Microeconomics 4
Screening

Spring 2023

Information Economics

Information plays important role in economic decision making

- Uncertainty, info asymmetries prevalent
- Adverse Selection
- Moral Hazard

In micro 3 we developed a pretty general framework to handle these problems.

Trade

Let's think about one of our canonical economic problems.

A monopolist is selling to a consumer.

- The monopolist produces good with quality q and sets a price t. Production is costly, $c(q) = q^2$.
- The consumer has utility $\theta q - t$ for buying a product at price t.
- The consumer can always choose to not buy anything. Let's make the utility from that 0.

Trade

So our monopolist solves

$$\max t - q^2$$
$$\text{s.t. } \theta q - t \geq 0$$

So the monopolist sets $q = \theta/2$, $t = \theta^2/2$.

Does the assumption that the monopolist knows θ seem reasonable?

Adverse Selection

Prevalent in many economic problems

- Screening
- Auctions
- Bilateral Trade
- Public goods

Two natural questions:

- What sort of inefficiencies arise due to adverse selection?
- What impact does it have on a monopolist's ability to extract rents?

Mechanism Design

- Game between an uninformed principal and informed agents.
- Principal commits to mechanism (game) the agents play.
- What is the optimal mechanism wrt to some objective.

Mechanism Design - Screening

Monopolist selling to single buyer

- Buyer has utility θq for the good, $\theta \in \{1, 2\}$.
- Buyer knows θ, monopolist doesn't
- Monopolist commits to menu of (q, t): quantities and prices.
- Buyer who chooses (q, t) gets utility $\theta q - t$, monopolist gets $t - q^2$.
- Buyer outside option: 0.
- $\alpha = Pr(\theta = 2)$

What is the optimal menu?

Mechanism Design - Screening

If there was no adverse selection, recall:

Monopolist solves

$$\max \theta q - q^2$$

so produces $\theta/2$ units of the good, sells for $\theta^2/2$,

Profit: $\theta^2/4$

Screening

If the monopolist didn't know type:

Clearly offers 2 quantities, find prices that make them make sense

$$2q_2 - t_2 \geq 2q_1 - t_1 \,(IC_{2,1})$$
$$q_1 - t_1 \geq q_2 - t_2 \,(IC_{1,2})$$
$$q_1 - t_1 \geq 0 \,(IR_1)$$
$$2q_2 - t_2 \geq 0 \,(IR_2)$$

- IC: Each type chooses what they are supposed to
- IR: No one wants to walk away

Screening

Reduce constraints to

$$
\begin{aligned}
&2q_2 - t_2 = 2q_1 - t_1 \, (IC_{2,1}) \\
&q_1 - t_1 = 0 \, (IR_1)
\end{aligned}
$$

So monopolist solves

$$\max (1-\alpha)(q_1 - q_1^2) + \alpha(2q_2 - q_1 - q_2^2)$$

So $q_2 = 1$ and $q_1 = \max\{0, \frac{1-2\alpha}{2-2\alpha}\}$.

Screening

Stuff to observe:

- Firm only sells to high types if $\alpha \geq 1/2$.
- Low type: No rents
- High type: Strictly prefers buying to not when both types buy

$$2q_2 - (2q_2 - q_1) = \frac{1 - 2\alpha}{2 - 2\alpha}$$

  - Info rent
- Seller still gains from price discrimination, but gains less

Screening

We solved this problem by solving directly for price as a function of quantity

Equivalent to a direct mechanism: buyer announces type, seller commits to type contingent quantity/transfer scheme

Screening

- Buyers type now $\theta \sim F$, supp $F = [\underline{\theta}, \bar{\theta}]$.
- Seller commits to space of messages M, and allocation $(q(m), t(m))$.

Theorem (Revelation Principal)

For any mechanism $\Gamma = (M, (q, t))$ and optimal strategy σ^*_Γ there is an incentive compatible direct mechanism $\hat{\Gamma} = (\Theta, (\hat{q}, \hat{t}))$ with the same outcome as mechanism Γ.

Goal: Solve

$$\max \int_{\underline{\theta}}^{\bar{\theta}} [t(\theta) - q(\theta)^2] f(\theta) d\theta$$

$$\text{s.t. } \theta q(\theta) - t(\theta) \geq \theta q(\theta') - t(\theta') \, (IC_{\theta,\theta'})$$

$$\theta q(\theta) - t(\theta) \geq 0 \, (IR_\theta)$$

Screening

Uh oh

- We have a lot of constraints.
- Lagrange multipliers not going to be much help.
- Remember two type case
- IR for lowest type, local IC

IC constraints

IC constraints are the real problem here.

