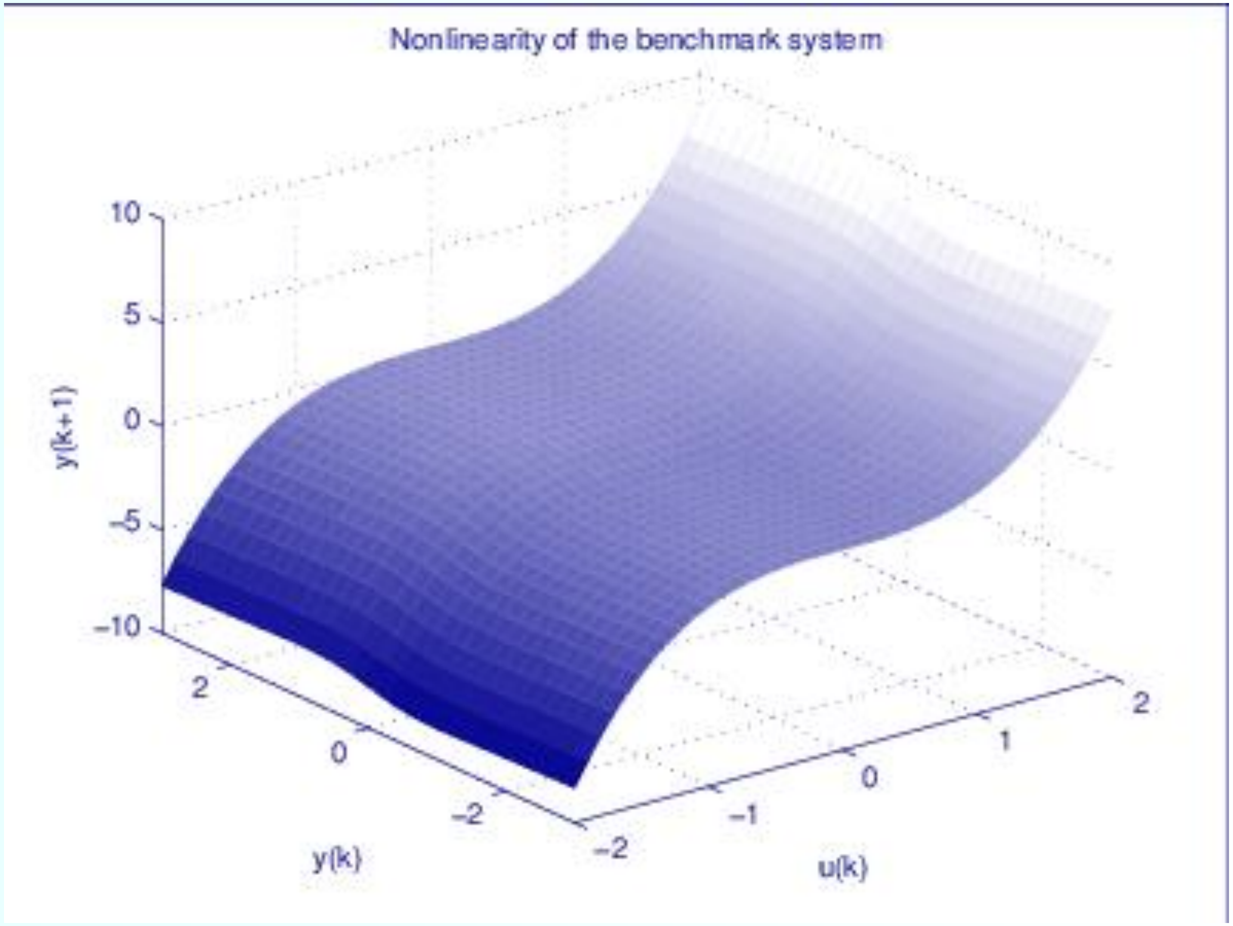
Aalto University
School of Science

Formulating nonlinear relationships



Nonlinear relationships

Up until now most economists have concerned themselves with linear systems, not because of any belief that the facts were so simple, but rather because of the mathematical difficulties involved in nonlinear systems...
[Linear systems are] mathematically simple, and exact solutions are known. But a high price is paid for this simplicity in terms of special assumptions which must be made.

