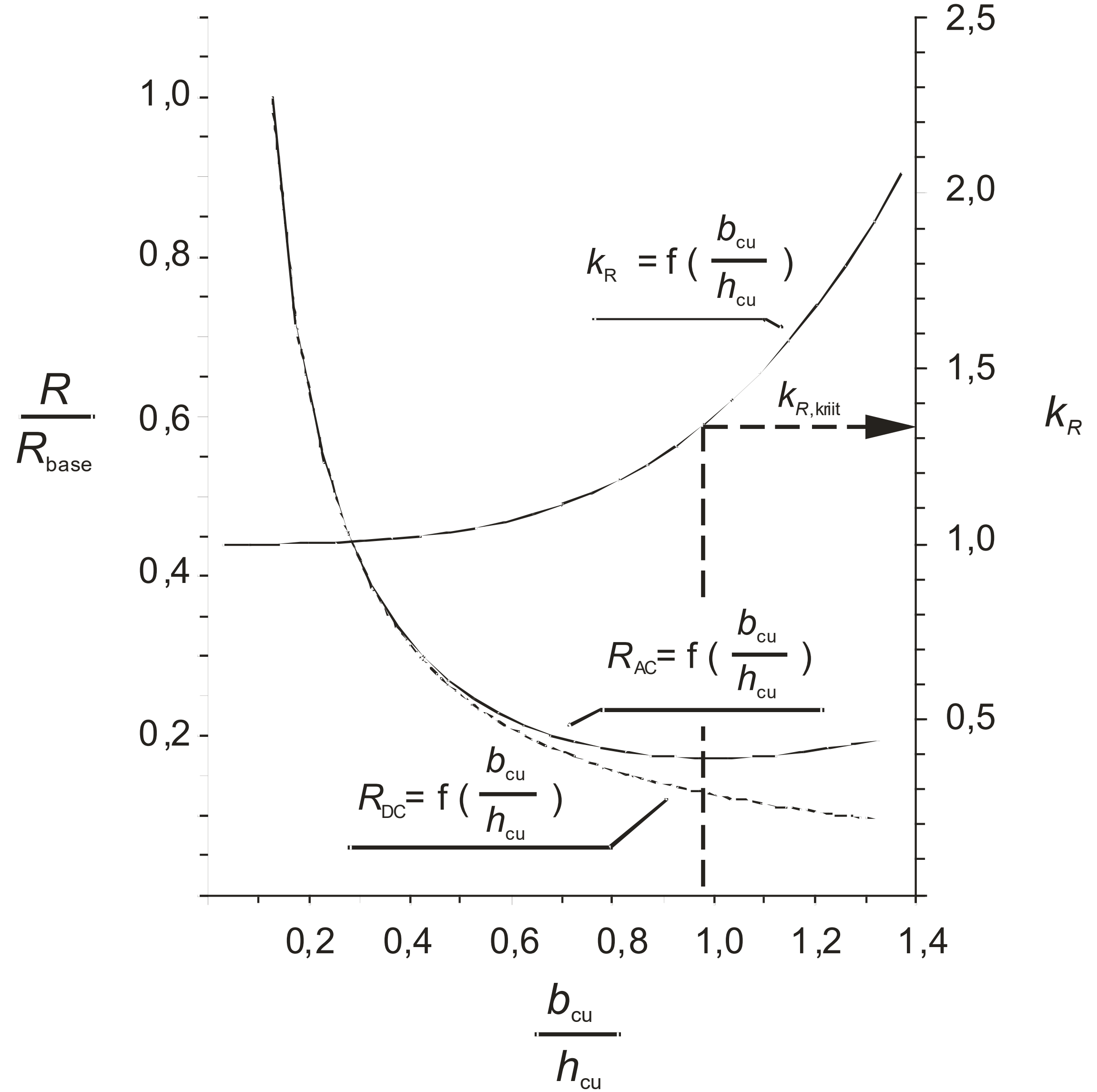
$$k_R \approx 1 + \frac{m^2 - 0.2}{9}\xi^4 \quad , \text{when } \; 0 \le \xi \le 1$$

