Parameters of the PM machine

Assuming sinusoidal flux distribution and sinusoidal time variation, an analytical expression can be derived for the torque of the machine

$$T_e = -\frac{3}{2}\frac{p}{\omega}\left[\frac{\hat{u}_s \hat{u}_p}{X_d}\sin\delta + \frac{\hat{u}_s^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta\right]$$

To be estimated

It includes three machine parameters u_p, X_d, X_q three other parameters u_s, ω, δ, which are related to the supply voltage and loading.

As we are used to calculating the voltages of a winding from the harmonic components of the air-gap flux, we shall,

1. calculate the voltages
2. and then extract the reactances from the voltages.

Calculating machine parameters

To find machine parameters u_p, X_d, X_q, we shall calculate

1. voltages induced by the flux of the permanent magnets (u_p)
2. Voltages induced by the flux of a current on d-axis $u_d = X_d i_d$
3. Voltages induced by the flux of a current on q-axis $u_q = X_d i_d$

1. Calculate voltages induced by PM (u_p)

At No load, $I_\mathrm{st} = 0$

This assumption is ok, since the magnets generate a main part of the flux. Find u_p, and flux densities in pole shoe, stator yoke and tooth.

2,3. Calculate voltages u_d, u_q
we freeze the permeabilities of material based on the no-load PM flux solved previously.

Reminder.. Calculate voltages induced by PM - 1

No load, $I_{st} = 0$

$$\begin{cases} \Psi_{pm} = \dfrac{\left(R_{ag} + R_{st} + R_{sy} + R_{rb}\right)}{\left[\left(R_{ag} + R_{st} + R_{sy} + R_{rb}\right)\left(R_{pm} + R_{ps} + R_{rb}\right) - R_{rb}R_{rb}\right]} \dfrac{B_r d_{pm}}{\mu_{pm}} \\ \Psi_{ag} = \dfrac{R_{rb}}{\left[\left(R_{ag} + R_{st} + R_{sy} + R_{rb}\right)\left(R_{pm} + R_{ps} + R_{rb}\right) - R_{rb}R_{rb}\right]} \dfrac{B_r d_{pm}}{\mu_{pm}} \end{cases}$$

$$\begin{cases} \Psi_{pm} = \Psi_{ps} \\ \Psi_{ag} = \Psi_{st} = \Psi_{sy} \\ \Psi_{rb} = \Psi_{pm} - \Psi_{ag} \end{cases}$$

Reminder.. Calculate voltages induced by PM - 2

No load, with PM flux,

- $\hat{B}_p = \frac{1}{\pi}\int_0^{2\pi} B_r(\varphi)\sin\varphi \mathrm{d}\varphi = \frac{2}{\pi}\int_{\alpha_1}^{\alpha_2} B_{\mathrm{ag}} \sin\varphi \mathrm{d}\varphi = 0.814\ \mathrm{T}$
- $k = \frac{1}{4}\left|1 + \mathrm{e}^{\mathrm{j}\pi/12} + \mathrm{e}^{\mathrm{j}\pi 2/12} + \mathrm{e}^{\mathrm{j}\pi 3/12}\right| \approx 0.958$
- $\hat{u}_{\mathrm{ph}} = p(2qN_{\mathrm{s}})k\left(\frac{\omega}{p}\frac{D_{\mathrm{st1}}}{2}l\right)\hat{B} = 2\cdot(2\cdot 4\cdot 12)\cdot 0.958\cdot\left(\frac{2\pi\cdot 50}{2}\cdot\frac{0.2}{2}\cdot 0.246\right)\hat{B}$

$$\approx 711\ \hat{B}\left[\frac{\mathrm{V}}{\mathrm{T}}\right] = 711\left[\frac{\mathrm{V}}{\mathrm{T}}\right]\cdot 0.814[\mathrm{T}]$$

$u_{\mathrm{ph}} = 409\ \mathrm{V}$

Air-gap flux density

$\boxed{\hat{u}_{\mathrm{pm}} = \hat{u}_{\mathrm{ph}}}$

Induced stator phase voltage (r.m.s)

Excitation voltage

Excitation voltage u_{pm}

This rms value of phase voltage 409 V would be quite alright if the machine was DELTA connected and supplied from a 400 V voltage source. However, there is a large 3rd harmonic component in the air-gap flux that would induce circulating currents in a delta connected winding. It is probably better to choose the star connection.

The stator of the original induction motor is STAR connected but the two pole pairs are connected in parallel. So, the machine has two parallel paths ($a = 2$). In parallel connection, the phase voltage is half of that calculated in series. So for STAR connected stator with $a = 2$, no-load phase voltage = 409/2= 205 V and the no-load line-to-line voltage 354 V. The machine would have a power factor somewhat smaller than one and inductive.

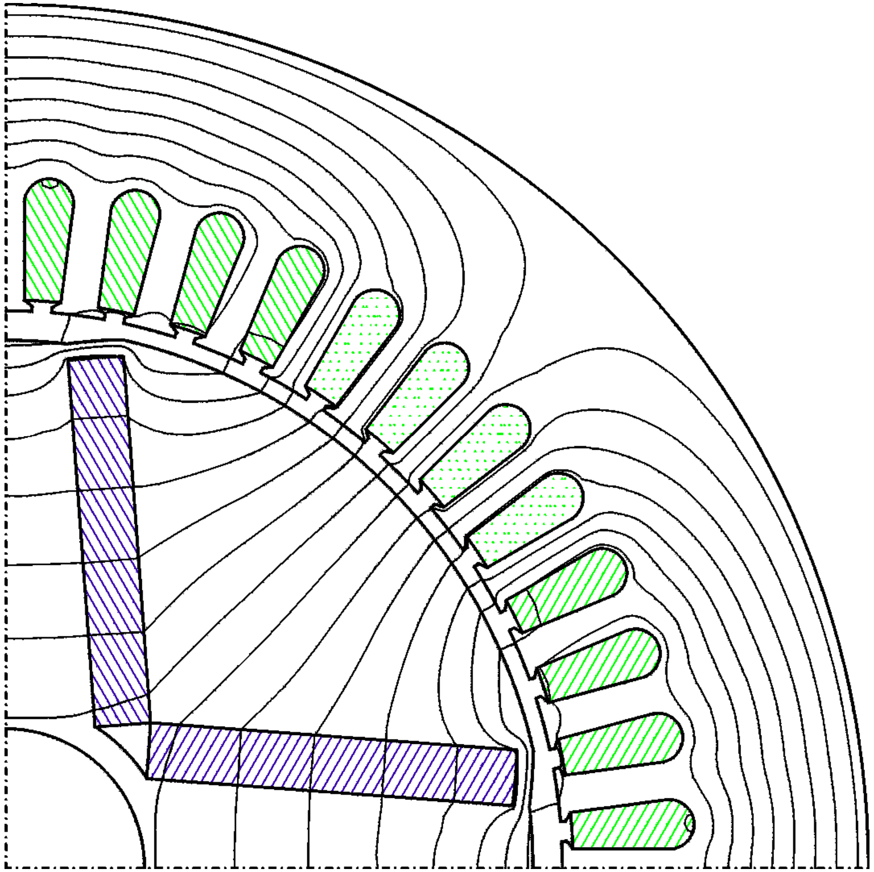
It is possible to easily increase the no-load voltage by increasing the number of turns. However, with this, the finite element analysis gave a somewhat larger air-gap flux density than the reluctance network. Let us stick with the original winding

Connection	STAR
Number of turns per slot	$N_s = 12$
Number of parallel paths	$a = 2$