[Paul A. Samuelson, 1947]

Nonlinear relationships

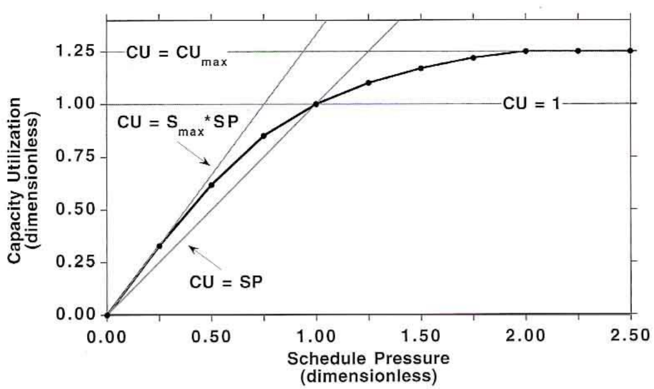
- Many relationships are nonlinear
  - Company ships at desired rate – untill inventory is inadequate
  - Improvements in nutrition ad health care boost life expectancy – up to a point
  - Demand for a product must tend to zero as availability or quality fall to zero, no matter how cheap it is
- Importance of nonlinearities has been recognized for centuries, but only since the advent of computer simulation, nonlinearity has become important in dynamic modelling

Nonlinear relationships

- Nonlinear relationships are not necessarily more difficult to model than linear
  - Linear approximations are not required, the actual real relationship can be used
- How to formulate realistic and robust nonlinear relationships for dynamic models?
- How to elicit shape information from experts?

Steps in Table functions

- Many formulations in SD models involve nolinear functions y = f(x)
- Sometimes these functions can be represented analytically
- More often, nonlinear relationships are represented through *table functions*
- Linear interpolation is used between the points
  - Some SD tools allow also other functions between points
- All possible sources to determine the points:
  - statistics, laws of physics, field work, interviews,



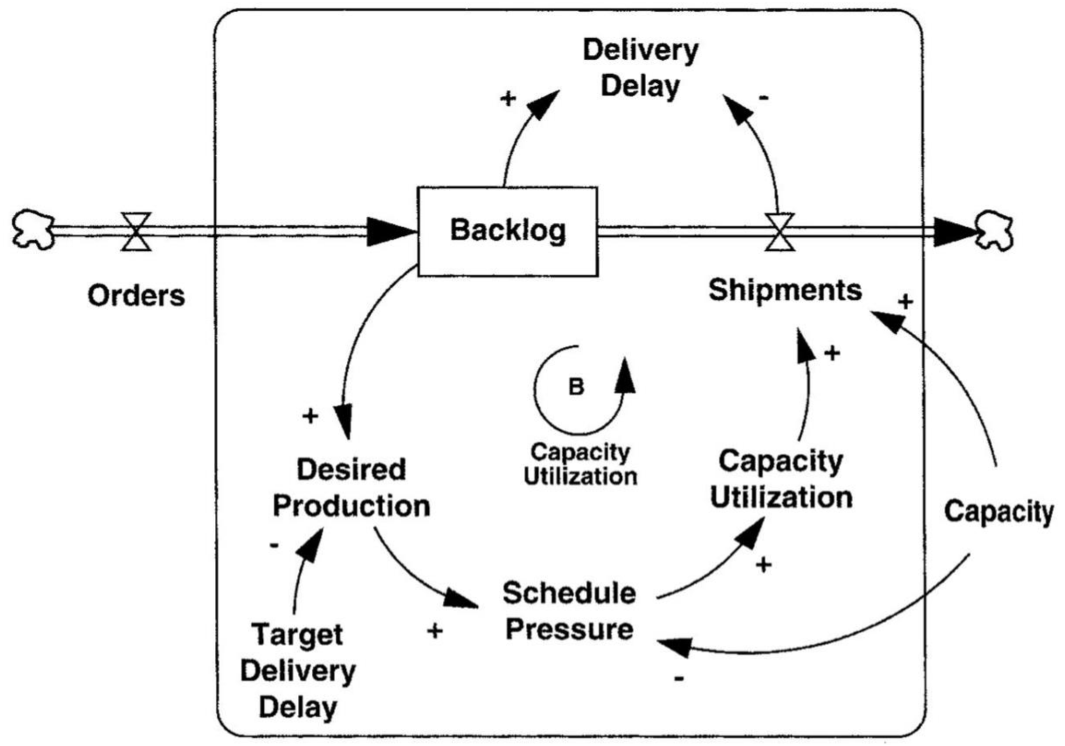
Schedule Pressure	Capacity Utilization	Increment
0.00	0.00	
0.25	0.33	0.33
0.50	0.62	0.29
0.75	0.85	0.23
1.00	1.00	0.15
1.25	1.10	0.10
1.50	1.17	0.07
1.75	1.22	0.05
2.00	1.25	0.03
2.25	1.25	0.00
2.50	1.25	0.00

