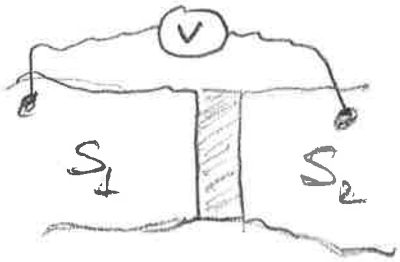
JOSEPHSON EFFECT

- What happens when we put a voltage across a <u>weak link</u> between two superconductors?

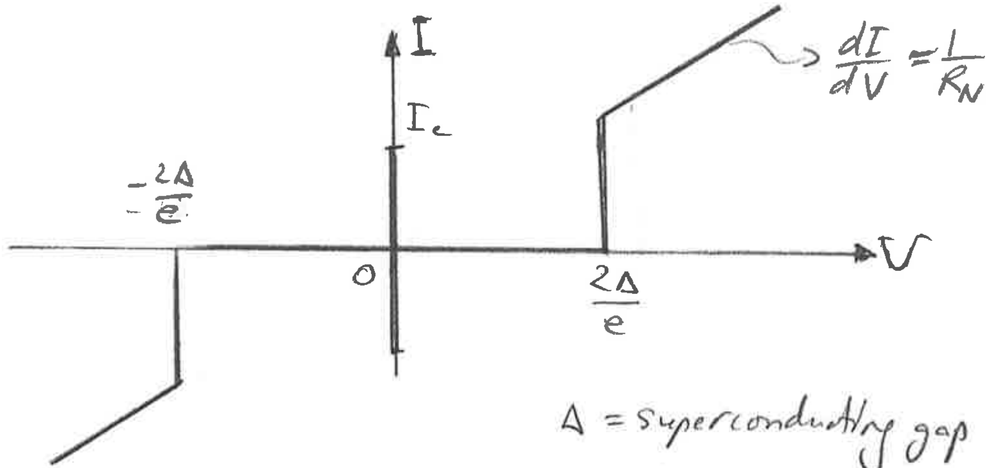
Weak link = can be a S-I-S (insulator between two superconductors)



S-N-S (a metal in-between)

S-s-S (a constriction)

- What we measure:



Δ = superconducting gap
$\Delta = 1.764\, k_B T_c$
T_c = critical temperature
(from BCS theory)

R_N = normal-state resistance

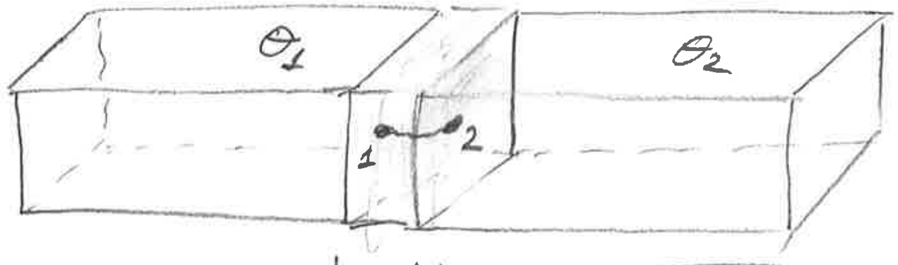
– Currents flowing for $|V| \geq \frac{2\Delta}{e}$ are no surprise – they are associated to breaking the Cooper pairs by the voltage.

But, at $V=0$ there is a current flowing, with max. value = I_c (critical current of the junction).

This is the Josephson effect.

– Circuit symbol 

The Josephson current-phase and phase-voltage relations



1] Current-phase.

Consider the gauge-invariant phase difference, obtained by integrating the gauge-invariant phase gradient,

$$\varphi = \int_1^2 d\vec{r}\left(\vec{\nabla}\theta + \frac{2\pi}{\phi_0}\vec{A}\right) = \theta_2 - \theta_1 + \frac{2\pi}{\phi_0}\int_1^2 d\vec{r}\,\vec{A}(\vec{r},t)$$

This is the only quantity which is gauge-invariant and includes the difference in phases $\theta_2 - \theta_1$ as we cross the insulator.

So perhaps $J_s = J_s(\varphi)$, that is, a function of φ.

