K1 $\quad x^2 + y^2 = a^2 \quad ; \quad z = y \quad ; \quad$ 1. oktantti

Kanta on ympyrän sektori : $0 \le \theta \le \pi/2$

$0 \le r \le a$

$z = y = r \sin\theta \longrightarrow$ korkeus

Tilavuudelle saadaan nyt kaava :

$$V = \int_0^{\pi/2} \int_0^a (r \sin\theta)\, r \, dr \, d\theta$$

$$= \int_0^{\pi/2} \sin\theta \, d\theta \int_0^a r^2 dr = \frac{1}{3} a^3$$

K2 $\quad A = \{ (x,y) \mid |x+2y| \leq 1, \ |x-2y| \leq 2 \}$

Muuttujanvaihto:

$$\begin{cases} u = x + 2y \\ v = x - 2y \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2}(u+v) \\ y = \frac{1}{4}(u-v) \end{cases}$$

Uusi alue : $B = \{ (u,v) \mid -1 \leq u \leq 1, \ -2 \leq v \leq 2 \}$

Määritetään Jacobin determinantti:

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} \end{vmatrix} = -\frac{1}{4}$$

Integraali on siis:

$$\iint_A (x+2y)^4 (x-2y)^6 \, dx\,dy$$

$$= \iint_B u^4 v^6 \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv$$

$$= \frac{1}{4} \int_{-2}^{2} v^6 \, dv \int_{-1}^{1} u^5 \, du = \frac{1}{140} \left(2^7 - (-2)^7\right)\left(1^5 - (-1^5)\right)$$

$$= \frac{128}{35}$$