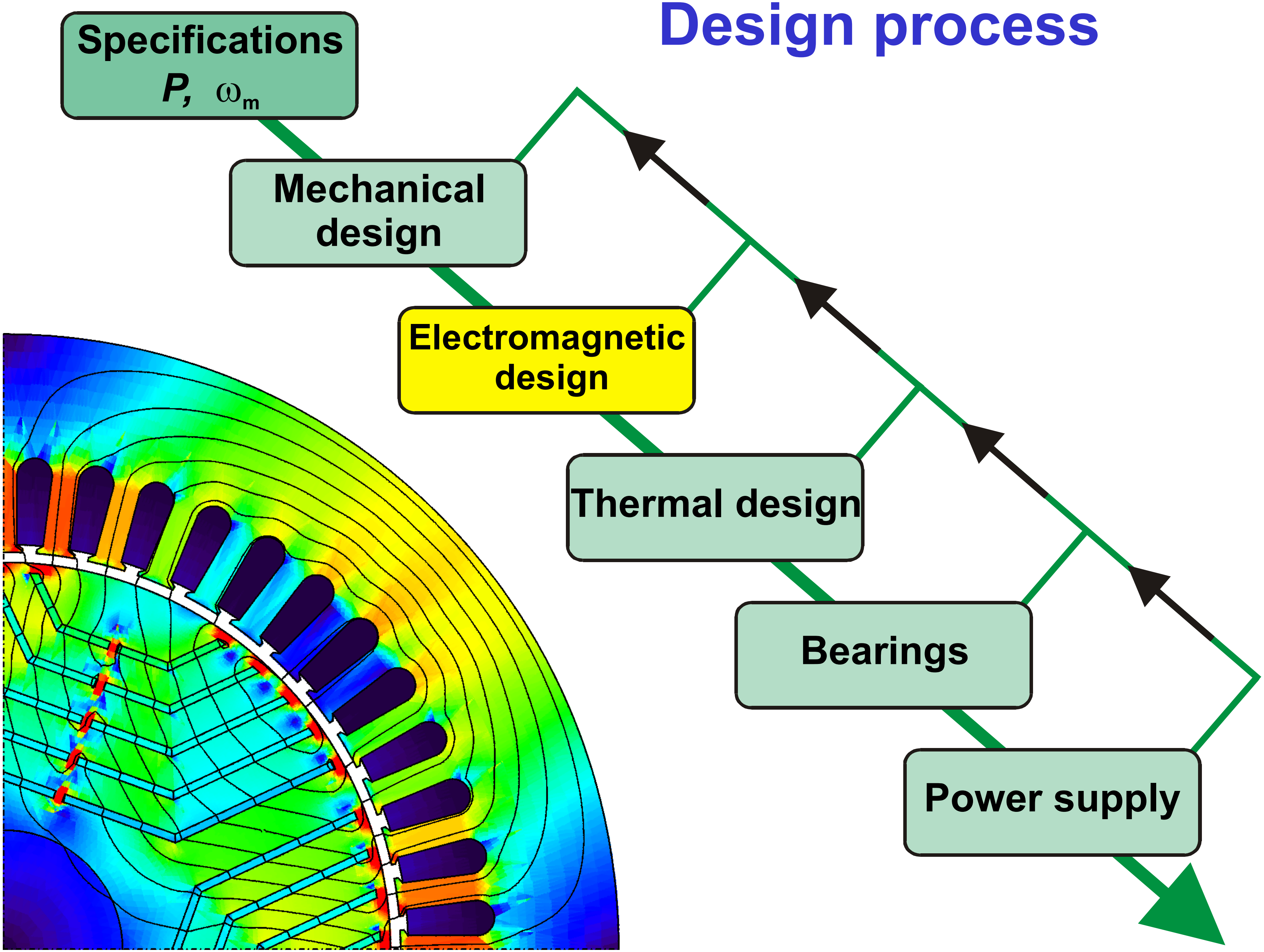
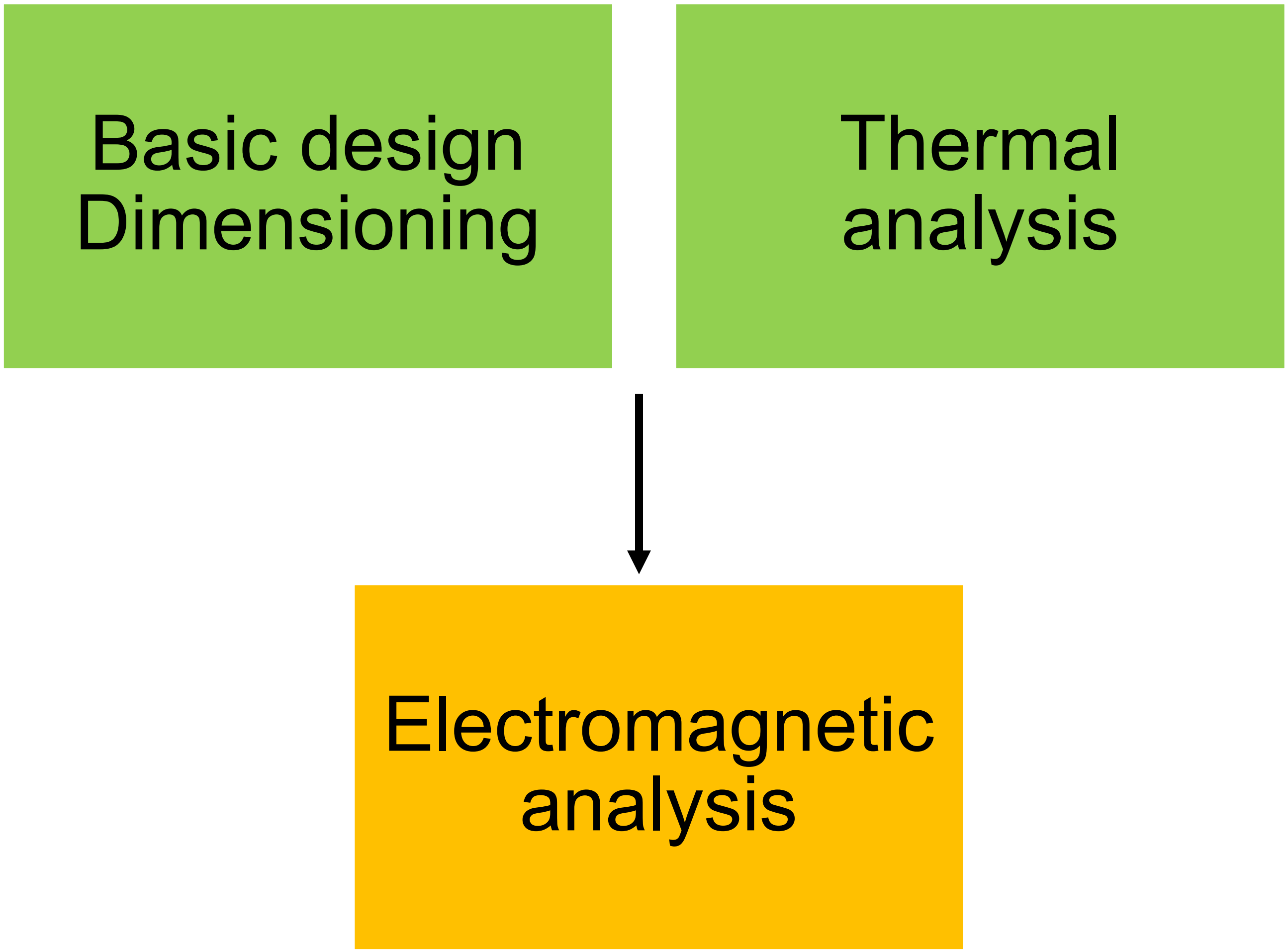
Design process



Task


Basic design Dimensioning
Thermal analysis
Electromagnetic analysis

Task

Design a 40 kW four-pole **PM motor** and estimate the parameters needed for the torque versus load angle equation

$$T_e = -\frac{3}{2}\frac{p}{\omega}\left[\frac{\hat{u}_s \hat{u}_p}{X_d}\sin\delta + \frac{\hat{u}_s^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta\right]$$

The motor will be connected to a balanced 50 Hz three-phase supply with a 400 V line-to-line voltage