Which one?

well, we should have also: 1) periodicity $J_s(\varphi) = J_s(\varphi + 2\pi n)$

2) $J_s(0) = 0$ (no current when there is no phase difference)

$$\Rightarrow \quad J_s(\varphi) = \underset{\substack{\text{constant,}\\ = \text{critical}\\ \text{Josephson current density.}}}{J_c} \sin\varphi + \underbrace{\sum_{m=2}^{\infty} J_m \sin(m\varphi)}_{\text{this can be neglected}}$$

Therefore, for a given device we will have

$$\boxed{I = I_c \sin\varphi}$$

I_c = critical Josephson current

2] Voltage-phase

Consider again the gauge-invariant phase difference

$$\varphi = \theta_2 - \theta_1 + \frac{2\pi}{\phi_0}\int_1^2 d\vec{r}\,\vec{A}(\vec{r},t)$$

Recall now the energy-phase relationship

$$-\hbar\frac{\partial\theta}{\partial t} = \frac{1}{2}\frac{\mu_0\lambda_L^2}{n_s}J_s^2 + q^* V$$ valid for the phases θ_1 and θ_2 inside the superconductors 1 and 2

$$\Rightarrow \frac{\partial\varphi}{\partial t} = -\frac{1}{\hbar}\frac{\mu_0\lambda_L^2}{2n_s}\left(J_s^2(2) - J_s^2(1)\right) - \frac{q^*}{\hbar}\left(V(2) - V(1)\right) + \frac{2\pi}{\Phi_0}\int_1^2 d\vec{r}\,\frac{\partial\vec{A}}{\partial t}$$

but $J_s(1) \equiv J_s(2)$ (conservation of charge or Kirchhoff's current law)

$q^* = -2e$

$$\Rightarrow \frac{\partial\varphi}{\partial t} = \frac{2\pi}{\phi_0}\int_1^2 d\vec{r}\left(\vec{\nabla}V + \frac{\partial\vec{A}}{\partial t}\right)$$

But $\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}$

$$\Rightarrow \frac{\partial\varphi}{\partial t} = -\frac{2\pi}{\phi_0}\int_1^2 d\vec{r}\cdot\vec{E}$$

Assume a short junction $\Rightarrow \int_1^2 d\vec{\ell}\,\vec{A} = 0 \Rightarrow$

$$\Rightarrow \boxed{\frac{\partial\varphi}{\partial t} = \frac{2\pi}{\phi_0}V}$$

where $V = V(2) - V(1)$

$$\Rightarrow \boxed{I = I_c \sin\varphi}$$ current-phase relation

$$\boxed{\frac{\partial \varphi}{\partial t} = \frac{2e}{\hbar} V}$$ phase-voltage relation

or: $V = \frac{\partial}{\partial t}\left(\frac{\phi_0}{2\pi}\cdot\varphi\right)$ flux

very similar to Faraday's law

- Consequences:

DC JOSEPHSON EFFECT

$$V = 0 \Rightarrow \frac{\partial \varphi}{\partial t} = 0 \Rightarrow \varphi = \text{const.}$$

$I = I_c \sin\varphi$ – The current can reach a max. value of I_c

AC JOSEPHSON EFFECT

$$V = \text{const} \neq 0 \Rightarrow \varphi = \frac{2e}{\hbar} V \cdot t$$

$$\Rightarrow I = I_c \sin\left(\frac{2e}{\hbar} V \cdot t\right) = I_c \sin\left(2\pi \frac{V}{\phi_0} t\right)$$

$f_J = \frac{V}{\phi_0}$ = Josephson frequency $= 483 \times 10^{12} V_0$ (Hz)

JOSEPHSON INDUCTANCE

$$\frac{\partial I}{\partial t} = \frac{\partial I}{\partial \varphi} \cdot \frac{\partial \varphi}{\partial t} = I_c \cos\varphi \cdot \frac{2\pi}{\phi_0} V$$

or $V = L_J(\varphi)\frac{\partial I}{\partial t}$ $\quad L_J(\varphi)$ = Josephson Inductance

$$L_J(\varphi) = \frac{\phi_0}{2\pi I_c \cos\varphi}$$

– depends on phase!
– can be ∞ if $\varphi = \frac{\pi}{2} + n\pi$

JOSEPHSON ENERGY

$$E_J = \int dt\, I \cdot V = \int d\varphi \cdot I_c \sin\varphi \cdot \frac{\phi_0}{2\pi}$$