We have a lot of them, maybe that helps us?

$$\theta q(\theta) - t(\theta) \geq \theta q(\theta') - t(\theta')$$
$$\theta' q(\theta') - t(\theta') \geq \theta' q(\theta) - t(\theta)$$

Combining $IC_{\theta',\theta}$ and $IC_{\theta,\theta'}$ gives

$$q(\theta)(\theta - \theta') \geq \underbrace{\theta q(\theta) - t(\theta)}_{:=V(\theta)} - \theta' q(\theta') + t(\theta') \geq q(\theta')(\theta - \theta')$$

IC Constraints

What does this mean:

- $V(\theta)$, type θ's utility from the mechanism is Lipschitz continuous
- $q(\theta)$ must be (weakly) increasing
- Moreover, we know what it's derivative is!

$$V'(\theta) = q(\theta)\,(a.e.)$$

  (We are using $q(\theta)$ increasing here)
- So:

$$V(\theta) - V(\underline{\theta}) = \int_{\underline{\theta}}^{\theta} q(s)ds$$

  by the fundamental theorem of calculus
  (Lipschitz-ness lets us do this)

IC Constraints

So let's replace our IC constraints with

$$V(\theta) = \int_{\underline{\theta}}^{\theta} q(s)ds + V(\underline{\theta})$$

and $q(\theta)$ increasing.

Rewrite first thing+ IR to give

$$t(\theta) = \theta q(\theta) - \int_{\underline{\theta}}^{\theta} q(s)ds.$$

IC constraints

So now solve

$$\max \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta q(\theta) - \int_{\underline{\theta}}^{\theta} q(s)ds - q(\theta)^2 \right) f(\theta)d\theta$$

Problems:

1. We dropped the increasing constraint
2. Only got rid of local constraints

Optimal Menu

Changing the order of integration

$$\begin{aligned}
&\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} q(s)\, ds\, f(\theta) d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \int_{s}^{\overline{\theta}} q(s) f(\theta)\, d\theta\, ds \\
&= \int_{\underline{\theta}}^{\overline{\theta}} (1 - F(s)) q(s)\, ds
\end{aligned}$$

So problem becomes

$$\max \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta q(\theta) - \frac{1 - F(\theta)}{f(\theta)} q(\theta) - q(\theta)^2 \right) f(\theta) d\theta$$

thus

$$q(\theta) = \max\{0, \frac{1}{2}\left(\theta - \frac{1 - F(\theta)}{f(\theta)} \right)\}$$

Need $\frac{1-F(\theta)}{f(\theta)}$ decreasing

Optimal Menu

Assume θ is unif $[0, 1]$.

$$q(\theta) = \max\{0, \theta - 1/2\}$$

$$t(\theta) = \begin{cases} 0 \text{ if } \theta < 1/2 \\ \frac{1}{2}\theta^2 - 1/8 \text{ o.w.} \end{cases}$$

$$V(\theta) = \begin{cases} 0 \text{ if } \theta < 1/2 \\ \frac{1}{2}\theta^2 - \frac{1}{2}\theta + \frac{1}{8} \text{ o.w.} \end{cases}$$

Optimal Menu

What changes because of asymmetric info:

Monopolist maximizes profits as-if he faces no incomplete info but agents have different types:

$$\max \int_{\underline{\theta}}^{\overline{\theta}} \left(\left(\theta - \frac{1 - F(\theta)}{f(\theta)} \right) q(\theta) - q(\theta)^2 \right) f(\theta) d\theta$$

- Virtual type: $\theta - \frac{1-F(\theta)}{f(\theta)}$
  - Quantities sold are distorted downwards.
  - Type θ is sold the optimal quantity for their virtual type.
  - No distortion at the top
- Info rent: $V(\theta) = \int_{\underline{\theta}}^{\theta} q(s)\, ds$
  - Payoff of type θ.
  - Increase in payoff due to asymmetric info.
  - Agents need to be compensated for info
  - Increasing in type

Finishing up

Two problems:

- We only used “local constraints”
  - Clearly not a problem under here, closer constraints imply further ones
  - What general property do we need for this?
- What if virtual type is not increasing
  - Regular case: virtual type increasing.
  - Holds for some standard distributions.
  - Ironing

General Single Agent Problem

Can redo this for any convex cost function (w/ unbounded 1st derivative)

$$\theta - \frac{1 - F(\theta)}{f(\theta)} = c'(q(\theta))$$

The agent side stuff is interesting

- Trick to simplify IC constraints: applies to other problems
- How much does linearity in types matter?

Envelope Theorem

Give agents utility $u(q;\theta)$, fix incentive compatible $q(\theta)$.