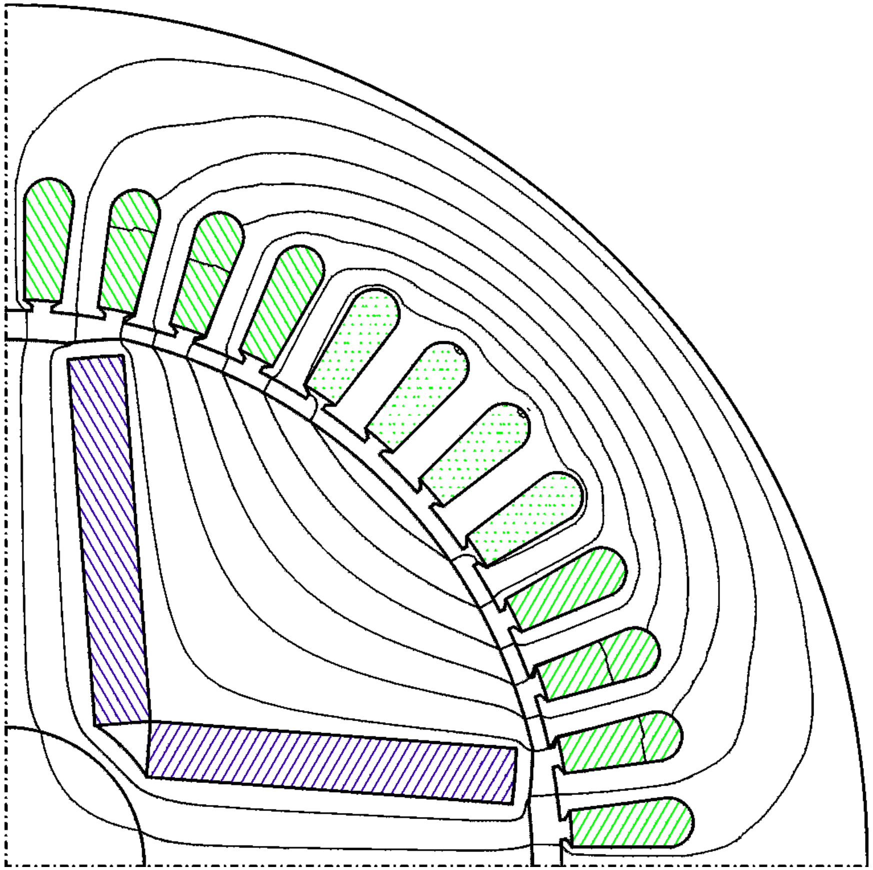
The excitation voltage (peak value of phase voltage) $\boxed{\hat{u}_{pm} = 205 \cdot \sqrt{2} = 290 \text{ V}}$

Calculate d-and q-axes voltages:
Reactances on the d- and q-axes

In FEM, when the flux components below were computed for the d- and q-axis, the permeability was freezed based on the no-load flux density and the remanence B_r of the magnets was set to zero.



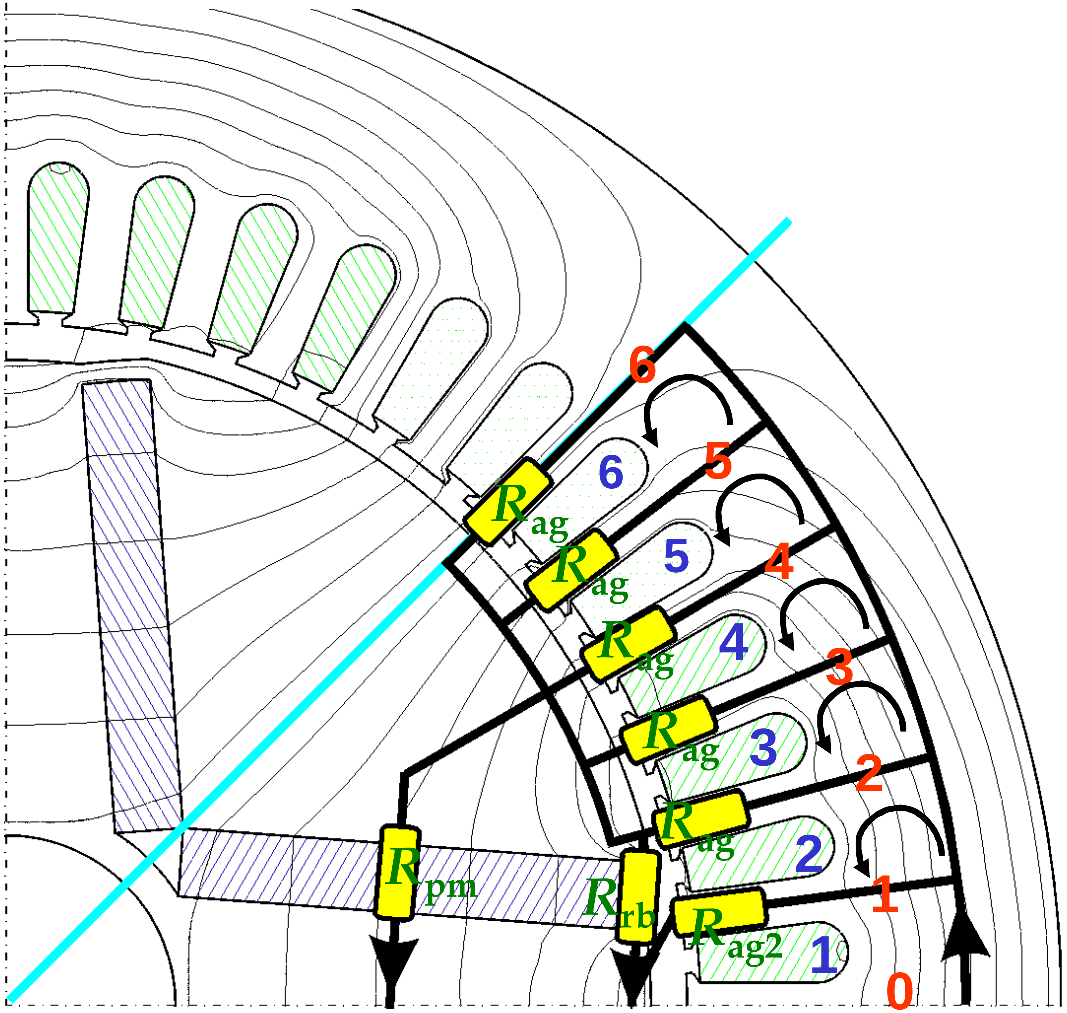
Stator current on d-axis



Stator current on q-axis

Flux on d-axis

The reluctance network shown is used to solve the relation between the flux and current on the d-axis. Here, we assume loaded condition. Flux is produced by a stator current which is distributed in the slots and the MMF over the air gap varies from tooth to tooth.