Steps in defining table functions

1. Normalization: $y/y^* = f(x/x^*)$
2. Identify *anchor points* where f(.) is know
3. Identify *reference policies*: slope of f(.) at some points
4. Consider extreme conditions, e.g. $x \rightarrow \pm\infty$
5. Specify domain of x
6. Identify plausible shapes of f()
7. Specify values for your best estimate of f()
8. Test model to see if f() works reasonably
9. Sensitivity analysis of results to plausible variation of f(): is more accurate data necessary?

Example: capacitated delivery delay in make-to-order system

- Delivery delay = Backlog/Shipments
- Backlog(t) = Backlog(0) + $\int_0^t$(Orders-Shipments)dt
- Desired production = Backlog/Target Delivery Delay
- Shipments = f(Desired Production)



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Step 1: Normalization

- Defining shipments as function of desired production is not robust, because if current capacity changes, function must be re-defined
- Therefore we define y/y* = f(x/x*)
  - Shipments = Capacity*Capacity Utilization
  - Capacity Utilization = f(Schedule Pressure)
  - Schedule Pressure = Desired Production/Capacity
- Now both Capacity Utilization and Schedule pressure are dimensionless ratios

Step 2: Anchor points

- Define x* and y*
  - Capacity is defined as ***normal*** production rate
    - not max possible output when heroic efforts are made
  - Given that definition, f must pass through point (1,1)
  - When Schedule Pressure = 1, Desired Production = Capacity

Step 3: Reference policies

- There are three reference policy lines for f()
  1. *Capacity Utilization = CU* = 1 when Shipments equal Capacity
  2. *45º line CU = SP* (=Schedule Pressure) corresponds to policy where utilization set to produce at desired rate
  3. $CU = S_{max} \cdot SP$, where S_{max} is max slope of f(), corresponds to producing as fast as possible:

$$Shipments_{max,SP} = Capacity \cdot Capacity\ Utilization$$
$$= C \cdot S_{max} \cdot SP = C \cdot S_{max} \cdot DP/C = S_{max} \cdot DP = S_{max} \cdot B/DD^*$$

$$Delivery\ delay = DDmin = Backlog/Shipments_{max,SP}$$
$$= B/(S_{max} \cdot B/DD^*) = DD^*/S_{max}$$