- Let $V(\theta) = u(q(\theta);\theta) - t(\theta)$.
- IC constraint

$$V(\theta) - V(\theta') + (u(q(\theta');\theta') - u(q(\theta');\theta)) \geq 0$$

- Combining

$$u(q(\theta);\theta) - u(q(\theta);\theta') \geq V(\theta) - V(\theta') \geq u(q(\theta');\theta) - u(q(\theta');\theta')$$

- What do we need
  - $u(q,\theta)$ diff in θ.
  - Derivatives are uniformly bounded (lets us use FTC)
- Then

$$V(\theta) - V(\underline{\theta}) = \int_{\underline{\theta}}^{\theta} \frac{\partial u}{\partial \theta}(q(s);s)\,ds.$$

Envelope Theorem

Theorem

Assume that X is compact, and $\Theta = [\underline{\theta}, \overline{\theta}]$ and $g : X \times \Theta \to \mathbb{R}$ is differentiable with uniformly bounded derivatives. Then if $x(\theta)$ solves

$$V(\theta) := \max_{x \in X} g(x; \theta)$$

then

$$V'(\theta) = g_\theta(x(\theta), \theta) \; (a.e)$$

and furthermore

$$V(\theta) = V(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} g_\theta(x(s), s) \, ds$$

Revenue Equivalence

Theorem (Revenue Equivalence)

Fix a function $q : \theta \to Q$*. Suppose that* $\Theta = [\underline{\theta}, \overline{\theta}]$*,* $u : Q \times \Theta \to \mathbb{R}$ *is differentiable with uniformly bounded derivatives and* Q *compact. Any incentive compatible mechanism that implements* $q(\theta)$ *gives agents payoff*

$$V(\theta) = V(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} u_\theta(q(s), s)\, ds$$

and transfers must satisfy

$$t(\theta) = u(q(\theta); \theta) - V(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} u_\theta(q(s), s)\, ds$$

Incentive compatibility + thing we want to implement pin down transfers

Principal gets same payoff in any mechanism that implements $q(\theta)$, up to lowest type payoff.

IC Constraints

We've shown any incentive compatible mechanism satisfies an envelope condition

When does this envelope condition pin down IC mechanisms?

Theorem

Suppose the conditions for Rev Equivalence hold and $\frac{\partial^2 u(q,\theta)}{\partial q \partial \theta} > 0$. *Then* $(q(\theta), t(\theta))$ *is IC iff* $q(\theta)$ *is non-decreasing and*

$$t(\theta) = u(q(\theta); \theta) - V(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} u_\theta(q(s), s)\, ds$$

Revenue Equivalence

Proof:

$$
\begin{aligned}
& u(q(\theta);\theta) - t(\theta) - [u(q(\theta');\theta) - t(\theta')] \\
&= \int_{\underline{\theta}}^{\theta} u_\theta(q(s),s)\,ds - \left(u(q(\theta'),\theta) - u(q(\theta'),\theta') + \int_{\underline{\theta}}^{\theta'} u_\theta(q(s),s)\,ds \right) \\
&= \int_{\theta'}^{\theta} (u_\theta(q(s),s) - u_\theta(q(\theta'),s))\,ds \\
&= \int_{\theta'}^{\theta} \int_{q(\theta')}^{q(s)} u_{q\theta}(z,s)\,dz\,ds
\end{aligned}
$$

The last term is non-negative all θ, θ' iff $q(\theta)$ is increasing.

If this was negative, then the mechanism wouldn't be IC.

Price Discrimination

Assume θ unif $[0, 1]$

- Mussa Rosen: $c(q) = cq^2$
  - Our example!
  - Possible interpretation, q is quality of good.
  - $t'(q)$ is increasing: sell quality at a premium
- Maskin-Riley: $c(q) = cq$, $u(q, \theta) = \theta v(q)$, v concave.
  - Can solve this using our tools

$$v'(q(\theta)) = \frac{c}{2\theta - 1}$$

  - Moreover, $\theta v'(q) = t'(q)$

$$t''(q) = \frac{1}{2} v''(q) \leq 0$$

  - Quantity discounts

Indivisible Goods

Can reinterpret problem as monopolist selling 1 indivisible good, constant cost of production c.

- q is now the probability of sale. θ is value for the good.
- Monopolist solves

$$\max \int_{\underline{\theta}}^{\overline{\theta}} [t(\theta) - cq(\theta)] f(\theta) d\theta$$

IC constraints:

$$\theta q(\theta) - t(\theta) \geq \theta q(\theta') - t(\theta')$$

IR:

$$\theta q(\theta) - t(\theta) \geq 0$$

Indivisible Goods

Solving this:

$$\max \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta - \frac{1 - F(\theta)}{f(\theta)} - c \right) q(\theta) f(\theta) d\theta$$

Optimal mechanism posted price:

- Sell to everyone whose virtual type is above cost
- Price is $c + \frac{1-F(\theta^*)}{f(\theta^*)}$ where θ^* solves

$$\theta^* - \frac{1 - F(\theta^*)}{f(\theta^*)} = c$$

- Don't use randomization

Recap

Tools we've developed

- Revelation principle
- Envelope theorem to deal with IC constraints

Results:

- Can solve for optimal mechanism
- Implementation: Fixing a $q(\theta)$ pins down IC transfer scheme

Caveats

Need some structure (beyond standard):

- Utility satisfies increasing differences
- Type distribution satisfies monotone hazard rate

Some subtle restrictions

- Types are single dimensional, drawn from interval in $\mathbb{R}$.
- Utility is quasilinear