=> $\hat{u}_s = 327 \text{ V}$

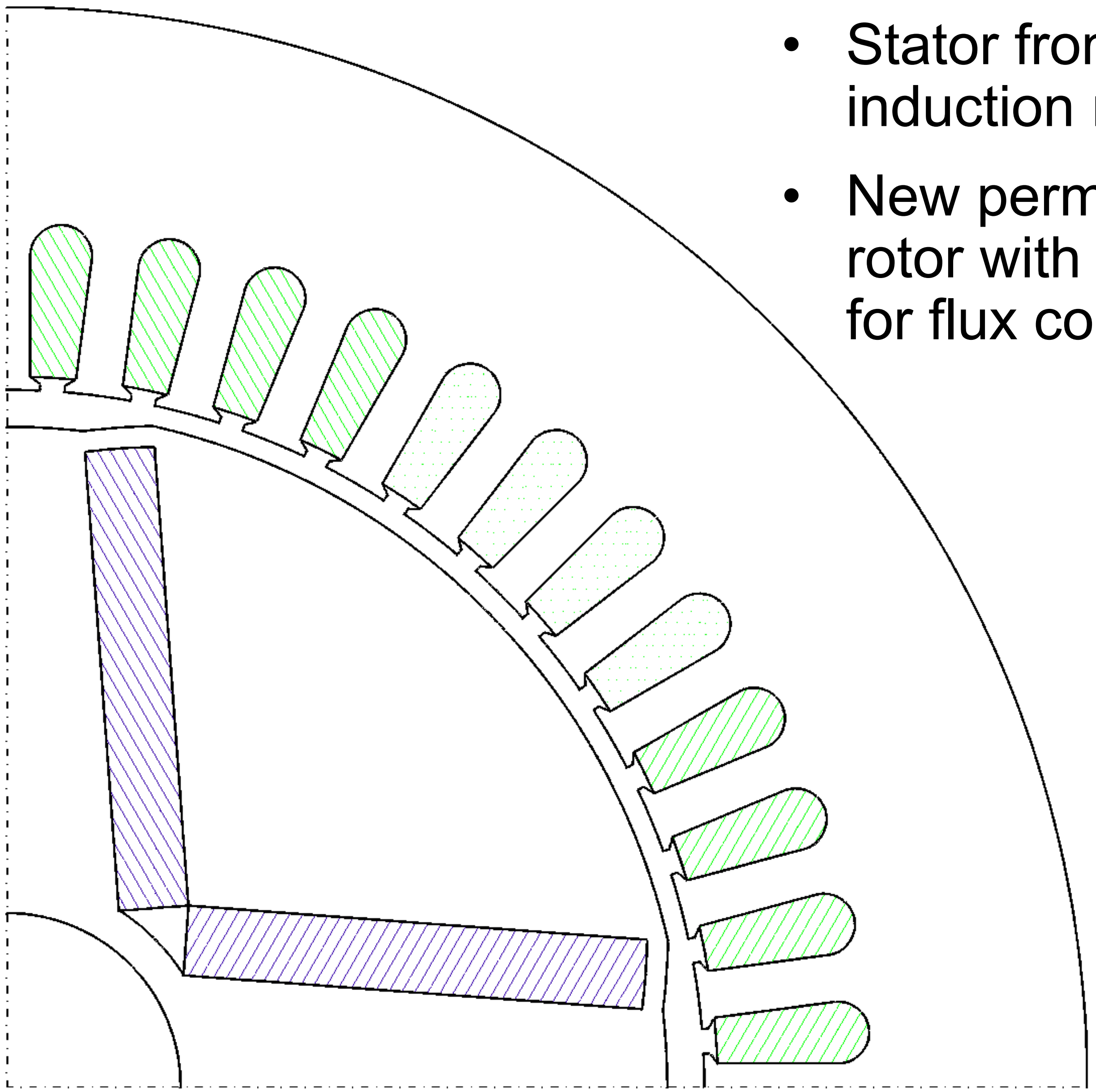
Simplifications

The stator is taken from a 37 kW cage induction machine.

The winding may have to be redesigned.

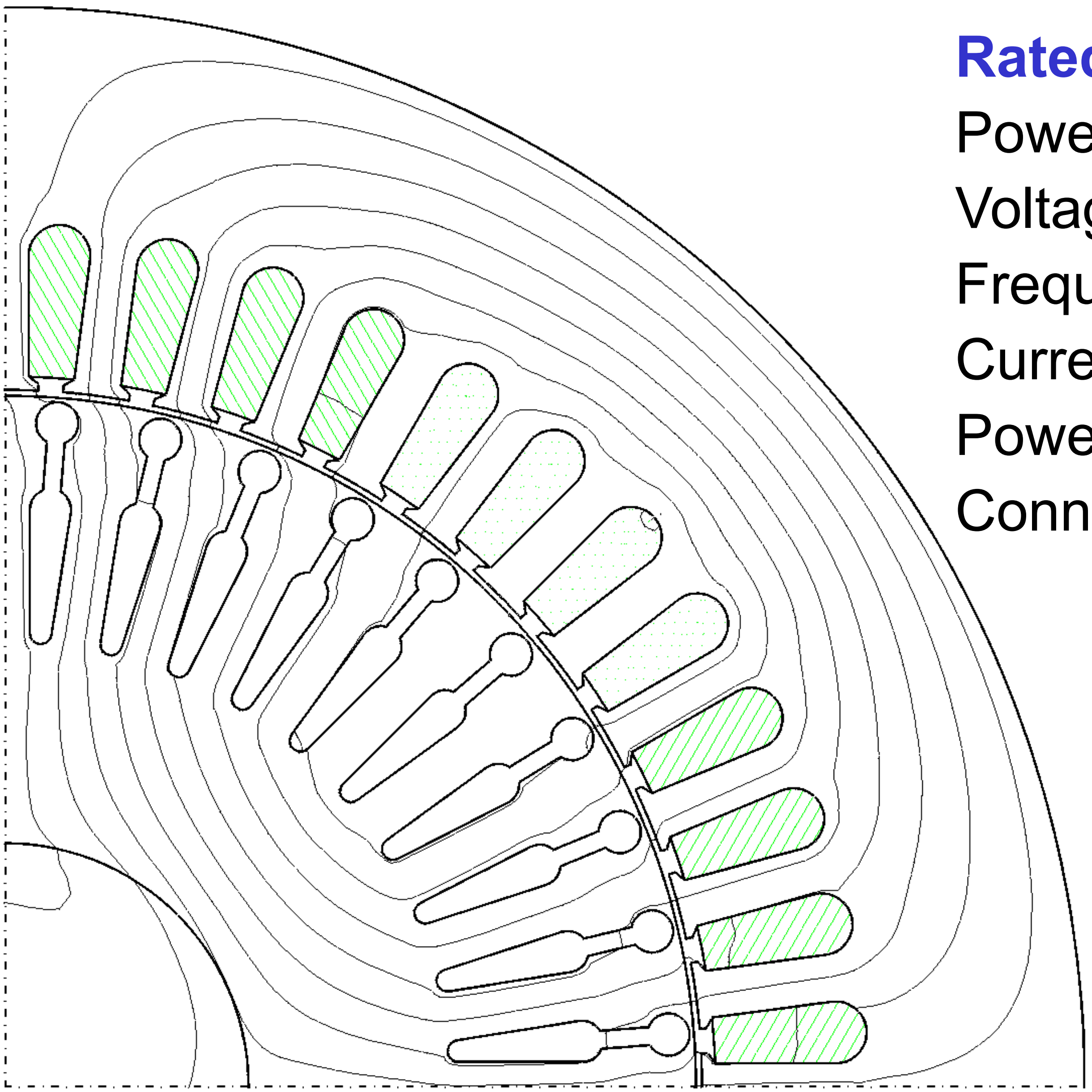
The rotor is of the embedded V-shape design.

