Hint: Problem set 3, Problem 2: Superconducting phase

The square of the term representing the Kinetic energy should be treated as a dot product. Consider the usual kinetic energy equation $K.E = \frac{1}{2}m\vec{v}^2$. Here the velocity $\vec{v}$ is a vector quantity, the square of which should give a scalar quantity. Thus, we alway consider square of a vector as a dot product.

$$\left(-i\hbar\vec{\nabla} - q^*\vec{A}\right)^2 = \left(-i\hbar\vec{\nabla} - q^*\vec{A}\right) \cdot \left(-i\hbar\vec{\nabla} - q^*\vec{A}\right) \tag{1}$$

When you evaluate equation 1, you will get following terms:

(a) $\vec{\nabla} \cdot \vec{\nabla}\Psi(\vec{r}, t) = \vec{\nabla} \cdot (i\Psi(\vec{r}, t)\vec{\nabla}\theta(\vec{r}, t))$

Here, you will use product rule for vector calculus (look up vector calculus identities on wikipedia).
$\vec{\nabla} \cdot (\psi\vec{A}) = \psi(\vec{\nabla} \cdot \vec{A}) + (\vec{\nabla}\psi) \cdot \vec{A}$

Remember to consider i as a constant when evaluating the product rule. With Coulomb gauge, we can apply the following identity: $\vec{\nabla}^2\theta = 0$.

(b) $\vec{\nabla} \cdot (\vec{A}\psi) = (\vec{\nabla} \cdot \vec{A})\psi + \vec{A} \cdot (\vec{\nabla}\psi)$

With Coulomb gauge, we have $\vec{\nabla} \cdot \vec{A} = 0$.

(c) $\vec{A} \cdot \vec{\nabla}\psi = \vec{A} \cdot (\vec{\nabla}\psi)$, that is, evaluate the gradient and leave the vector A as it is.

(d) $(q^*\vec{A}) \cdot (q^*\vec{A}) = (q^*\vec{A})^2$. You can leave it as it is.

With these, you should be able to complete this exercise. We have used few identities by invoking Coulomb gauge. It would be a good idea to check what that is and why we can use these identities (you can find these information on Wikipedia). If possible, add these explanations in your solution.