DD = Target Delivery Delay*

Step 4: Extreme conditions

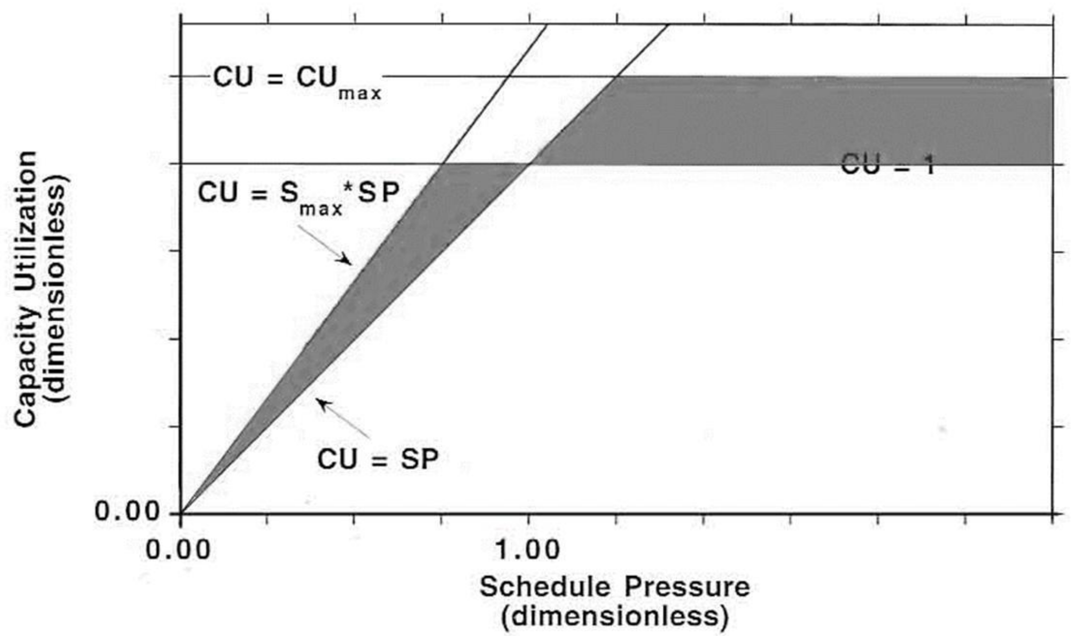
- The utilization function must pass through (0,0) because Shipments must = 0 when Schedule Pressure=0 (else backlog would become negative)
- At other extreme CU must saturate at CU_{max} when pressure is high

Step 5: Domain for independent variable

- Domain for input should be wide enough to capture full range of possible values
- Schedule Pressure cannot be less than zero, but can in principle be infinite
- Because CU saturates at maximum value for large values of Schedule Pressure, function need only be specified for values large enought to ensure saturation point is reached
- Interval [0, 2.5] is selected

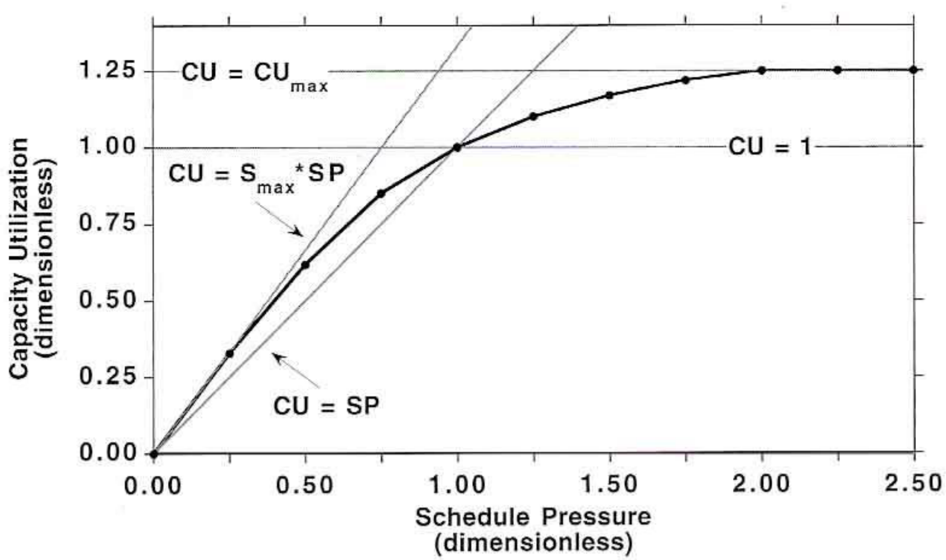
Step 6: Plausible shapes for function

- Reference policies and extreme conditions define narrow region where function must lie
- Function must be below $CU = S_{max} \cdot SP$, because orders cannot be delivered faster than min delay
- Function must be below $CU = CU_{max}$
- Left of (1,1) f() must be below CU=1
- Right of (1,1) f() must be above CU=1