Basic geometry



- Stator from an existing cage induction motor
- New permanent magnet rotor with V-shaped magnets for flux concentration

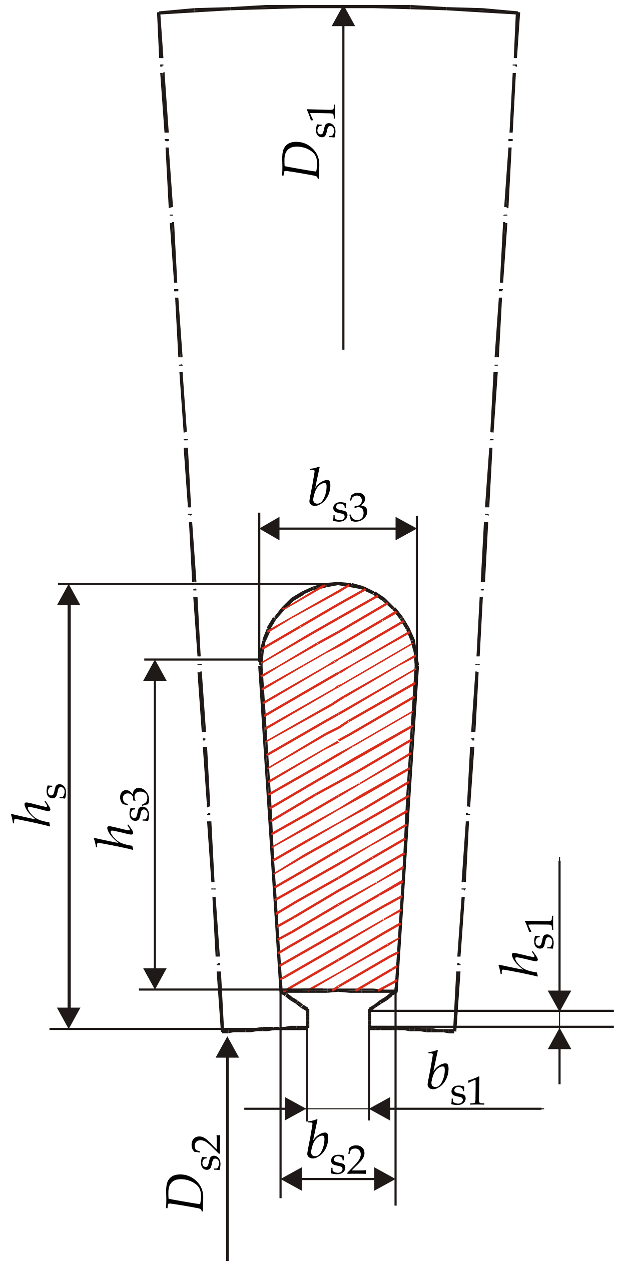
Original induction motor



Rated parameters

Power	37 kW
Voltage	400 V
Frequency	50 Hz
Current	72 A
Power factor	0.800
Connection	STAR

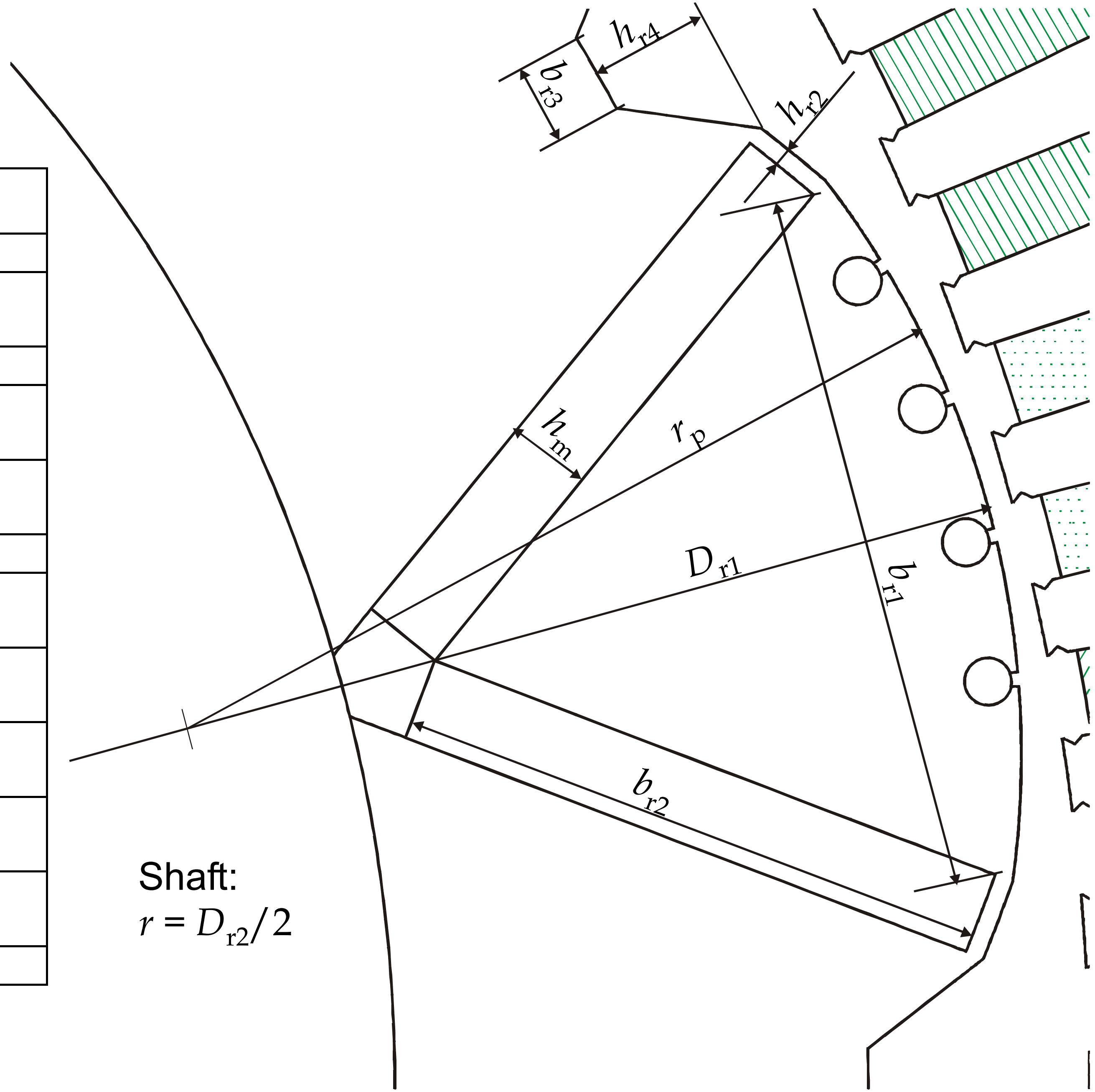
Main dimensions of the stator



	[mm]
Length l	246
D_{s1}	310
D_{s2}	200
b_{s1}	3.5
b_{s2}	6.5
b_{s3}	8.8
h_s	23.9
h_{s1}	1.0
h_{s3}	17.5

Main dimensions of the rotor

	[mm]
Length l	246
D_{r1}	194
D_{r2}	50
b_{r1}	100
b_{r2}	66
h_m	10.0
h_{r2}	3.0
r_p	98



Shaft:
$r = D_{r2}/2$

Space-vector diagram for a PM synchronous machine

$$\begin{cases} u_d = R_s i_d - \omega\psi_q \\ u_q = R_s i_q + \omega\psi_d \end{cases}$$