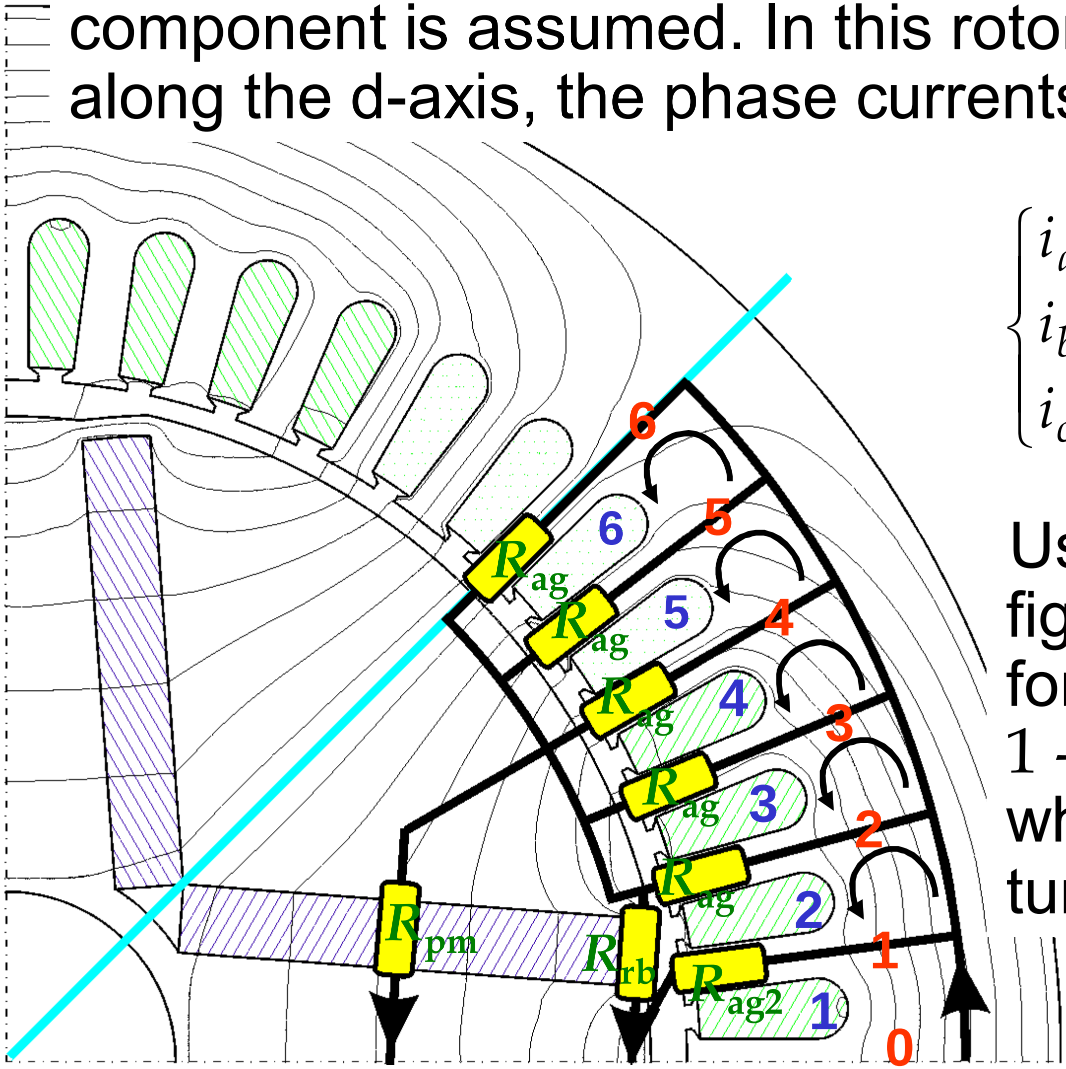
The permeability should be frozen based on the common flux produced by the stator and rotor.

In this machine, there is rotor and stator excitation.

As the magnets generate a main part of the flux, we freeze the permeability based on the no-load PM flux solved previously.

Flux on d-axis II

The order of the phases in the stator slots from the right bottom to the left upper corner is $a, -c, b$. Four slots belong to a same phase. A balanced three-phase current without zero-component is assumed. In this rotor position, to direct the flux along the d-axis, the phase currents should be



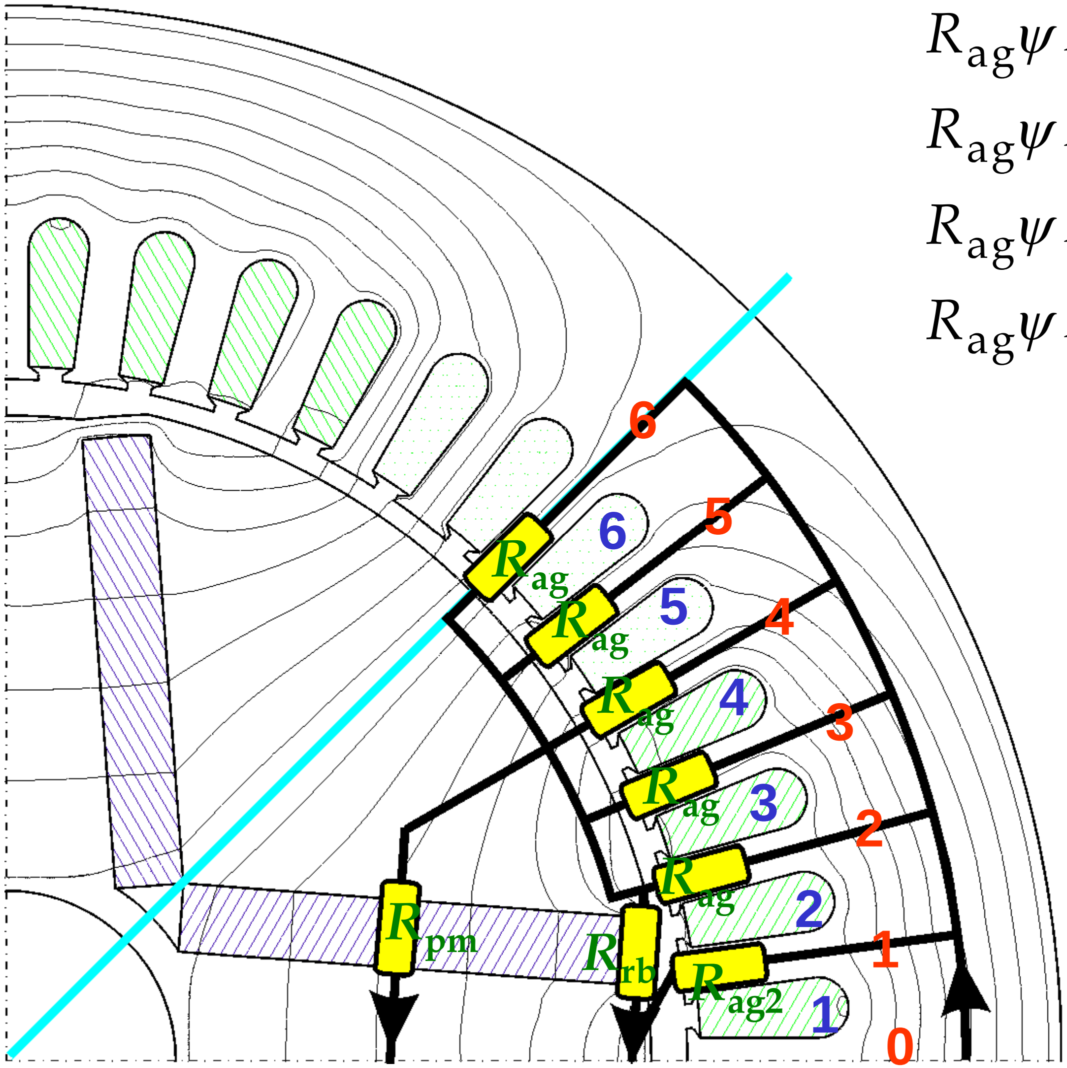
$$\begin{cases} i_a = i \\ i_b = -i \\ i_c = 0 \end{cases} \quad \text{as} \quad i_a + i_b + i_c = 0$$

Using the slot indices of the figure, the magnetomotive forces produced by the slots 1 – 6 are $N_s i$, $N_s i$, $N_s i$, $N_s i$, 0, 0, where N_s is the number of turns in a slot.

