K1 $\quad V = \{ (x,y,z) \mid x \geq 0,\ y \geq 0,\ z \geq 0,\ x+y+z \leq 1 \}$

(a) $$\int_V (1-x-y)^5 \, dV$$

$$= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} (1-x-y)^5 \, dz\, dy\, dx$$

$$= \int_0^1 \int_0^{1-x} (1-x-y)^6 \, dy\, dx \qquad \text{(ovclea!)}$$

$$= \int_0^1 \Big/_0^{1-x} -\frac{1}{7}(1-x-y)^7 \, dx$$

$$= \int_0^1 -\frac{1}{7}\Big((1-x-1+x)^7 - (1-x)^7 \Big) dx$$

$$= \frac{1}{7}\int_0^1 (1-x)^7 dx = -\frac{1}{56} \Big/_0^1 (1-x)^8$$

$$= \frac{1}{56}$$

(b) $\int_V x y z^4 \, dV =$

$$= \int_0^1 \int_0^{1-z} \int_0^{1-z-y} x y z^4 \, dx \, dy \, dz$$

$$= \frac{1}{2} \int_0^1 \int_0^{1-z} y z^4 \Big/_0^{1-z-y} x^2 \quad dy \, dz$$

$$= \frac{1}{2} \int_0^1 \int_0^{1-z} y z^4 \left(1 - z - y\right)^2 dy \, dz$$

Kirjoitetaan auki:

$$= \frac{1}{2} \int_0^1 z^4 \int_0^{1-z} \left(y^3 - 2(1-z)y^2 + (1-z)^2 y\right) dy \, dz$$

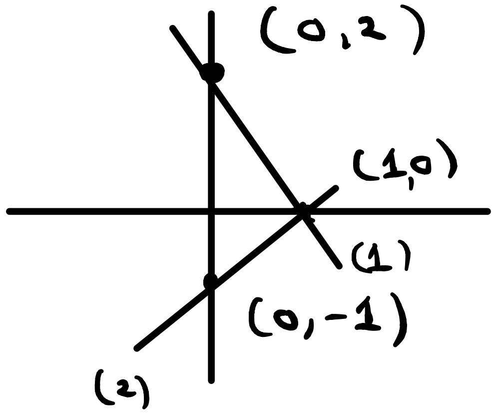
$$= \frac{1}{2} \int_0^1 z^4 \left(\frac{1}{4}(1-z)^4 - \frac{2}{3}(1-z)^4 + \frac{1}{2}(1-z)^4 \right)$$

$$= \frac{1}{24} \int_0^1 z^4 (1-z)^4 \, dz$$

$$= \frac{1}{24} \int_0^1 \left(z^8 - 4z^7 + 6z^6 - 4z^5 + z^4 \right) dz$$

$$= \frac{1}{24} \left(\frac{1}{9} - \frac{1}{2} + \frac{6}{7} - \frac{2}{3} + \frac{1}{5} \right) = \frac{1}{15120}$$

K2



(1) $y = 2 - 2x$

(2) $y = x - 1$

$$\iint_K 2x \, dA = \int_0^1 \int_{x-1}^{2-2x} 2x \, dy \, dx$$

$$= \int_0^1 2x \Big/_{x-1}^{2-2x} y \, dx$$

$$= \int_0^1 \Big(2x(2-2x) - 2x(x-1) \Big) dx$$

$$= \int_0^1 \left(6x - 6x^2 \right) dx =$$

$$= \Big/_0^1 3x^2 - 2x^3 = 1$$