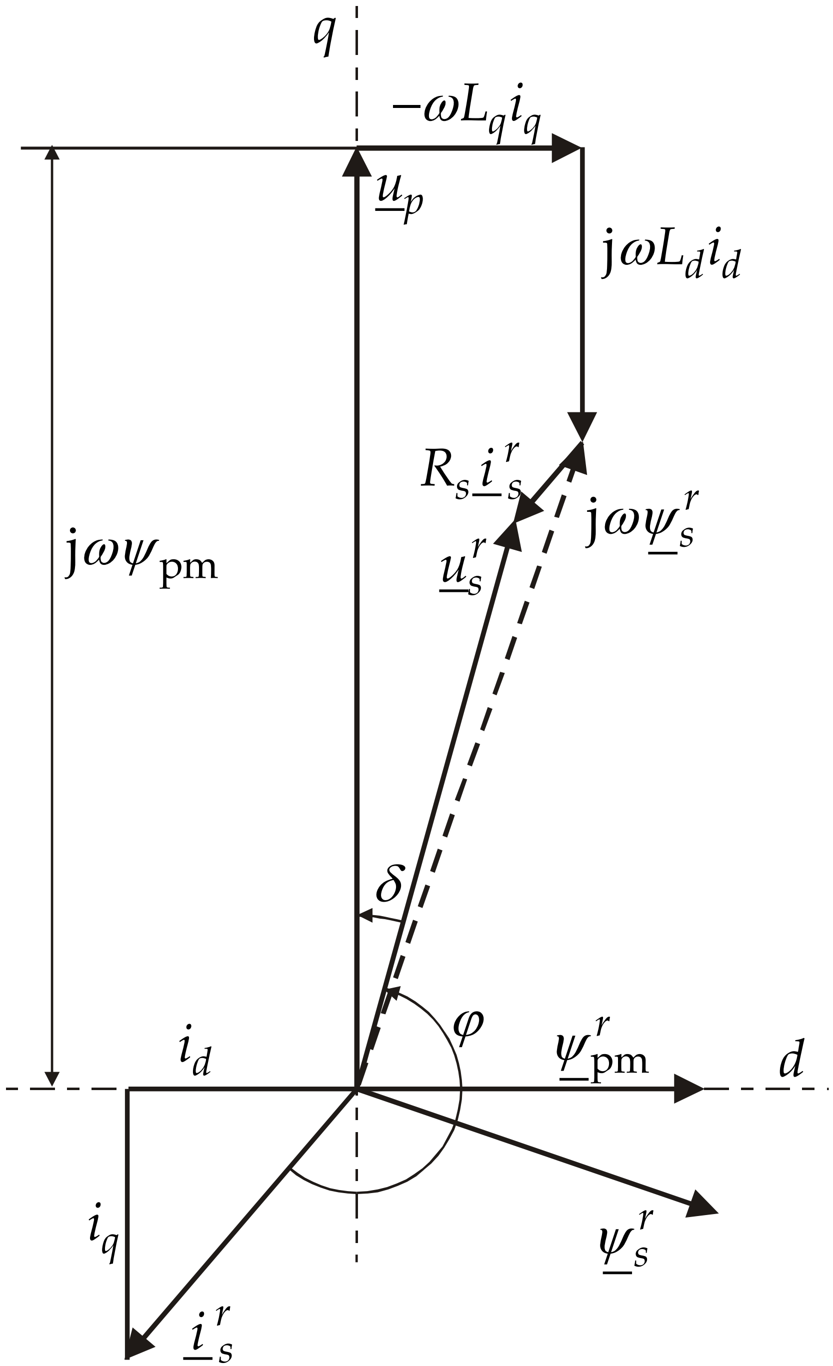
$$\begin{cases} \psi_d = L_d i_d + \psi_{\mathrm{pm}} \\ \psi_q = L_q i_q \end{cases}$$

$$\underline{u}_s^r = R_s \underline{i}_s^r + \mathrm{j}\omega L_d i_d - \omega L_q i_q + \underline{u}_p$$

Complex phasor of voltage

$$\underline{u}_p = \mathrm{j}\hat{u}_p = \mathrm{j}\omega\psi_{\mathrm{pm}}$$

$$\begin{cases} i_d = \hat{i}_s \sin(\delta + \varphi) \\ i_q = \hat{i}_s \cos(\delta + \varphi) \end{cases}$$



Parameters of the PM machine

The voltage of the stator winding presented as a space vector is

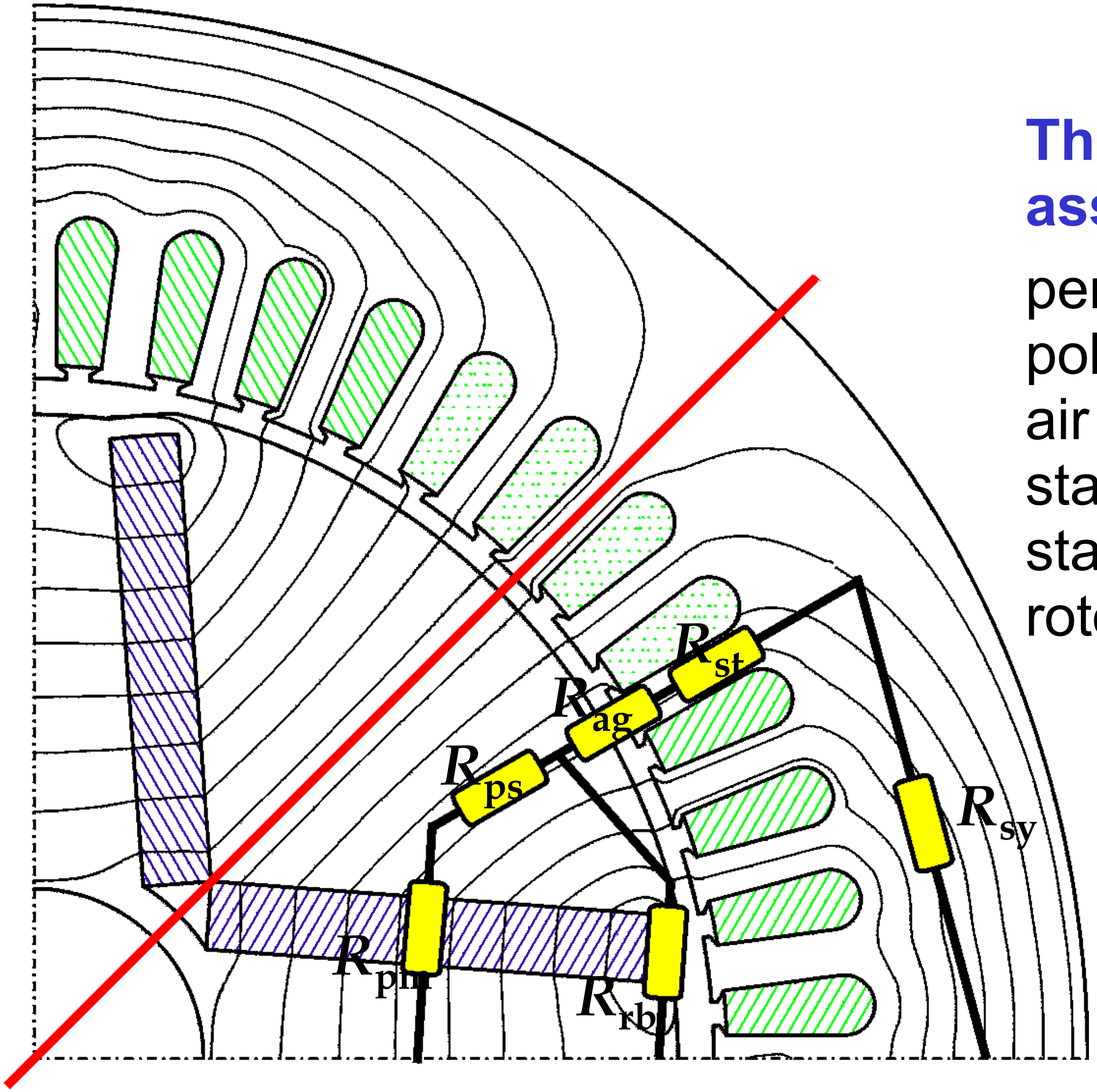
$$\underline{u}_s^r = R_s \underline{i}_s^r + \mathrm{j}\omega L_d i_d - \omega L_q i_q + \underline{u}_p; \qquad \underline{u}_p = \mathrm{j}\hat{u}_p = \mathrm{j}\omega\psi_{\mathrm{pm}}$$

When deriving the expression for the torque, the stator resistance was neglected and stator reactances were used instead of the inductances

$$\underline{u}_s^r \approx \mathrm{j}X_d i_d - X_q i_q + \underline{u}_p; \qquad \underline{u}_p = \mathrm{j}\hat{u}_p = \mathrm{j}\omega\psi_{\mathrm{pm}}$$

To get the machine parameters u_{p}, X_{d}, X_{q}, we shall calculate one by one the voltages induced by the flux of the permanent magnets (u_{p}), by the flux of a current on d-axis ($u = X_{\mathrm{d}} i_{\mathrm{d}}$) and by the flux of a current on q-axis ($u = X_{\mathrm{q}} i_{\mathrm{q}}$).