The resistance ratio for circular conductors (wires) is

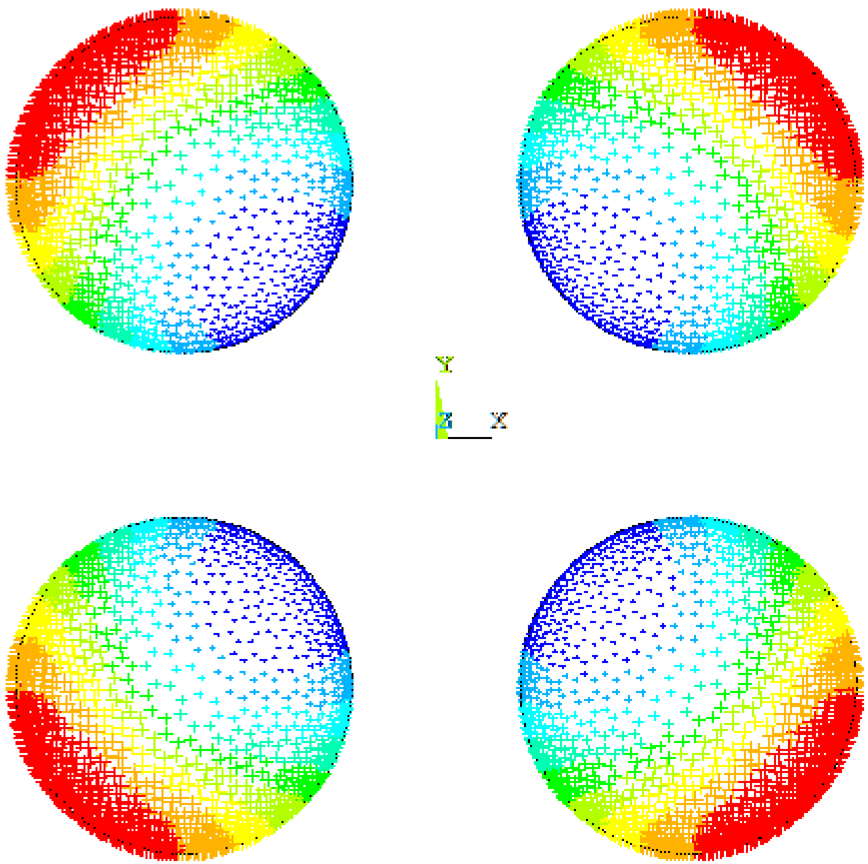
$$k_R \approx 1 + 0.59\,\frac{m^2 - 0{,}2}{9}\,\xi^4$$

Critical conductor height

When the height of the conductor is increased, the dc resistance decreases but the resistance ratio k_R increases. There is an optimal height at about $k_R = 1.33$, which gives the minimum ac loss.



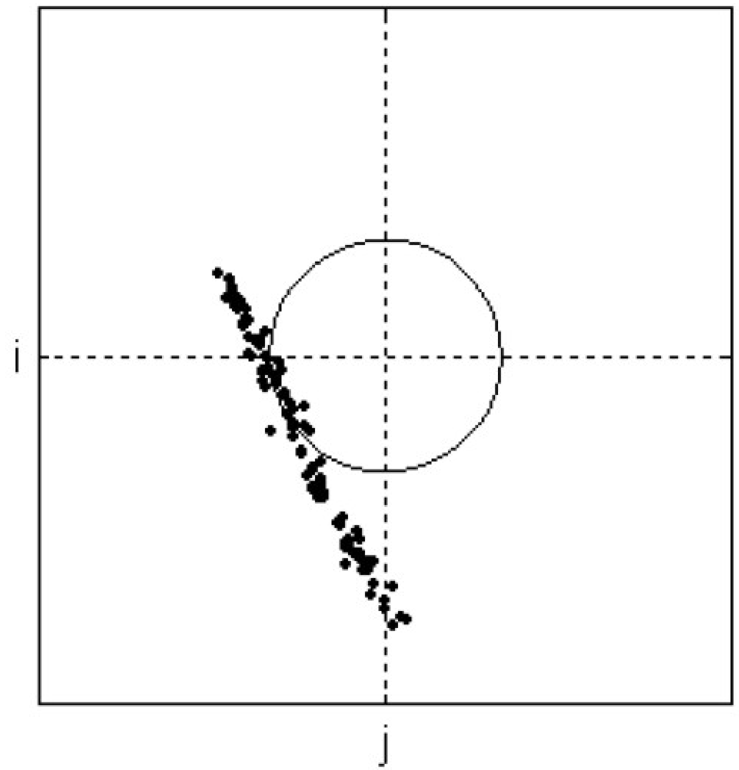
Eddy current and circulating current



The eddy-current loss can be reduced by dividing the conductor into several thin strands, which are connected in parallel. However, the strands see somewhat different flux linkages because of the leakage flux. This induces slightly different electromotive forces in the strands, and produces circulating currents flowing between the strands.

The strands should be perfectly transposed from slot to slot to avoid the circulating currents but this is difficult, especially, if there are tens of parallel strands. In some special cases, like high-speed machines, this effect may double the resistive loss.