Flux on d-axis

Loop equations around slots 1, 3, 4, 5 and 6

$$R_{ag2}\psi_{t1} = N_s i$$
$$R_{ag}\psi_{t3} - R_{ag}\psi_{t2} = N_s i$$
$$R_{ag}\psi_{t4} - R_{ag}\psi_{t3} = N_s i$$
$$R_{ag}\psi_{t5} - R_{ag}\psi_{t4} = 0$$
$$R_{ag}\psi_{t6} - R_{ag}\psi_{t5} = 0$$



=>

$$\psi_{t1} = N_s i / R_{ag2}$$
$$\psi_{t2} = \psi_{t4} - 2N_s i / R_{ag}$$
$$\psi_{t3} = \psi_{t4} - N_s i / R_{ag}$$
$$\psi_{t4} = \psi_{t5} = \psi_{t6}$$

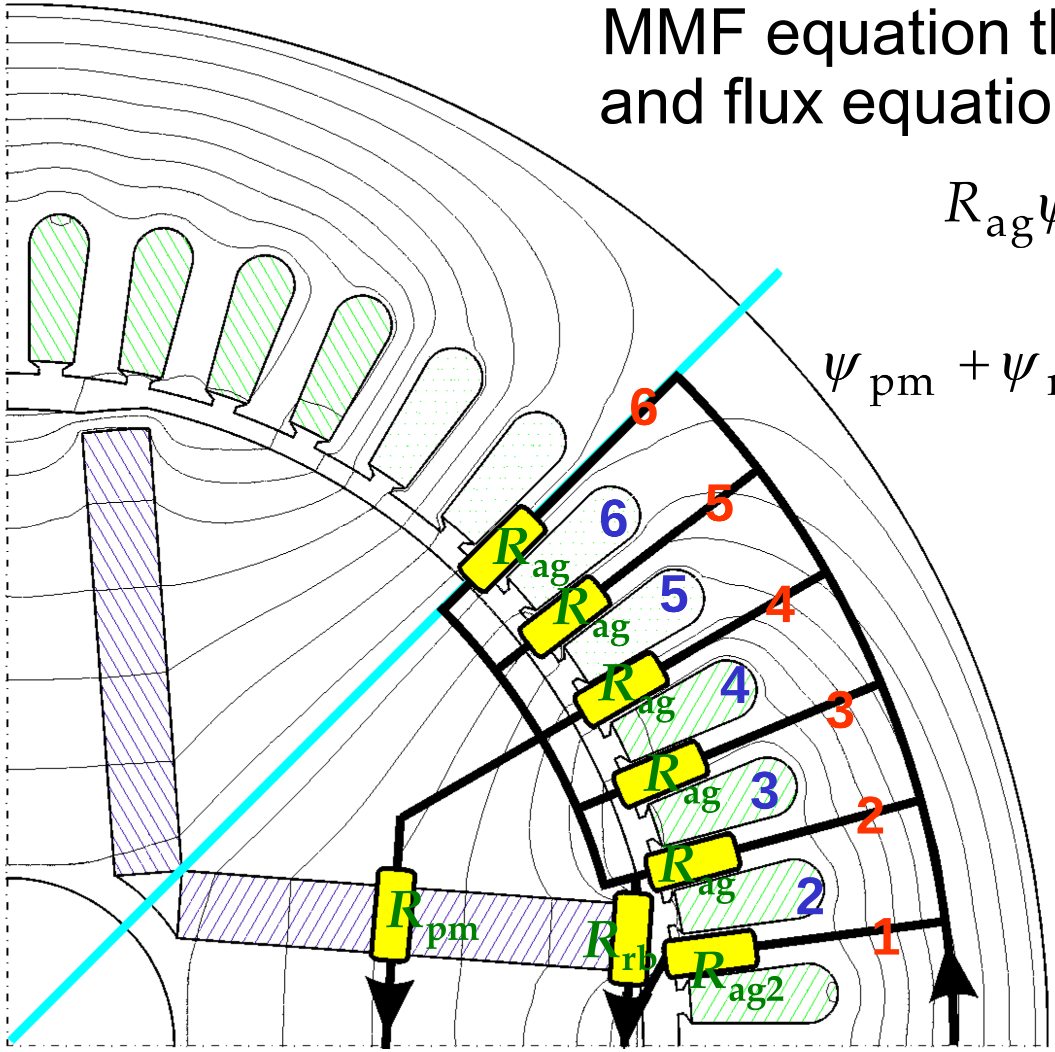
Flux on d-axis III

Reluctance of the rotor $$R_{rt} = \frac{R_{pm} R_{rb}}{R_{pm} + R_{rb}}$$

MMF equation through tooth 4 to the rotor, and flux equation

$$R_{ag}\psi_{t4} + R_{rt}\underbrace{\left(\psi_{pm} + \psi_{rb}\right)} = 4N_s i$$

$$\psi_{pm} + \psi_{rb} = \psi_{t2} + \psi_{t3} + \psi_{t4} + \psi_{t5} + \tfrac{1}{2}\psi_{t6}$$



Flux on d-axis IV

Combining the last two equations gives

$$R_{ag}\psi_{t4} + R_{rt}\left(\tfrac{1}{2}\psi_{t4} + \psi_{t4} + \psi_{t4} + \psi_{t4} - {}^{N_s i}\!\big/_{R_{ag}} + \psi_{t4} - {}^{2N_s i}\!\big/_{R_{ag}}\right)$$
$$= 4N_s i$$

=> $$\left(R_{ag} + 4.5R_{rt}\right)\psi_{t4} = \left(4 + 3\frac{R_{rt}}{R_{ag}}\right)N_s i$$

=> $$\psi_{t4} = \frac{4R_{ag} + 3R_{rt}}{R_{ag}\left(R_{ag} + 4.5R_{rt}\right)}N_s i$$

Coefficients for the tooth and rotor fluxes (in units T/A)

Ψ0	Ψ1	Ψ2	Ψ3	Ψ4	Ψ5	Ψ6	Ψpm	Ψrb
0.00E+00	7.67E-07	-3.07E-07	9.54E-07	2.22E-06	2.22E-06	2.22E-06	5.40E-06	1.90E-06

Flux on d-axis V

As we know how to calculate the voltage induced in a phase winding by a fundamental field of the air gap, we shall calculate the flux linkage from this voltage. The fundamental air-gap flux density must be integrated numerically from the tooth fluxes, for instance, by using the trapezoidal rule. In the spatial polar coordinates,

$$\hat{B}_p = \frac{1}{\pi}\int_0^{2\pi} B_r(\varphi)\sin p\varphi \,\mathrm{d}\varphi = \frac{8}{\pi}\sum_{n=0}^{6}\frac{2\pi k_n}{Q_\mathrm{s}}\frac{\Psi_{\mathrm{t}n}}{\tau_\mathrm{s} l}\sin\left(np\alpha_\mathrm{s}\right)$$

where k_n is $0{,}5$ for the first and last tooth, and 1 otherwise. The numerical values are presented on the worksheet on the next slide. The fundamental air-gap flux density is

$$\boxed{\hat{B}_p = 0.631\, N_\mathrm{s} i \ {}^{\mathrm{mT}}\!/_{\mathrm{A}}}$$