$$E_J(\varphi) = -\frac{I_c \phi_0}{2\pi}\cos\varphi = -E_J \cos\varphi$$

$$E_J = \frac{\phi_0 I_c}{2\pi} = \text{Josephson energy}$$

The Josephson junction as a circuit element

Josephson energy $E_J(\varphi) = -E_J \cos\varphi$

Energy stored in the capacitor that shunts the junction

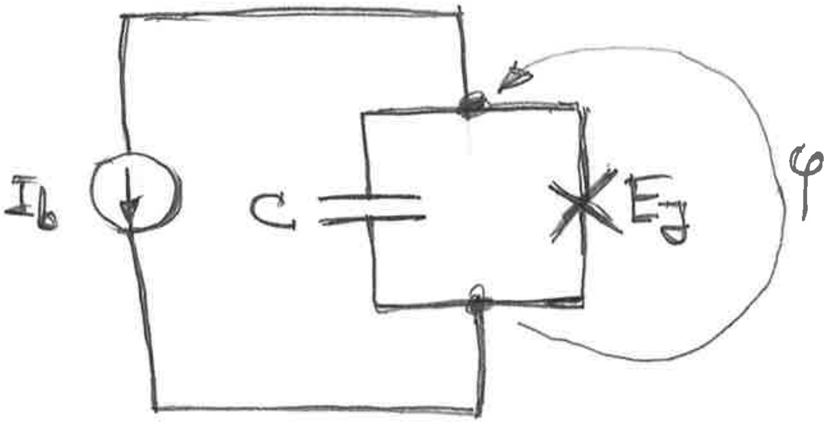
$$E_C = \int dt \cdot I_C \cdot V = \int dt \, \frac{d(CV)}{dt} \cdot V = C \int dV \cdot V = \frac{1}{2} C V^2$$ ← usual formula..

but $V = \frac{\phi_0}{2\pi} \frac{d\varphi}{dt}$ $\Rightarrow$

$$\underline{E_C = \frac{1}{2} C \left(\frac{\phi_0}{2\pi}\right)^2 \left(\frac{d\varphi}{dt}\right)^2}$$

Energy associated with the phase difference across the I_b bias

$$E_b = \int dt \, I_b \cdot V = I_b \int dt \, \frac{\phi_0}{2\pi} \cdot \frac{d\varphi}{dt} = \frac{\phi_0}{2\pi} I_b \cdot \varphi$$



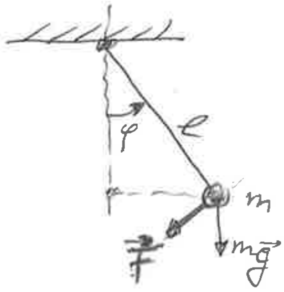
Have you seen these before?

Analogy: pendulum driven by a constant torque

$-mgl\cos\varphi$ is the potential energy

$\frac{1}{2} m l^2 \left(\frac{d\varphi}{dt}\right)^2$ is the kinetic energy

$F \cdot l \cdot \varphi$ is the energy due to the force F

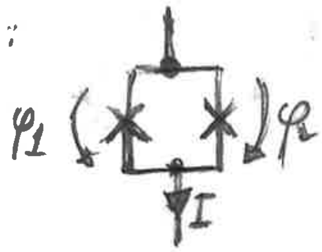


$v = l \cdot \frac{d\varphi}{dt}$

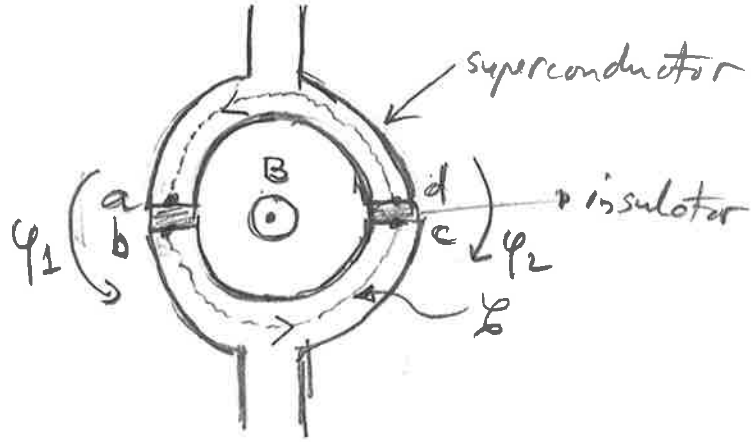
Application: the dc-SQUID

(superconducting quantum interference device)

⇒ Schematic:



How it is fabricated:



$$\oint_{\mathcal{C}} \vec{\nabla}\theta\cdot d\vec{r} = 2\pi n$$

$$= \underbrace{(\theta_b-\theta_a)}_{\varphi_1 - \frac{2\pi}{\phi_0}\int_a^b \vec{A}\,d\vec{r}} + (\theta_c-\theta_b) + \underbrace{(\theta_d-\theta_c)}_{-\varphi_2 - \frac{2\pi}{\phi_0}\int_c^d \vec{A}\,d\vec{l}} + (\theta_a-\theta_d)$$

$$\int_b^c d\vec{r}\cdot\vec{\nabla}\theta = \underbrace{-\frac{2\pi}{\phi_0}\mu_0\lambda_L^2\int_b^c d\vec{r}\,\vec{J}_s}_{\text{zero}} - \frac{2\pi}{\phi_0}\int_b^c d\vec{r}\,\vec{A}$$

zero, $J_s = 0$ inside the superconductor

$$\int_d^a d\vec{r}\cdot\vec{\nabla}\theta = \underbrace{-\frac{2\pi}{\phi_0}\mu_0\lambda_L^2\int_d^a \vec{J}_s\,d\vec{r}}_{\text{zero}} - \frac{2\pi}{\phi_0}\int_d^a d\vec{r}\,\vec{A}$$

zero

$$= \varphi_1 - \varphi_2 - \frac{2\pi}{\phi_0}\underbrace{\oint_{\mathcal{C}} d\vec{r}\cdot\vec{A}}_{=\Phi}$$

$= \Phi$ (magnetic flux piercing the SQUID)

$$\Rightarrow \boxed{\varphi_1 - \varphi_2 = 2\pi n + \frac{2\pi\Phi}{\phi_0}}$$

So $I = I_1 + I_2 = I_c \sin\varphi_1 + I_c \sin\varphi_2$

(we assume that the junctions are identical)

$$= 2I_c \sin\frac{\varphi_1+\varphi_2}{2} \cos\frac{\varphi_1-\varphi_2}{2}$$

Let $\varphi = \frac{\varphi_1+\varphi_2}{2} \Rightarrow$

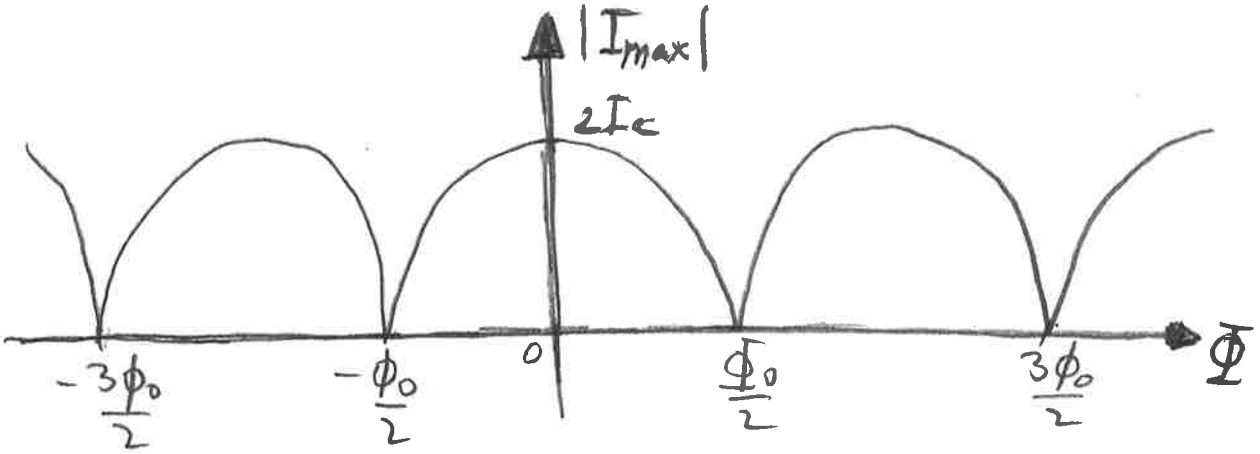
$$I = 2I_c \sin\varphi \cdot \cos\left(\frac{\pi\Phi}{\Phi_0} + \pi n\right)$$

$$I \equiv I_{max}(\Phi) \cdot \sin\varphi$$

where

$$I_{max}(\Phi) = 2I_c \cos\left(\frac{\pi\Phi}{\Phi_0} + \pi n\right)$$

The SQUID behaves as a single Josephson junction with critical current controlled by the magnetic flux.



- I_{max} never exceeds $2I_c$
- I_{max} can be zero! This can be understood as destructive interference of the currents in the two branches of the SQUID.

References

- Terry P. Orlando and Kevin A. Delin – – Foundations of Applied Superconductivity
- D.R. Tilley and J. Tilley – Superfluidity and Superconductivity
- Antonio Barone and Gianfranco Paternò – – Physics and Applications of the Josephson Effect