Circulating currents

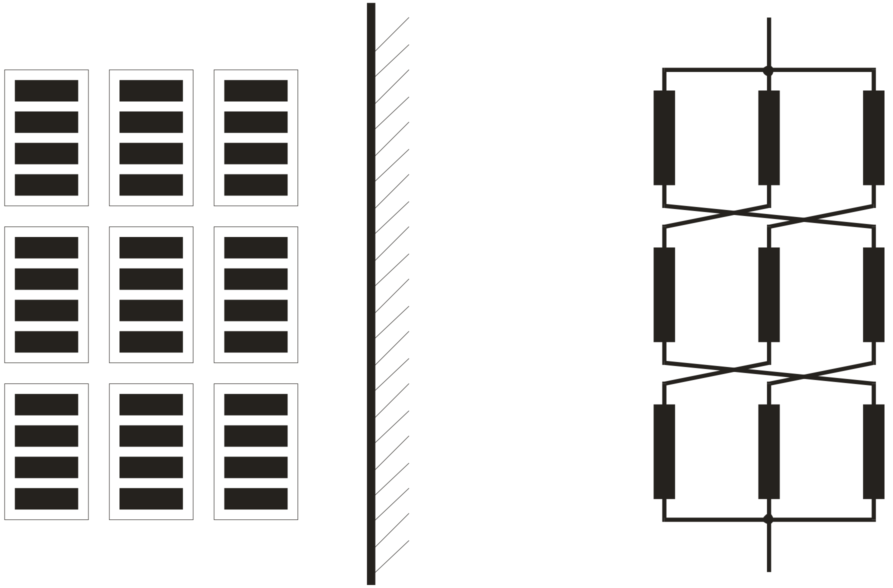


Strand currents measured from a high-speed motor and presented as phasors on complex plane. Without the circulating currents, all the strands would have an equal current and this would give a much smaller resistive loss than the distribution on the figure.

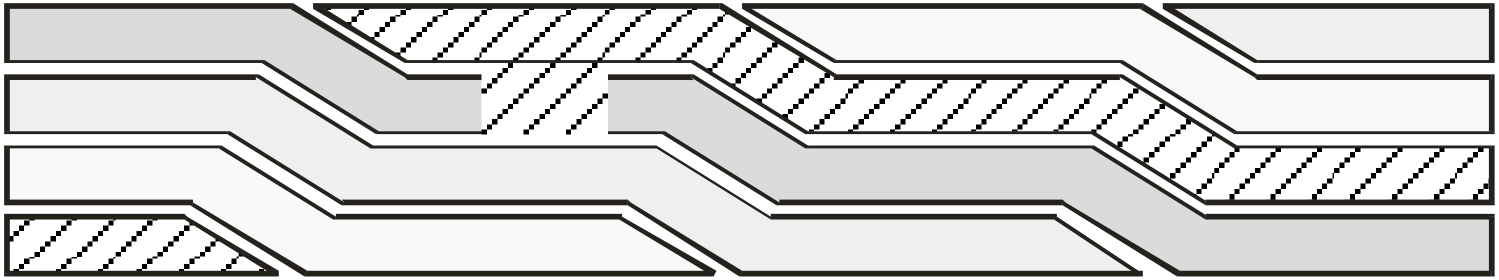
Lähteenmäki J., 2002, Design and voltage supply of high-speed induction machines. Dissertations of Helsinki University of Technology. Available at http://lib.tkk.fi/Diss/2002/isbn951226224X/

Eddy current and circulating current

In form-wound windings, the conductors can be transposed systematically. The transposition of a transformer winding is shown to the right.



The transposition of an electrical machine winding is done at the end winding or in a slot as for the Röbel-bar of a large synchronous machine shown below.



Assignment 1

Design 3 phase winding Ns = 48, Np = 4. One layer concentric winding.

a) Draw the base winding of the one-layer concentric winding.

b) Draw the total magnetomotive force produced by all the sinusoidal phase currents at a time instant when one of the phase currents has its peak value.

$$\begin{cases} i_{\mathrm{mA},1}(t) = \hat{i}_1 \cos(\omega t) \\ i_{\mathrm{mB},1}(t) = \hat{i}_1 \cos\left(\omega t - \frac{2\pi}{3}\right) \\ i_{\mathrm{mC},1}(t) = \hat{i}_1 \cos\left(\omega t - \frac{4\pi}{3}\right) \end{cases}$$

• c) Define the winding factors of the harmonic waves v= 1, 3 and 5 by drawings

Assignment 2

Design a three-phase wye-connected (Y) winding for a four-pole (p = 2) squirrel-cage motor when the number of slots per phase per pole is q = 4, short pitching by 2 slot, and the winding is a two layer diamond winding.
a) Draw the base winding of the two-layer diamond winding.
b) Draw the total magnetomotive force produced by all the sinusoidal phase currents at a time instant when one of the phase currents has its peak value.

$$\begin{cases} i_{\mathrm{mA},1}(t) = \hat{i}_1 \cos\left(\omega t\right) \\ i_{\mathrm{mB},1}(t) = \hat{i}_1 \cos\left(\omega t - \dfrac{2\pi}{3}\right) \\ i_{\mathrm{mC},1}(t) = \hat{i}_1 \cos\left(\omega t - \dfrac{4\pi}{3}\right) \end{cases}$$

- c) Define the winding factors of the harmonic waves v= 1, 3 and 5 by drawings