Reluctance network for the magnetic circuit



The reluctances are associated with:

permanent magnet (pm),
pole shoe (ps),
air gap (ag),
stator tooth (st),
stator yoke (sy) and
rotor bridge (rb)

Reluctance network for the magnetic circuit; no-load => $I_{\mathrm{st}} = 0$

Magnetotive force equation

$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I; \quad I = 0 \quad => \quad \sum_{i=1}^{n} H_i d_i = 0$$

Passive materials

$$B_i = \mu_i H_i \quad => \quad H_i d_i = \frac{d_i}{\mu_i} B_i = \frac{d_i}{\mu_i A_i} \Psi_i$$

Permanent magnet

$$B_{\mathrm{pm}} = B_{\mathrm{r}} + \mu_{\mathrm{pm}} H_{\mathrm{pm}}$$

$$=> \quad H_{\mathrm{pm}} d_{\mathrm{pm}} = \frac{d_{\mathrm{pm}}}{\mu_{\mathrm{pm}}} \left(B_{\mathrm{pm}} - B_{\mathrm{r}} \right) = \frac{d_{\mathrm{pm}} \Psi_{\mathrm{pm}}}{\mu_{\mathrm{pm}} A_{\mathrm{pm}}} - \frac{d_{\mathrm{pm}} B_{\mathrm{r}}}{\mu_{\mathrm{pm}}}$$

mmv equation

$$\sum_{i=1}^{n} H_i d_i = \sum_{i=1}^{n} \frac{d_i}{\mu_i A_i} \Psi_i - \frac{B_{\mathrm{r}} d_{\mathrm{pm}}}{\mu_{\mathrm{pm}}} = 0$$

Reluctance network for the magnetic circuit II

Flux equation with the reluctance coefficients

$$\sum_{i=1}^{n} \frac{d_i}{\mu_i A_i} \Psi_i = \sum_{i=1}^{n} R_i \Psi_i = \frac{B_\mathrm{r} d_\mathrm{pm}}{\mu_\mathrm{pm}}$$

Conservation of flux

$$\sum_{i=1}^{k} \Psi_i = 0$$

Flux equations for the PM motor

$$\begin{cases} R_\mathrm{pm}\Psi_\mathrm{pm} + R_\mathrm{ps}\Psi_\mathrm{ps} + R_\mathrm{rb}\Psi_\mathrm{rb} = \dfrac{B_\mathrm{r} d_\mathrm{pm}}{\mu_\mathrm{pm}} \\ R_\mathrm{ag}\Psi_\mathrm{ag} + R_\mathrm{st}\Psi_\mathrm{st} + R_\mathrm{sy}\Psi_\mathrm{sy} - R_\mathrm{rb}\Psi_\mathrm{rb} = 0 \end{cases}$$

$$\begin{cases} \Psi_\mathrm{pm} = \Psi_\mathrm{ps} \\ \Psi_\mathrm{ag} = \Psi_\mathrm{st} = \Psi_\mathrm{sy} \\ \Psi_\mathrm{rb} = \Psi_\mathrm{pm} - \Psi_\mathrm{ag} \end{cases}$$

Application to the PM motor geometry

$$\begin{cases} R_{\mathrm{pm}}\Psi_{\mathrm{pm}} + R_{\mathrm{ps}}\Psi_{\mathrm{ps}} + R_{\mathrm{rb}}\Psi_{\mathrm{rb}} = \dfrac{B_{\mathrm{r}} d_{\mathrm{pm}}}{\mu_{\mathrm{pm}}} \\ R_{\mathrm{ag}}\Psi_{\mathrm{ag}} + R_{\mathrm{st}}\Psi_{\mathrm{st}} + R_{\mathrm{sy}}\Psi_{\mathrm{sy}} - R_{\mathrm{rb}}\Psi_{\mathrm{rb}} = 0 \end{cases}$$

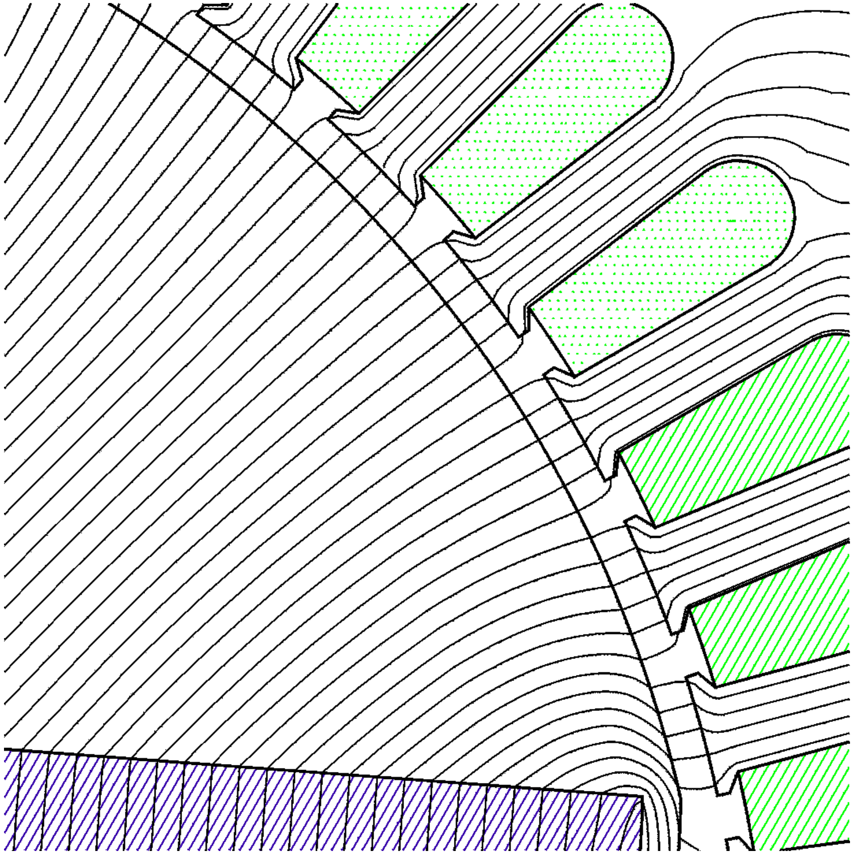
$$\begin{cases} \Psi_{\mathrm{pm}} = \Psi_{\mathrm{ps}} \\ \Psi_{\mathrm{ag}} = \Psi_{\mathrm{st}} = \Psi_{\mathrm{sy}} \\ \Psi_{\mathrm{rb}} = \Psi_{\mathrm{pm}} - \Psi_{\mathrm{ag}} \end{cases}$$

$$\begin{cases} \left(R_{\mathrm{pm}} + R_{\mathrm{ps}} + R_{\mathrm{rb}}\right)\Psi_{\mathrm{pm}} - R_{\mathrm{rb}}\Psi_{\mathrm{ag}} = \dfrac{B_{\mathrm{r}} d_{\mathrm{pm}}}{\mu_{\mathrm{pm}}} \\ R_{\mathrm{rb}}\Psi_{\mathrm{pm}} - \left(R_{\mathrm{ag}} + R_{\mathrm{st}} + R_{\mathrm{sy}} + R_{\mathrm{rb}}\right)\Psi_{\mathrm{ag}} = 0 \end{cases}$$

$$\begin{cases} \Psi_{\mathrm{pm}} = \dfrac{\left(R_{\mathrm{ag}} + R_{\mathrm{st}} + R_{\mathrm{sy}} + R_{\mathrm{rb}}\right)}{\left[\left(R_{\mathrm{ag}} + R_{\mathrm{st}} + R_{\mathrm{sy}} + R_{\mathrm{rb}}\right)\left(R_{\mathrm{pm}} + R_{\mathrm{ps}} + R_{\mathrm{rb}}\right) - R_{\mathrm{rb}}R_{\mathrm{rb}}\right]} \dfrac{B_{\mathrm{r}} d_{\mathrm{pm}}}{\mu_{\mathrm{pm}}} \\ \Psi_{\mathrm{ag}} = \dfrac{R_{\mathrm{rb}}}{\left[\left(R_{\mathrm{ag}} + R_{\mathrm{st}} + R_{\mathrm{sy}} + R_{\mathrm{rb}}\right)\left(R_{\mathrm{pm}} + R_{\mathrm{ps}} + R_{\mathrm{rb}}\right) - R_{\mathrm{rb}}R_{\mathrm{rb}}\right]} \dfrac{B_{\mathrm{r}} d_{\mathrm{pm}}}{\mu_{\mathrm{pm}}} \end{cases}$$

Reluctance of the air gap

Because of the stator slotting, the air gap represents a slightly larger reluctance for the flux than a smooth air gap would represent. In a conventional analysis, this is taken into account by multiplying the physical air-gap length by a Carter's factor.