Flux on d-axis VI

Integration of the fundamental harmonic

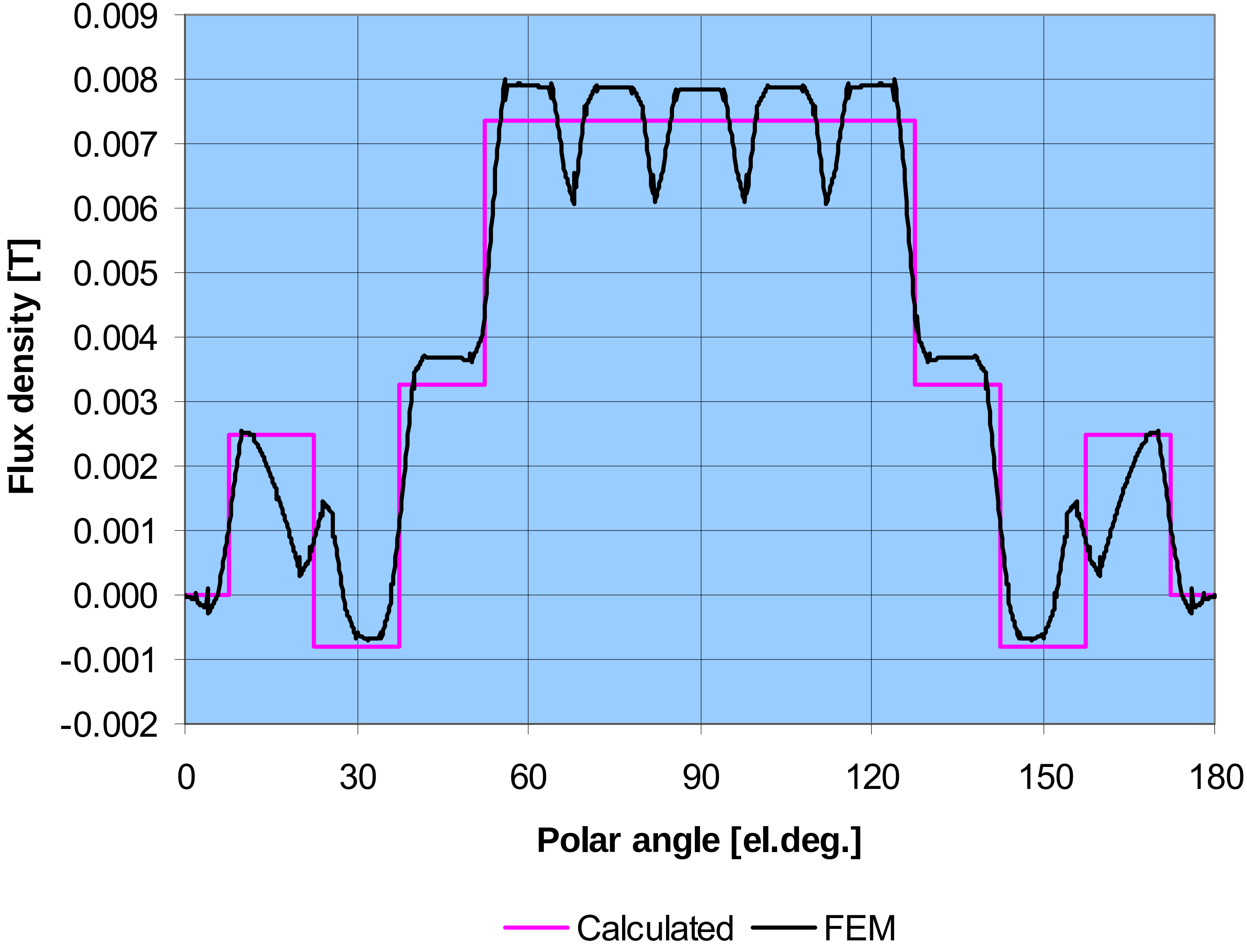
Br	1.1
μ0	1.26E-06
Length l	0.2460
Ds1	0.3100
Ds2	0.2000
bs1	0.0035
bs2	0.0065
bs3	0.0088
hs	0.0239
hs1	0.0010
hs3	0.0175
Dr1	0.1940
Dr2	0.0500
br1	0.1000
br2	0.0660
hm	0.0100
hr2	0.0030
rp	0.0980
γ	0.189
τ	0.013
kC	1.053

	B est	μ	d	A	R
PM	0.80	1.37E-06	0.0100	0.01624	449659
air gap	0.80	1.26E-06	0.0032	0.00317	792765
rotor bridge	2.10	1.16E-05	0.0120	0.00074	1395836
Rotor					340099
air-gap 2		1.26E-06	0.0052	0.00317	1304615

Analytical

Ψ0	Ψ1	Ψ2	Ψ3	Ψ4	Ψ5	Ψ6
0.00E+00	7.67E-07	-2.47E-07	1.01E-06	2.28E-06	2.28E-06	2.28E-06
Ψpm	Ψrb					
5.74E-06	1.85E-06					
Integration						
0	1	2	3	4	5	6
0.00E+00	6.16E-05	-3.84E-05	2.23E-04	6.12E-04	6.83E-04	7.07E-04
					B =	**6.31E-04**

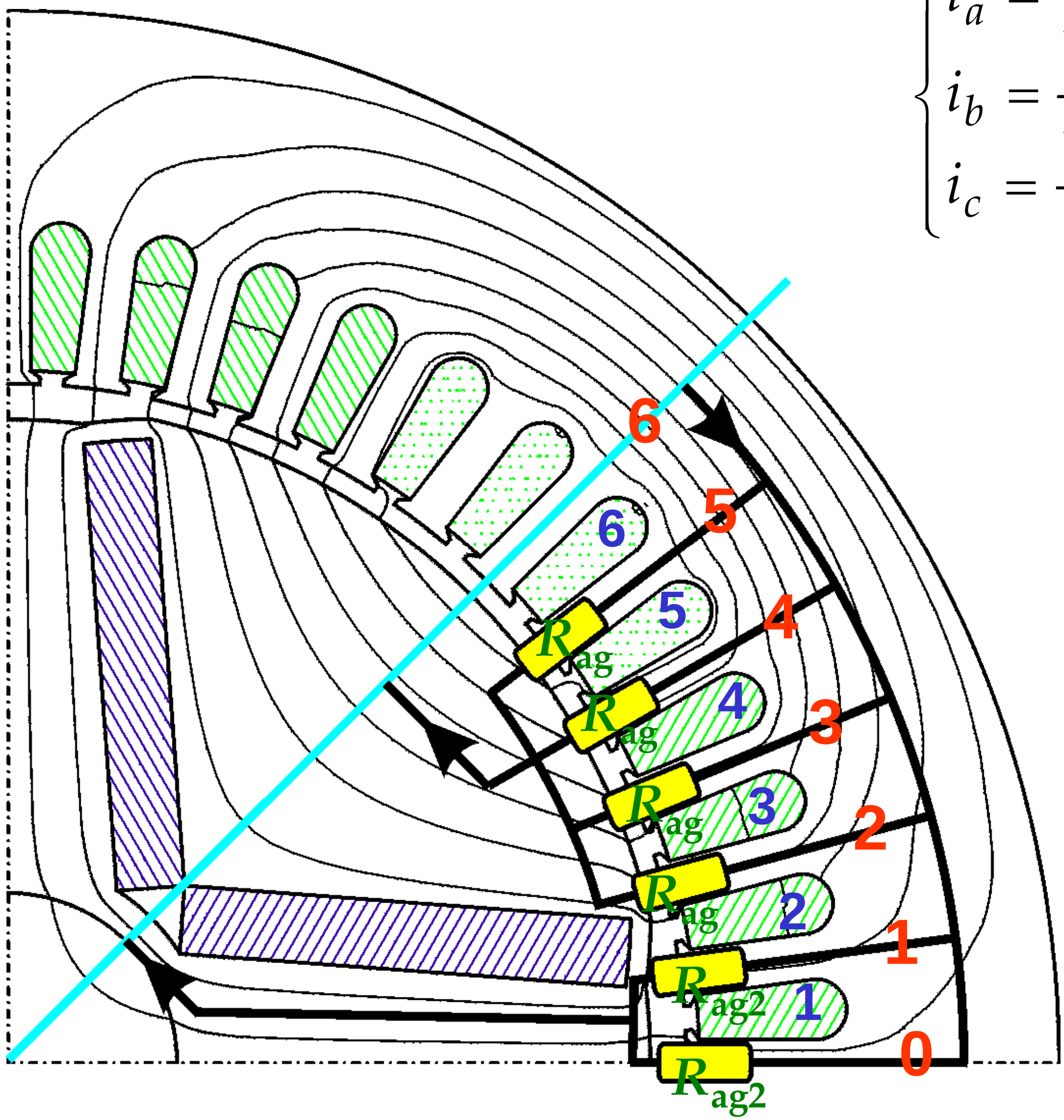
Flux on d-axis VII



Flux on q-axis

For a balanced system, to focus the flux on the q-axis, the phase currents should be

$$\begin{cases} i_a = \frac{1}{2} i \\ i_b = \frac{1}{2} i \\ i_c = -i \end{cases} \quad \text{as} \quad i_a + i_b + i_c = 0$$



From tooth 0 to tooth 6, the magnetomotive forces driving the flux over the air-gap are $4N_s i$, $3{,}5N_s i$, $3N_s i$, $2{,}5N_s i$, $2N_s i$, $N_s i$, 0, where N_s is the number of turns in a slot.