Step 7: Specifying values for the function

- Personnel estimates normal CU is 80% of Cu_{max}
- Delivery delay can never be less than 75% of normal: $S_{max} = 1/0.75 = 1.333$
- Function is assumed monotonic



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1.50	1.17	0.07
1.75	1.22	0.05
2.00	1.25	0.03
2.25	1.25	0.00
2.50	1.25	0.00

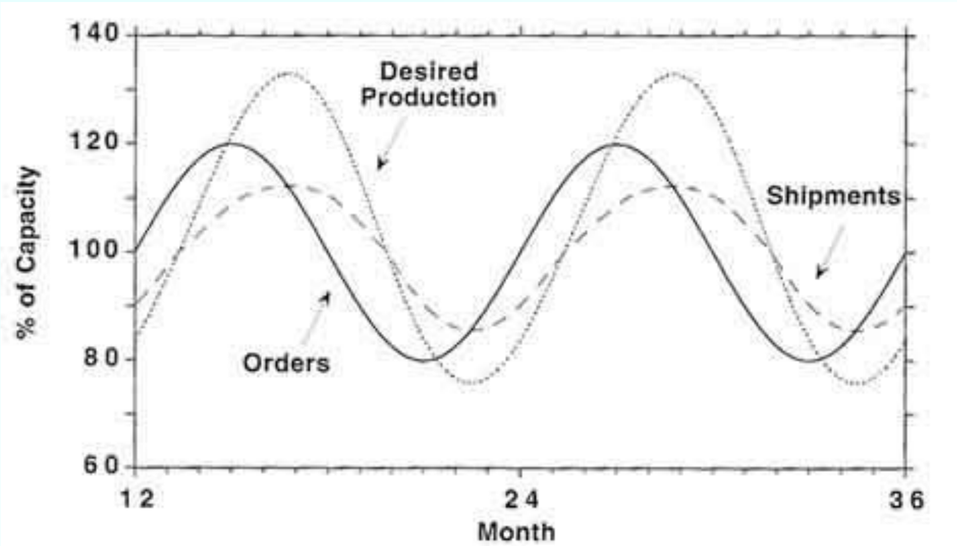
Step 8: Testing the formulation

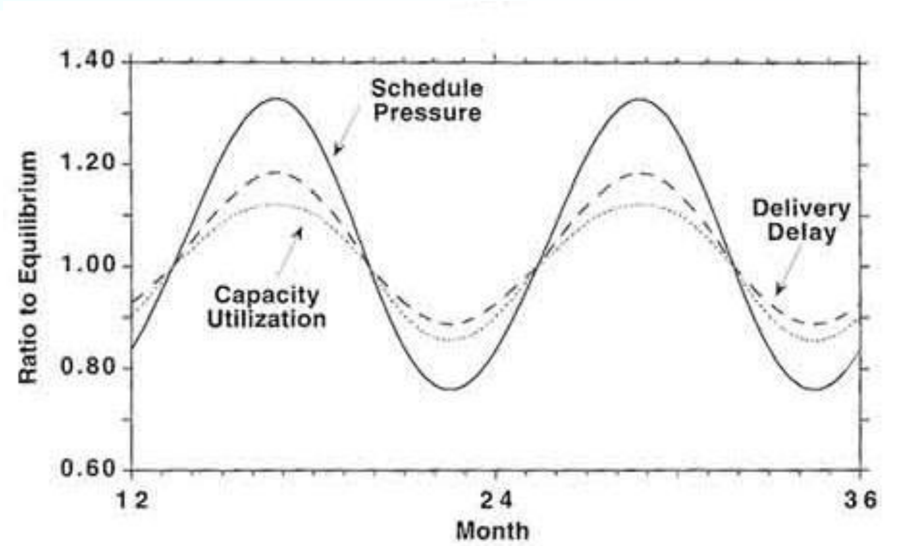
- Assume orders vary cyclically with amplitude A=0.20 and period P=12 months