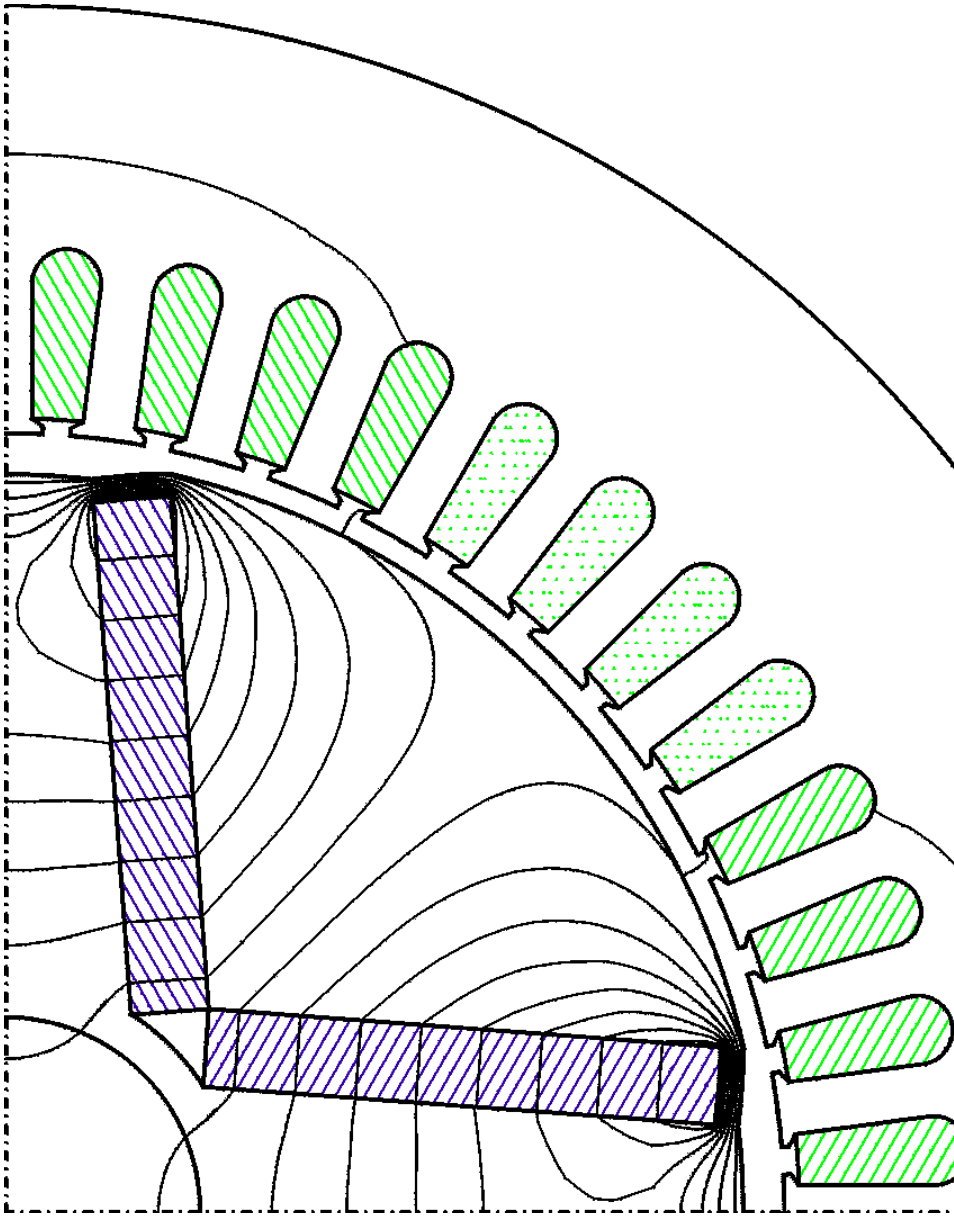
Carter's factor is defined as the ratio of the maximum and average flux densities

$$k_\mathrm{C} = \frac{B_\mathrm{max}}{B_\mathrm{ave}}$$

over a slot pitch. It depends on the ratio of the slot opening and air-gap length

$$k_\mathrm{C} = \frac{\tau_\mathrm{s}}{\tau_\mathrm{s} - \gamma b_\mathrm{s1}}; \qquad \gamma \approx \frac{1}{1 + 5\dfrac{\delta}{b_\mathrm{s1}}}$$

Saturation of the iron bridge

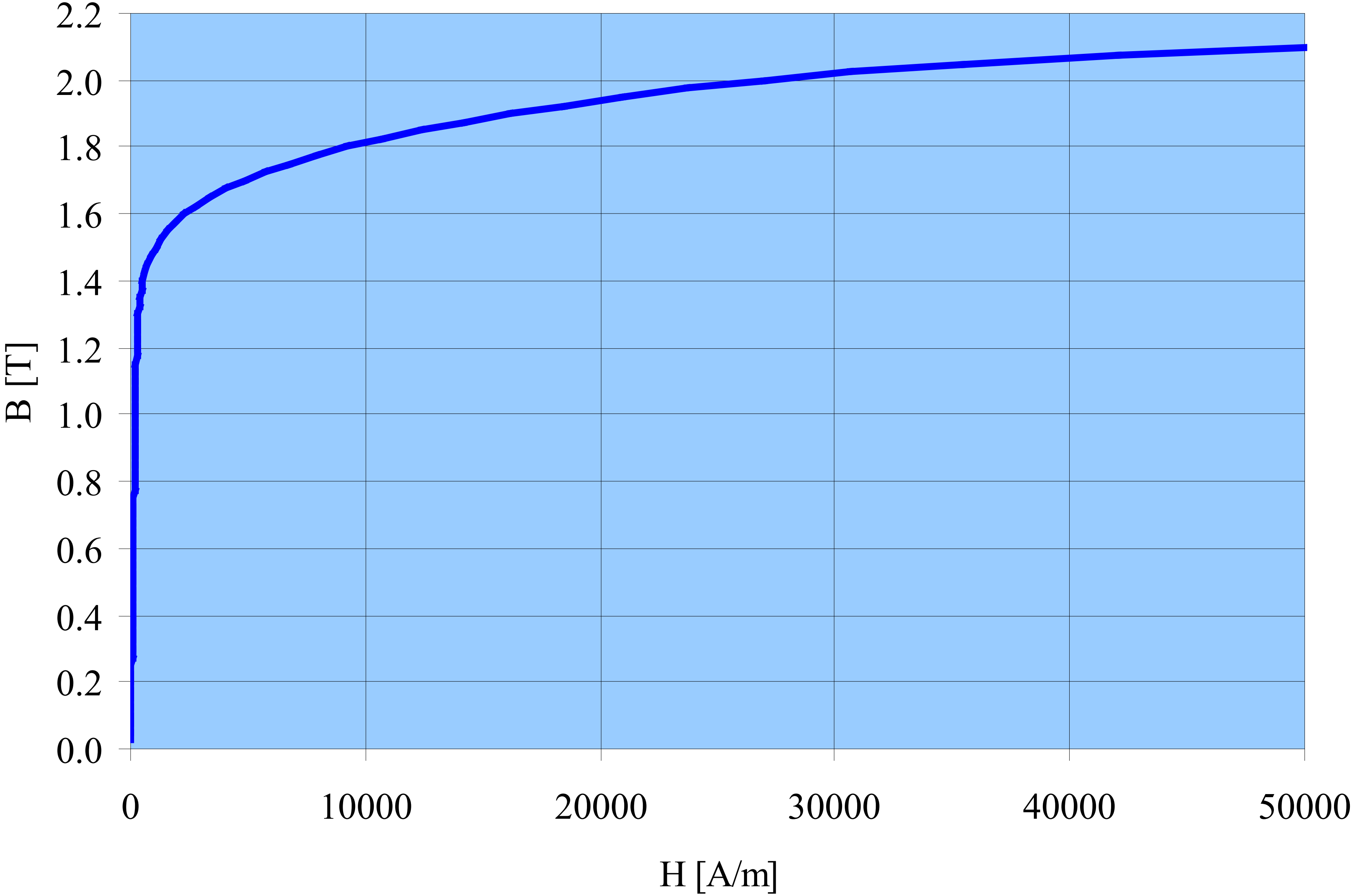


It is essential to model the saturation of the iron bridge. Without saturation, the main flux would flow through the bridge without crossing the air gap.

The figure shows the flux if a constant relative permeability of 1000 is assumed for all the iron parts.

In this case, the no-load voltage of the original stator winding is 63 V, only, when it should be about 400 V.