Flux on q-axis II

The tooth fluxes from the middle to the right are

$$\Psi_{\mathrm{t0}} = \frac{4N_{\mathrm{s}}i}{R_{\mathrm{ag2}}}; \quad \Psi_{\mathrm{t1}} = \frac{4N_{\mathrm{s}}i}{R_{\mathrm{ag2}}}; \quad \Psi_{\mathrm{t2}} = \frac{3,5N_{\mathrm{s}}i}{R_{\mathrm{ag}}}; \quad \Psi_{\mathrm{t3}} = \frac{2,5N_{\mathrm{s}}i}{R_{\mathrm{ag}}};$$

$$\Psi_{\mathrm{t4}} = \frac{2N_{\mathrm{s}}i}{R_{\mathrm{ag}}}; \quad \Psi_{\mathrm{t5}} = \frac{N_{\mathrm{s}}i}{R_{\mathrm{ag}}}; \quad \Psi_{\mathrm{t6}} = 0$$

The fundamental air-gap flux density must be integrated numerically from the tooth fluxes, for instance, using the trapezoidal rule. In the spatial polar coordinates

$$\hat{B}_p = \frac{1}{\pi}\int_0^{2\pi} B_r(\varphi)\cos p\varphi\,\mathrm{d}\varphi = \frac{8}{\pi}\sum_{n=0}^{6}\frac{2\pi k_n}{Q_{\mathrm{s}}}\frac{\Psi_{\mathrm{t}n}}{\tau_{\mathrm{s}}l}\cos(np\alpha_{\mathrm{s}})$$

where k_n is 0.5 for the first and last tooth, and 1 otherwise. The fundamental flux density is

$$\boxed{\hat{B}_p = 1.16\,N_{\mathrm{s}}i\ \mathrm{mT}/\mathrm{A}}$$

Flux on q-axis III

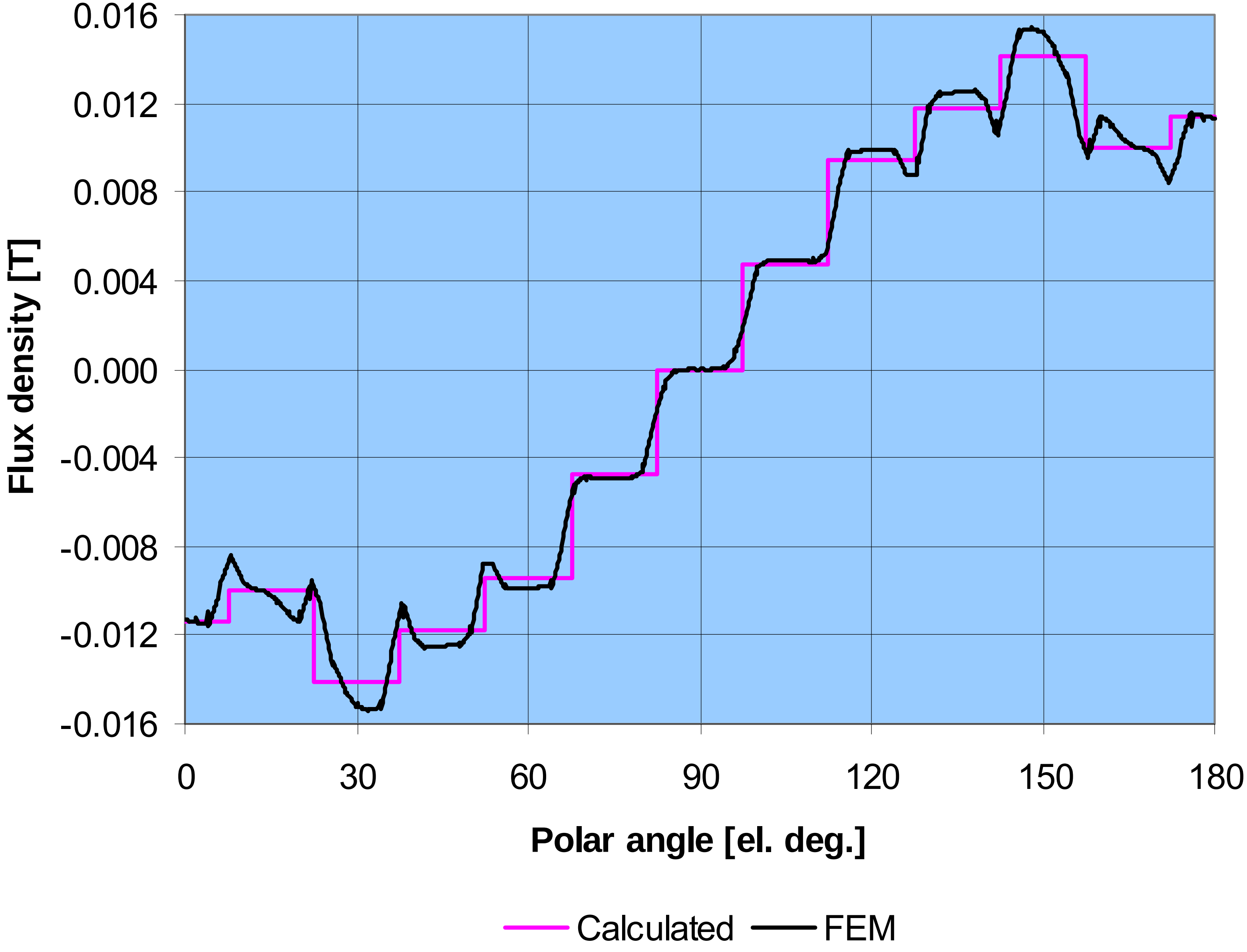
Integration of the fundamental harmonic

Br	1.1
μ0	1.26E-06
Length l	0.2460
Ds1	0.3100
Ds2	0.2000
bs1	0.0035
bs2	0.0065
bs3	0.0088
hs	0.0239
hs1	0.0010
hs3	0.0175
Dr1	0.1940
Dr2	0.0500
br1	0.1000
br2	0.0660
hm	0.0100
hr2	0.0030
rp	0.0980
γ	0.189
τ	0.013
kC	1.053

	B est	μ	d	A	R
PM	0.80	1.26E-06	0.0100	0.01624	490129
air gap	0.80	1.26E-06	0.0032	0.00317	792765
rotor bridge	2.10	1.16E-05	0.0120	0.00074	1395836
Rotor					362753
air-gap 2		1.26E-06	0.0052	0.00317	1304615

Ψ0	Ψ1	Ψ2	Ψ3	Ψ4	Ψ5	Ψ6
3.07E-06	2.68E-06	3.78E-06	3.15E-06	2.52E-06	1.26E-06	0.00E+00
Integration						
0	1	2	3	4	5	6
9.52E-04	8.05E-04	1.02E-03	6.92E-04	3.92E-04	1.01E-04	0.00E+00
					B =	**1.16E-03**

Flux on q-axis IV



Stator reactances of the d- and q-axis

To produce the flux on the d-axis, we used currents

$$\begin{cases} i_a = i \\ i_b = -i \\ i_c = 0 \end{cases}$$

or as a space vector

$$\underline{i}_\mathrm{s} = \frac{2}{3}\left(i_\mathrm{a} + \underline{a}i_\mathrm{b} + \underline{a}^2 i_\mathrm{c}\right) = \frac{2}{3}\left(i - \underline{a}i\right)$$

$$= \frac{2}{3}\left[1 - \left(-\frac{1}{2} + \mathrm{j}\frac{\sqrt{3}}{2}\right)\right]i = \left(1 - \mathrm{j}\frac{\sqrt{3}}{3}\right)i$$

$$\Rightarrow \left|\underline{i}_\mathrm{s}\right| \approx 1.155i$$

For the q-axis, the currents were

$$\begin{cases} i_a = \frac{1}{2}i \\ i_b = \frac{1}{2}i \\ i_c = -i \end{cases}$$

$$\underline{i}_\mathrm{s} = \frac{2}{3}\left(i_\mathrm{a} + \underline{a}i_\mathrm{b} + \underline{a}^2 i_\mathrm{c}\right) = \frac{2}{3}\left(\frac{1}{2}i + \underline{a}\frac{1}{2}i - \underline{a}^2 i\right)$$

$$= \frac{2}{3}\left[\frac{1}{2} + \frac{1}{2}\left(-\frac{1}{2} + \mathrm{j}\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2} - \mathrm{j}\frac{\sqrt{3}}{2}\right)\right]i = \frac{1}{2}\left(1 + \mathrm{j}\sqrt{3}\right)i$$

$$\Rightarrow \left|\underline{i}_\mathrm{s}\right| = i$$

Thus, we used a somewhat larger space-vector current on the d-axis than on the q-axis. This has to be kept in mind.