  Orders = Capacity·[1 + A sin(2πt/P)]

  Target Delivery Delay=1 month, Capacity=100 units/month

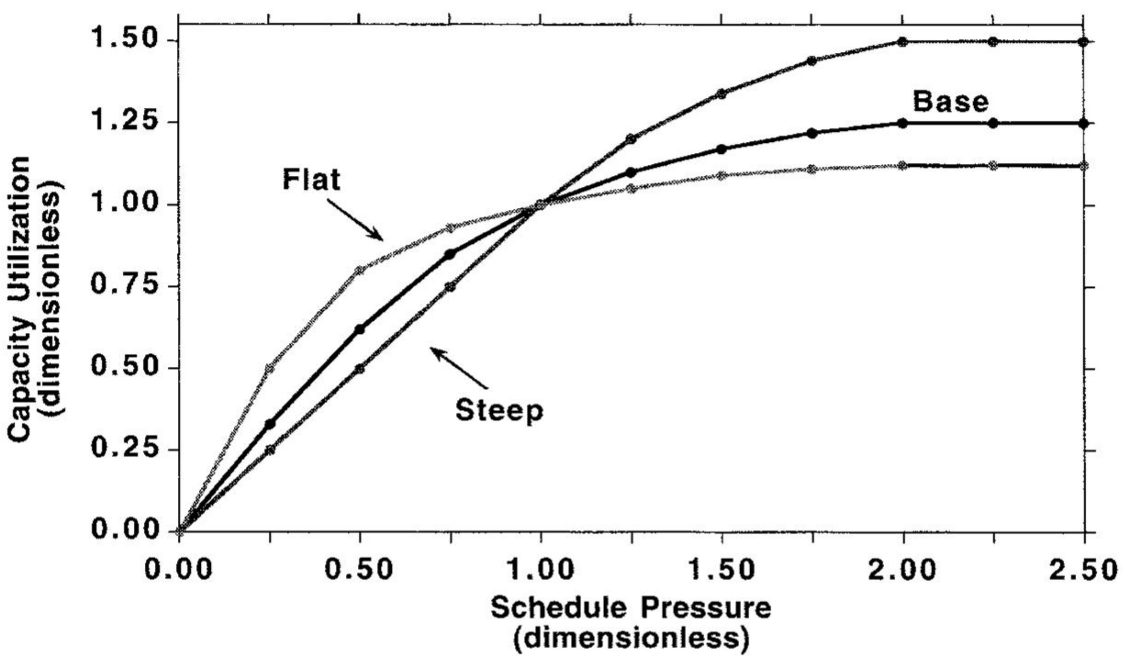
  – Average Schedule Pressure=1.05, Delivery Delay=1.04