Magnetisation curve for the core material



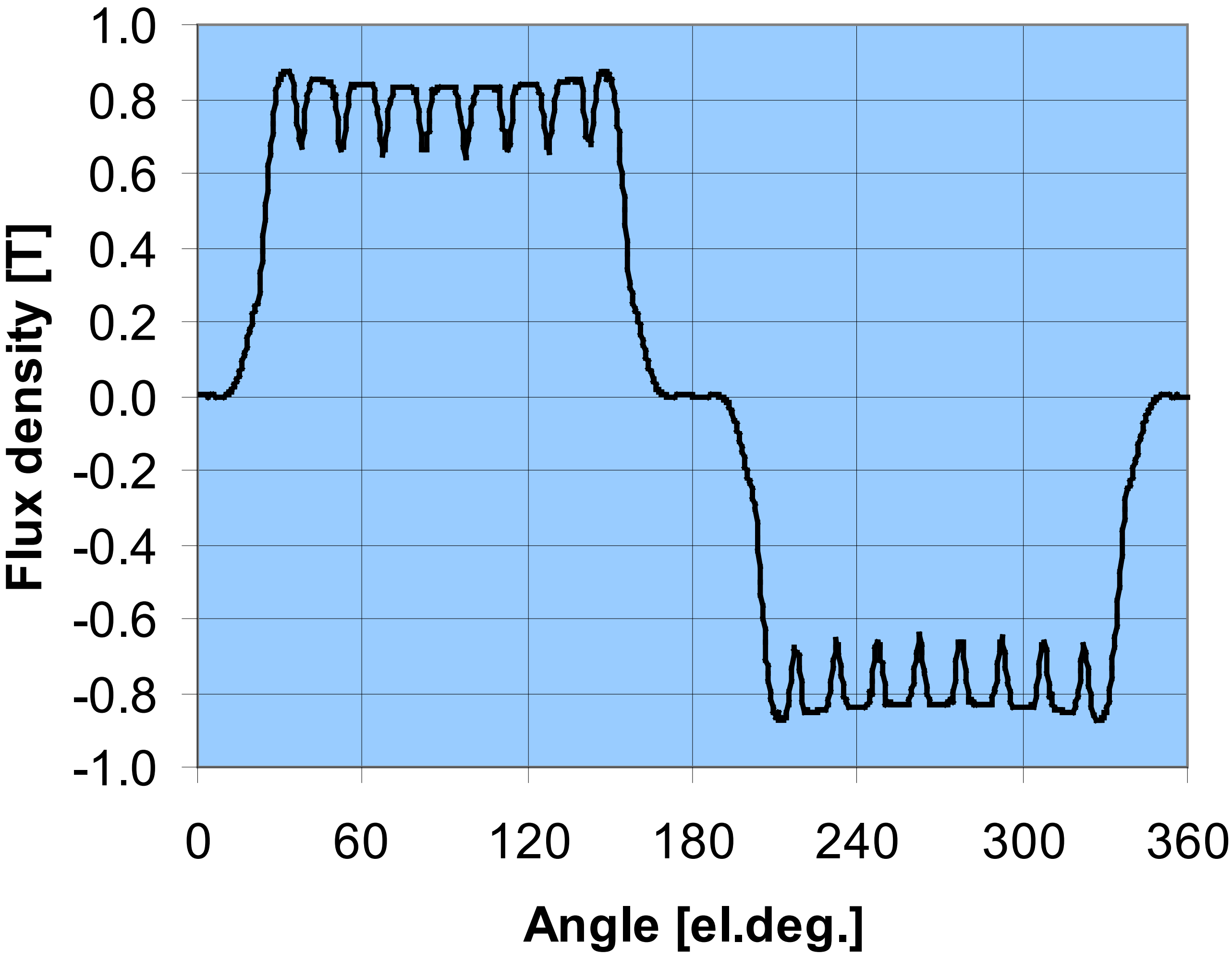
Calculation result

Br	1.1
μ0	1.26E-06
Length l	0.2460
Ds1	0.3100
Ds2	0.2000
bs1	0.0035
bs2	0.0065
bs3	0.0088
hs	0.0239
hs1	0.0010
hs3	0.0175
Dr1	0.1940
Dr2	0.0500
br1	0.1000
br2	0.0660
hm	0.0100
hr2	0.0030
rp	0.0980
γ	0.189
τ	0.013
kC	1.053

	B est	μ	d	A	R
PM	0.80	1.37E-06	0.0100	0.01624	449659
pole shoe	1.00	1.26E-03	0.0233	0.01148	1617
air gap	0.80	1.26E-06	0.0032	0.01392	180641
stator teeth	1.50	1.26E-03	0.0239	0.00758	2508
stator yoke	1.50	1.26E-03	0.0548	0.00765	5696
rotor bridge	2.10	1.16E-05	0.0120	0.00074	1395836

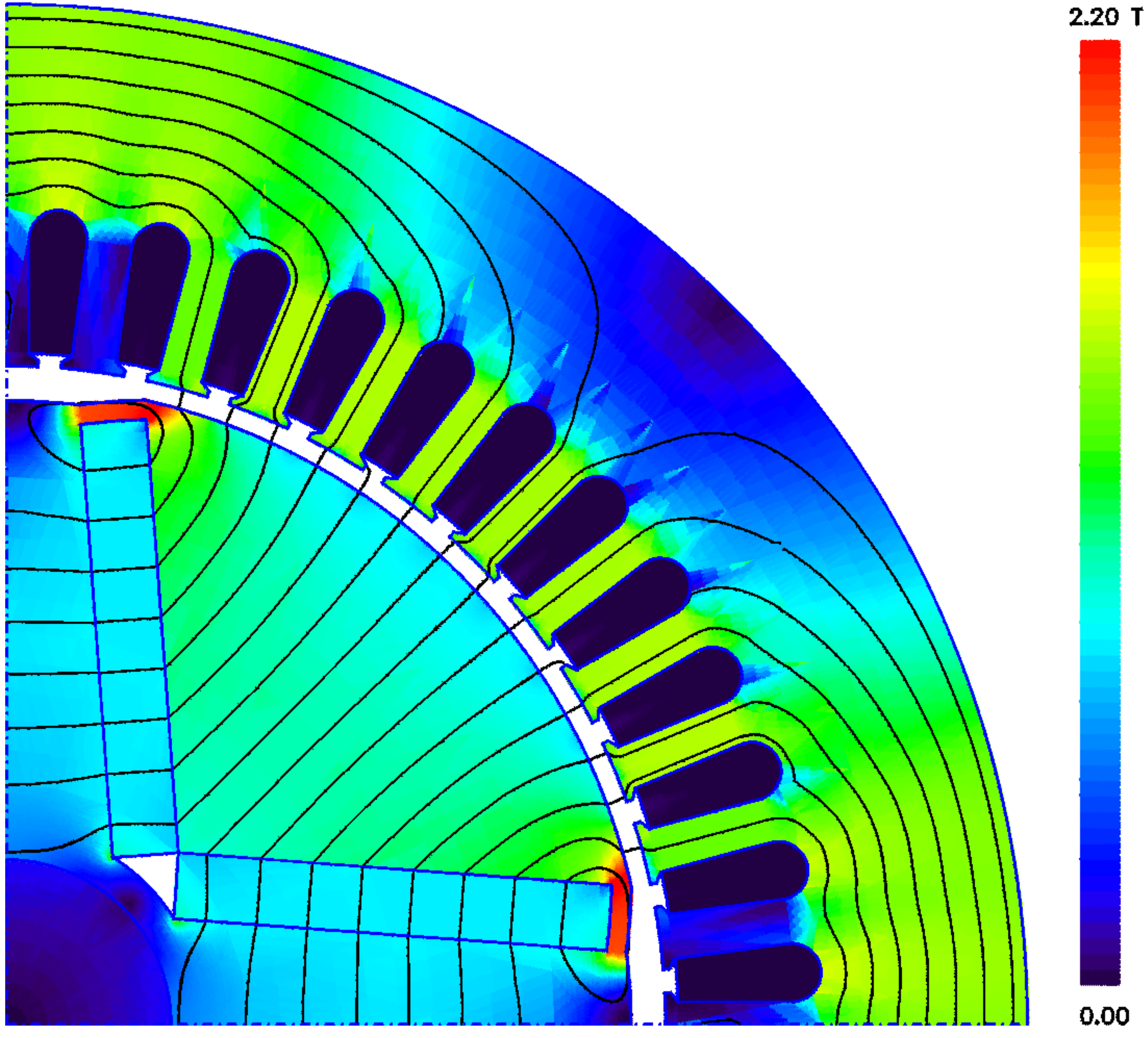
Ψag	**0.0115**	**Bag**	**0.705**
Ψpm	**0.0130**	**Bpm**	**0.801**
Ψrb	**0.0015**	**Brb**	**2.100**