Reactances of the d and q-axis II

We have already derived equation

$$\hat{u}_{\mathrm{ph}} = p(2qN_{\mathrm{s}})k\left(\frac{\omega}{p}\frac{D_{\mathrm{st1}}}{2}l\right)\hat{B} = 2\cdot(2\cdot 4\cdot 12)\cdot 0.958\cdot\left(\frac{2\pi\cdot 50}{2}\cdot\frac{0.2}{2}\cdot 0.246\right)\hat{B}\left[\frac{\mathrm{V}}{\mathrm{T}}\right]$$

$$\approx 711\ \hat{B}\left[\frac{\mathrm{V}}{\mathrm{T}}\right]$$

which gives the relation between the peak values of the air-gap flux density and stator phase voltage. The peak value of a phase voltage is also the amplitude of voltage space vector.

Substituting the equations of the flux densities for the d- and q-axes gives

$$\hat{u}_{\mathrm{sd}} = 711\ \hat{B} = 711\cdot 0.631\cdot 10^{-3}\ N_{\mathrm{s}}i = \frac{711\cdot 0.631\cdot 10^{-3}\cdot 12}{1.155}\hat{i}_{\mathrm{s}} = 4.66\ \hat{i}_{\mathrm{s}}\left[\frac{\mathrm{V}}{\mathrm{A}}\right]$$

$$\hat{u}_{\mathrm{sq}} = 711\ \hat{B} = 711\cdot 1.16\cdot 10^{-3}\cdot 12\ \hat{i}_{\mathrm{s}} = 9.90\ \hat{i}_{\mathrm{s}}\left[\frac{\mathrm{V}}{\mathrm{A}}\right]$$

Reactances of the d and q-axis III

However, the equation of phase voltage was derived for a series connected winding and we already decided to use two parallel paths, i.e. to connect the two pole pairs of the machine in parallel. In this case, the current in the terminals of the machine will be twice as large as the current of a series connected winding and the voltage only half of the series connected winding. This means that the reactances have to be divided by four. We finally obtain

$$\boxed{\begin{cases} X_{\mathrm{d}} = 1.17\ \Omega \\ X_{\mathrm{q}} = 2.48\ \Omega \end{cases}}$$

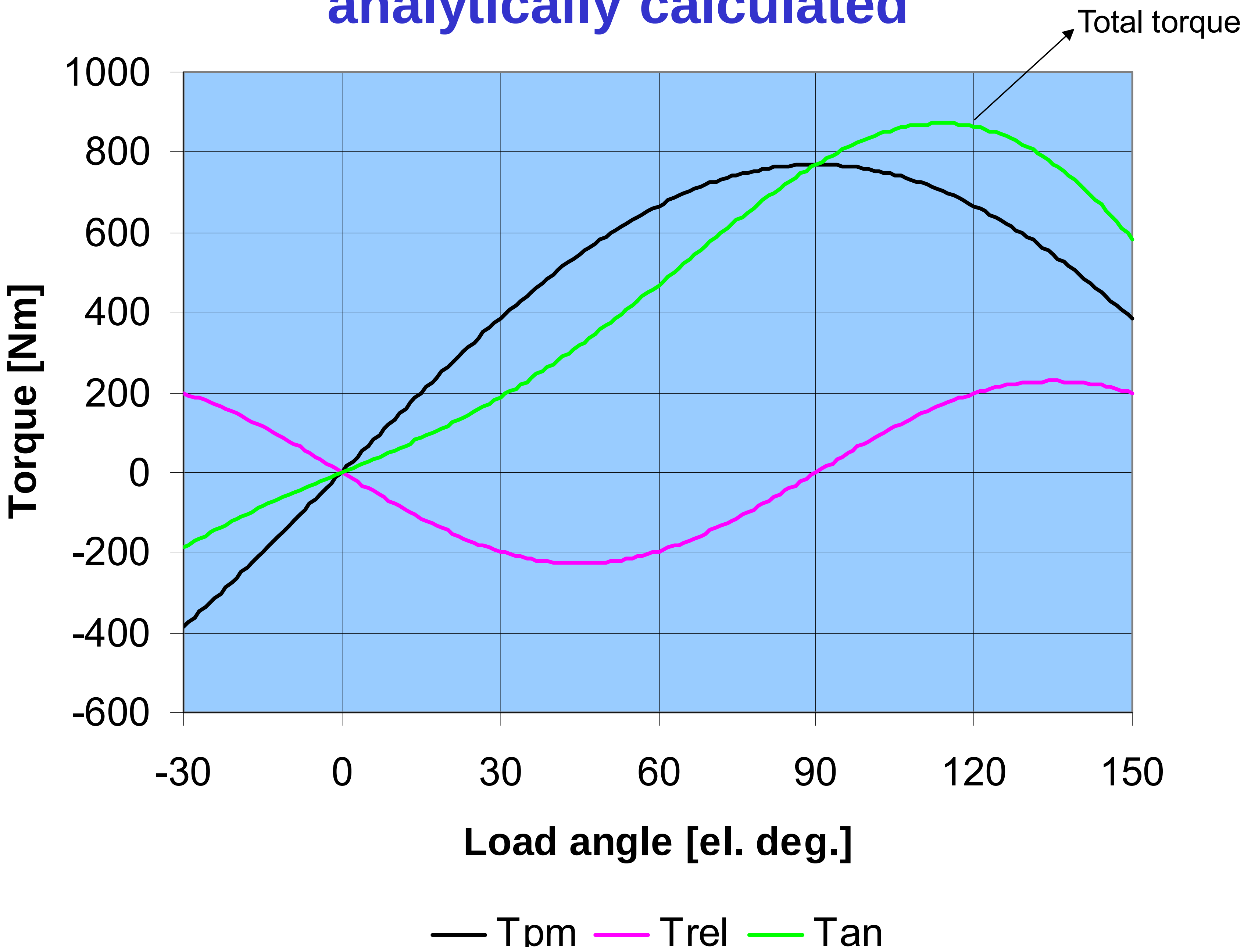
The FEM model gives

$$\begin{cases} X_{\mathrm{d}} = 1.31\ \Omega \\ X_{\mathrm{q}} = 2.54\ \Omega \end{cases}$$

