# Class exam: Model implied correlations

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| --- | --- | --- |
| Model | Question | Answer here |
|   | Cor(x1,y) |  |
| Var(y) |  |
|  | Cor(x,m) |  |
| Cor(x,y) |  |
| Cor(m,y) |  |
| Var(y) |  |
|  | Cor(x,y) |  |
| Cor(z,y) |  |
| Var(y) |  |

# Tracing rules

https://en.wikipedia.org/wiki/Path\_analysis\_(statistics)#Path\_tracing\_rules

Path tracing rules

In order to validly calculate the relationship between any two boxes in the diagram, Wright (1934) proposed a simple set of path tracing rules, for calculating the correlation between two variables. The correlation is equal to the sum of the contribution of all the pathways through which the two variables are connected. The strength of each of these contributing pathways is calculated as the product of the path-coefficients along that pathway.

The rules for path tracing are:

1. You can trace backward up an arrow and then forward along the next, or forwards from one variable to the other, but never forward and then back. Another way to think of this rule is that you can never pass out of one arrow head and into another arrowhead: heads-tails, or tails-heads, not heads-heads.
2. You can pass through each variable only once in a given chain of paths.
3. No more than one bi-directional arrow can be included in each path-chain.

Again, the expected correlation due to each chain traced between two variables is the product of the standardized path coefficients, and the total expected correlation between two variables is the sum of these contributing path-chains.

**NB**: Wright's rules assume a model without feedback loops: the [directed graph](https://en.wikipedia.org/wiki/Directed_graph) of the model must contain no [cycles](https://en.wikipedia.org/wiki/Cycle_%28graph_theory%29), i.e. it is a [directed acyclic graph](https://en.wikipedia.org/wiki/Directed_acyclic_graph), which has been extensively studied in the [causal analysis framework](https://en.wikipedia.org/wiki/Causality) of [Judea Pearl](https://en.wikipedia.org/wiki/Judea_Pearl).

**Path tracing in unstandardized models**[[edit](https://en.wikipedia.org/w/index.php?title=Path_analysis_(statistics)&action=edit&section=4)]

If the modeled variables have not been standardized, an additional rule allows the expected covariances to be calculated as long as no paths exist connecting dependent variables to other dependent variables.

The simplest case obtains where all residual variances are modeled explicitly. In this case, in addition to the three rules above, calculate expected covariances by:

1. Compute the product of coefficients in each route between the variables of interest, tracing backwards, changing direction at a two-headed arrow, then tracing forwards.
2. Sum over all distinct routes, where pathways are considered distinct if they contain different coefficients, or encounter those coefficients in a different order.

Where residual variances are not explicitly included, or as a more general solution, at any change of direction encountered in a route (except for at two-way arrows), include the variance of the variable at the point of change. That is, in tracing a path from a dependent variable to an independent variable, include the variance of the independent-variable except where so doing would violate rule 1 above (passing through adjacent arrowheads: i.e., when the independent variable also connects to a double-headed arrow connecting it to another independent variable). In deriving variances (which is necessary in the case where they are not modeled explicitly), the path from a dependent variable into an independent variable and back is counted once only.