Radial flux density in the air gap



Harmonic	Amplitude
1	0.9340
3	0.1368
5	0.0757
7	0.1234
9	0.0852
11	0.0196
13	0.0279
15	0.0393
17	0.0236
19	0.0000
21	0.0191
23	0.0518
25	0.0435
27	0.0128

Flux density at no load

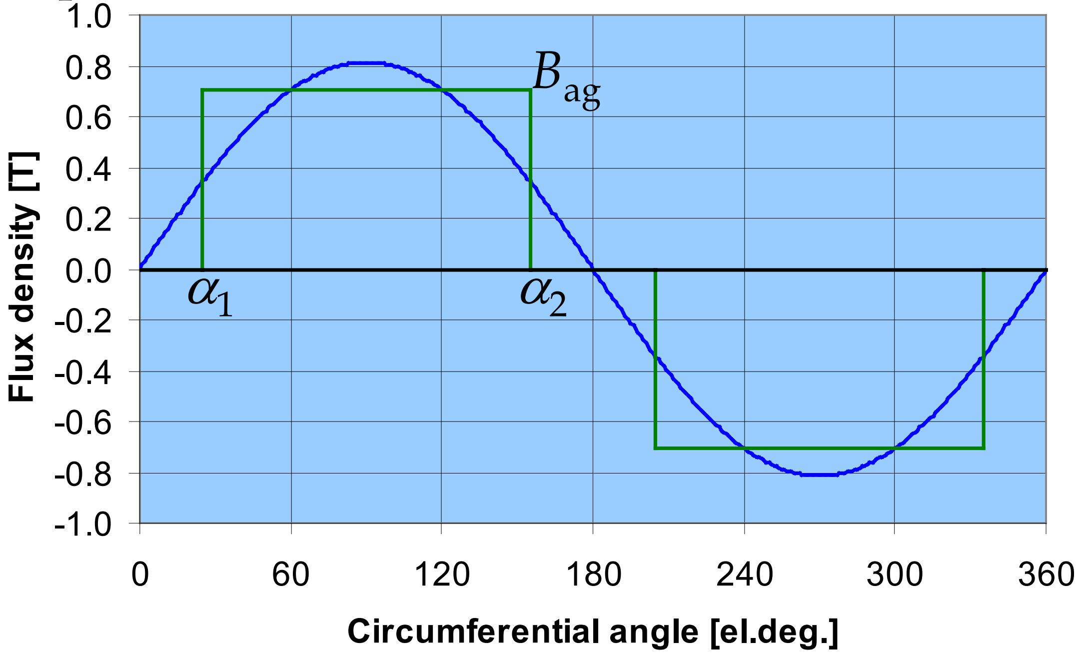


Excitation voltage u_{pm}

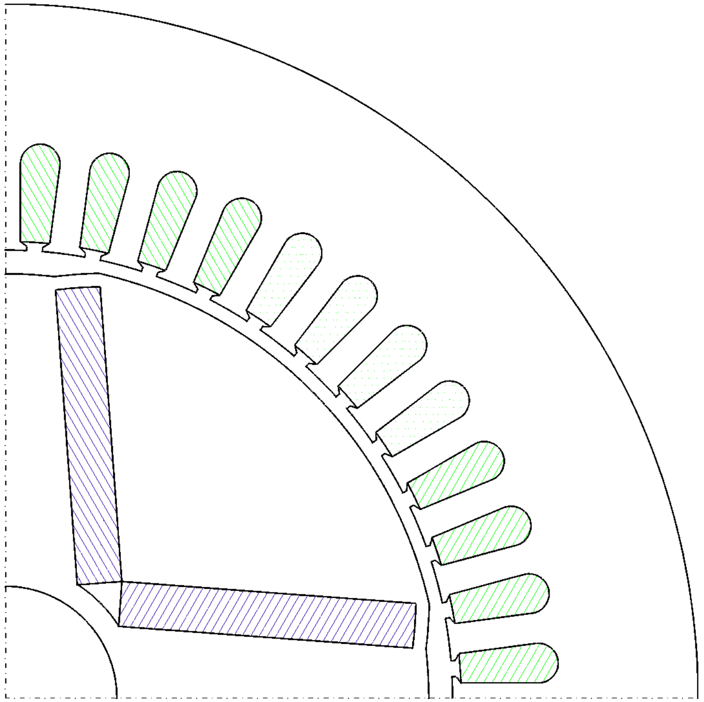
We obtained from the reluctance network that the flux-density in the air gap under the pole shoe was $B_{\mathrm{ag}} = 0.705\ \mathrm{T}$. The width of the pole shoe is approximately 130 electrical degrees. These values give for the fundamental component of the air-gap flux density

$$\hat{B}_p = \frac{1}{\pi}\int_0^{2\pi} B_r(\varphi)\sin\varphi \mathrm{d}\varphi = \frac{2}{\pi}\int_{\alpha_1}^{\alpha_2} B_{\mathrm{ag}}\sin\varphi \mathrm{d}\varphi = \frac{2B_{\mathrm{ag}}}{\pi}\left(\cos\alpha_1 - \cos\alpha_2\right)$$

$$= 0.814\ \mathrm{T}$$

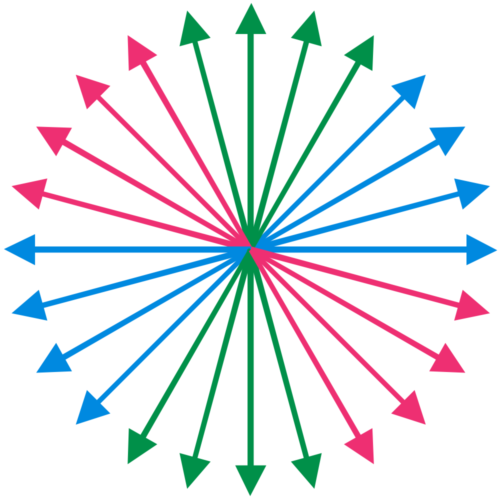


Excitation voltage u_{pm} II



The original stator winding has two pole pairs (p = 2), four slots per phase and pole (q = 4), twelve turns in each slot (N_s = 12), and let us first assume that all the turns are connected in series.

Constructing the slot-star, we obtain for the winding factor k and induced phase voltage U_{ph} (rms value)



$$k = \frac{1}{4}\left|1 + \mathrm{e}^{\mathrm{j}\pi/12} + \mathrm{e}^{\mathrm{j}\pi 2/12} + \mathrm{e}^{\mathrm{j}\pi 3/12}\right| \approx 0.958$$

$$U_{\mathrm{ph}} = p\left(2qN_{\mathrm{s}}\right)k\left(\frac{\omega}{p}\frac{D_{\mathrm{st1}}}{2}lB\right)$$

$$= 2\cdot\left(2\cdot 4\cdot 12\right)\cdot 0.958\cdot\left(\frac{2\pi\cdot 50}{2}\cdot\frac{0.2}{2}\cdot 0.246\cdot\frac{0.814}{\sqrt{2}}\right)\mathrm{V} = 409\ \mathrm{V}$$

Excitation voltage u_{pm} III

An rms value of phase voltage 409 V would be quite alright if the machine were DELTA connected and supplied from a 400 V voltage source. However, there is a large third harmonic component in the air-gap flux that would induce circulating currents in a delta connected winding. It is probably better to choose the star connection.

The stator of the original induction motor is STAR connected but the two pole pairs are connected in parallel. The machine has two parallel paths ($a = 2$). If we make the same connection, the no-load phase voltage would be about 205 V and the no-load line-to-line voltage 354 V. The machine would have a power factor somewhat smaller than one and inductive.

The no-load voltage could be easily increased by increasing the number of turns. However, the finite element analysis gave a somewhat larger air-gap flux density than the reluctance network. Let us choose the original winding

Connection	STAR
Number of turns per slot	$N_{\mathrm{s}} = 12$
Number of parallel paths	$a = 2$

The excitation voltage (peak value of phase voltage) becomes $\boxed{\hat{u}_{\mathrm{pm}} = 290\ \mathrm{V}}$