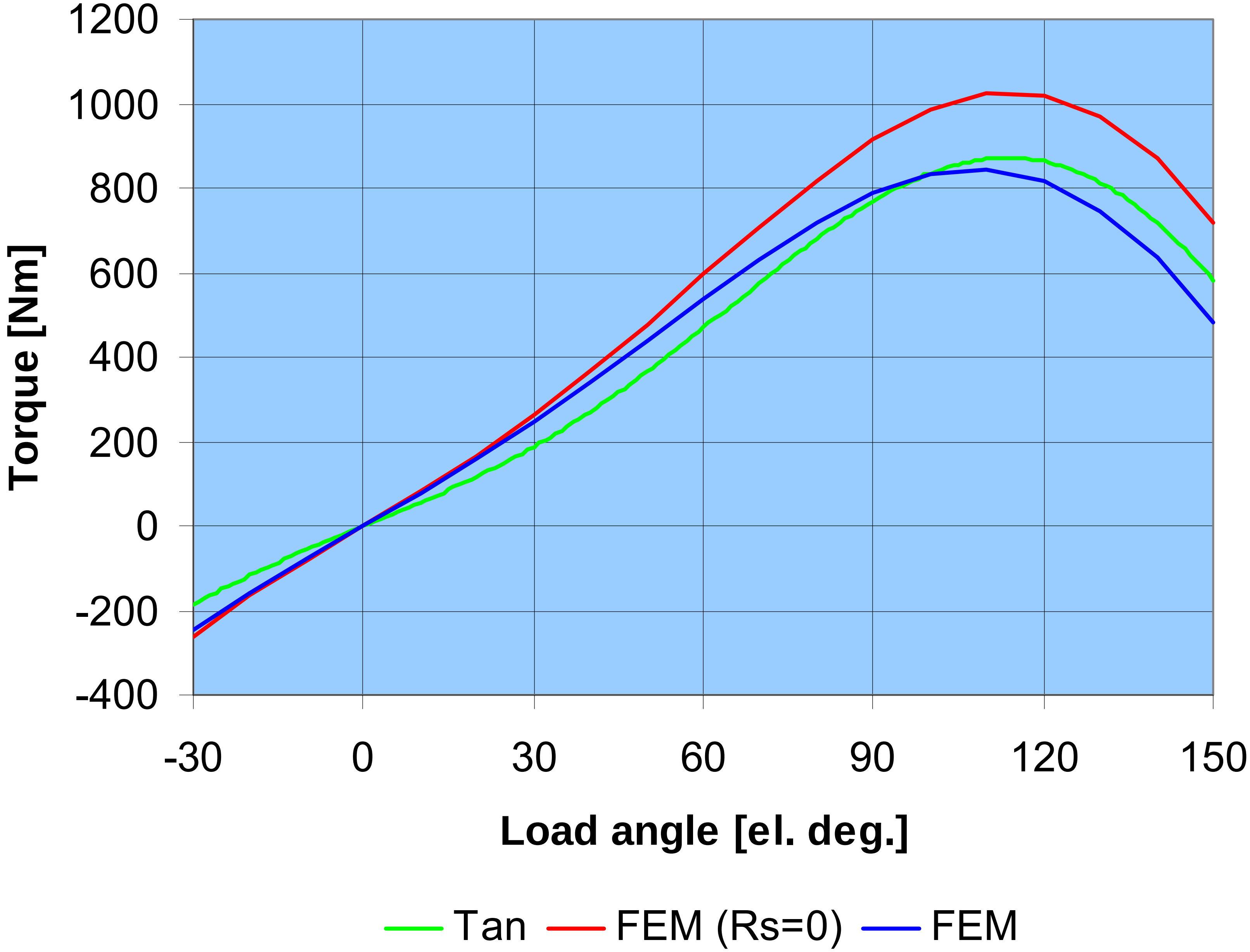
Torque equation

$$T_e = -\frac{3}{2}\frac{p}{\omega}\left[\frac{\hat{u}_s \hat{u}_p}{X_d}\sin\delta + \frac{\hat{u}_s^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta\right]$$

PM torque and reluctance torque, analytically calculated



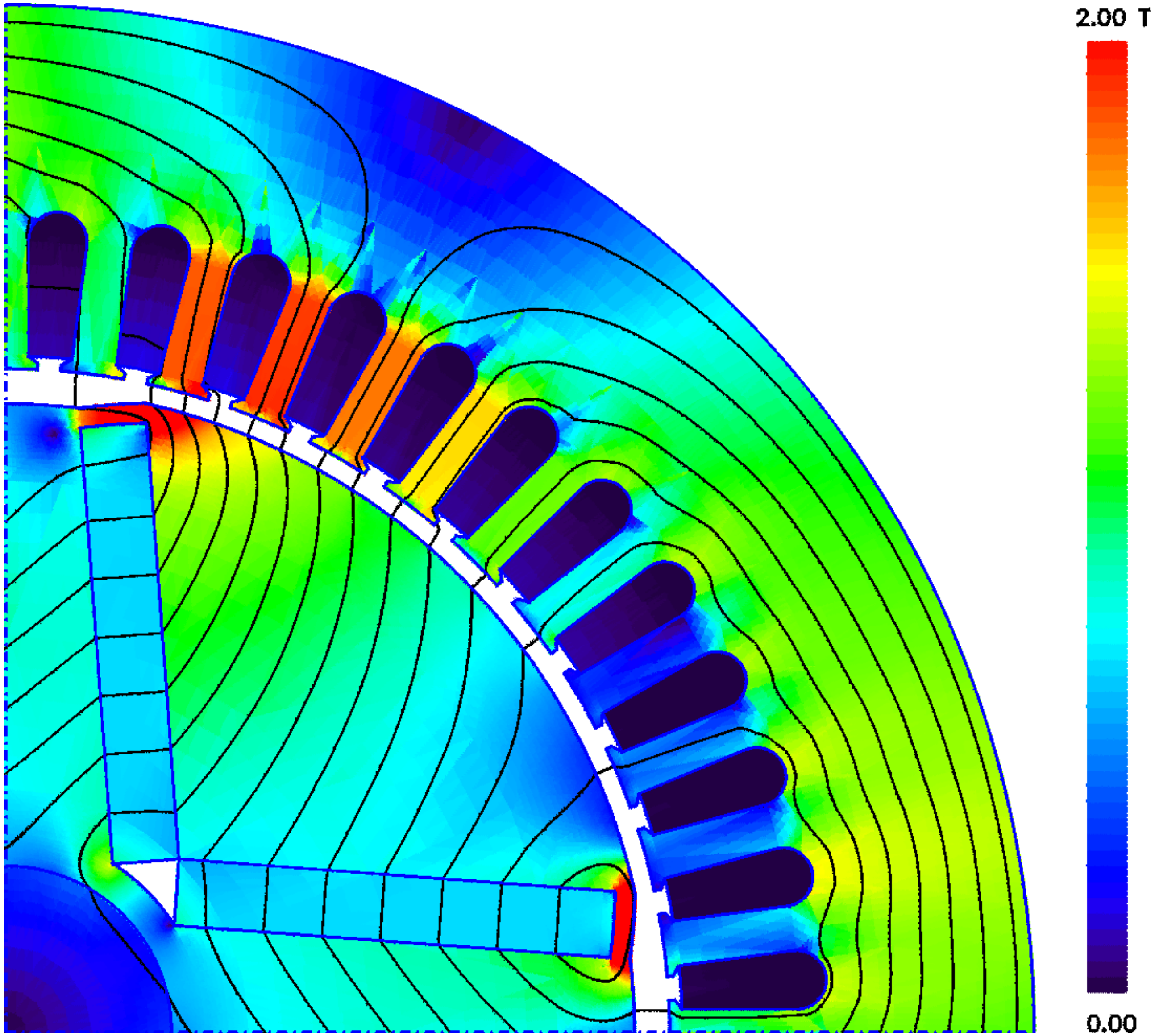
Comparison with FEM results



Armature reaction

In a loaded machine (with stator winding flux and PM flux), the d-axis component of the stator current reduces the air-gap flux somewhat. The current on q-axis decreases the flux on one side of a pole but increases it on the other side (see the figure on right).

This effect is called **armature reaction**. It affects the machine characteristics by saturating the iron core (see the teeth facing the left side of the pole).



Magnetic field of a motor loaded by the rated torque.