Step 9: Sensitivity analysis

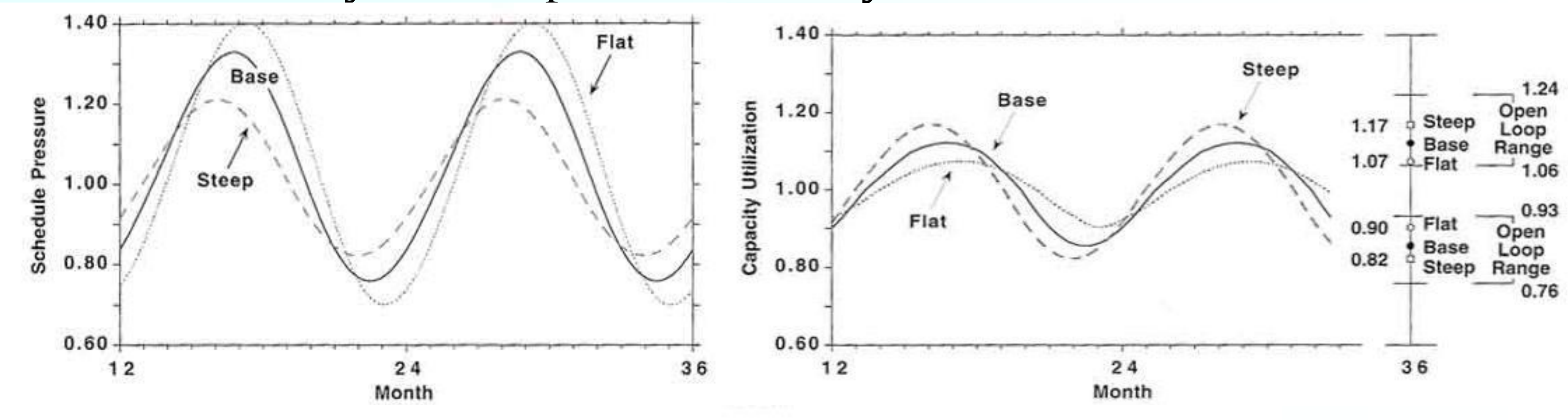
- Vary function parameters to see effect on model
  - CU_{max}, Delivery Delay
- Variation of parameters causes modest variation in CU ⇒ model seems fairly robust



Schedule Pressure	Flat Case	Base Case	Steep Case
0.00	0.00	0.00	0.00
0.25	0.50	0.33	0.25
0.50	0.80	0.62	0.50
0.75	0.93	0.85	0.75
1.00	1.00	1.00	1.00
1.25	1.05	1.10	1.20
1.50	1.09	1.17	1.34
1.75	1.11	1.22	1.44
2.00	1.12	1.25	1.50
2.25	1.12	1.25	1.50
2.50	1.12	1.25	1.50

Step 9: Sensitivity analysis

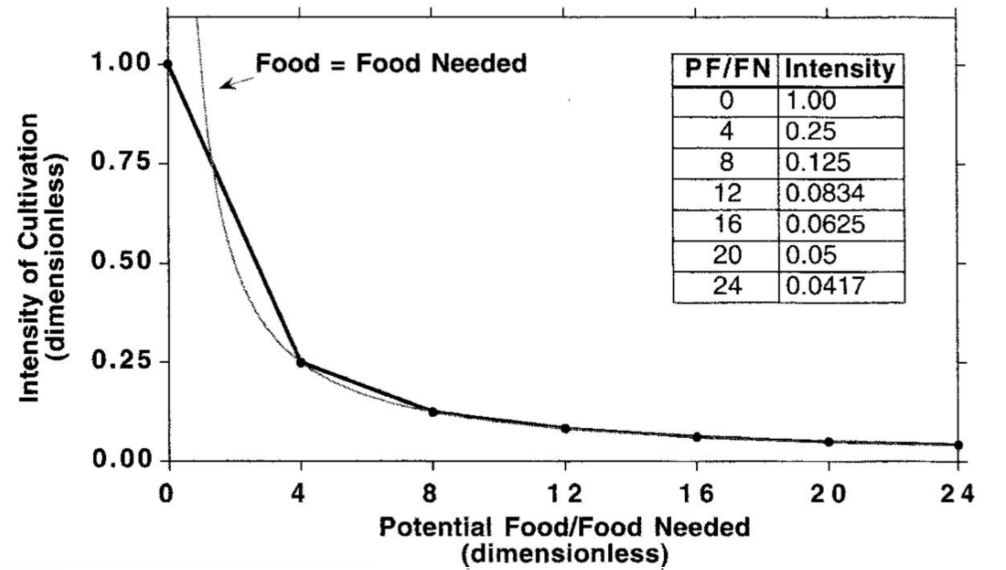
- The response of a complex system is often insensitive to large variations in many parameters
  - Only certain parameters may be critical

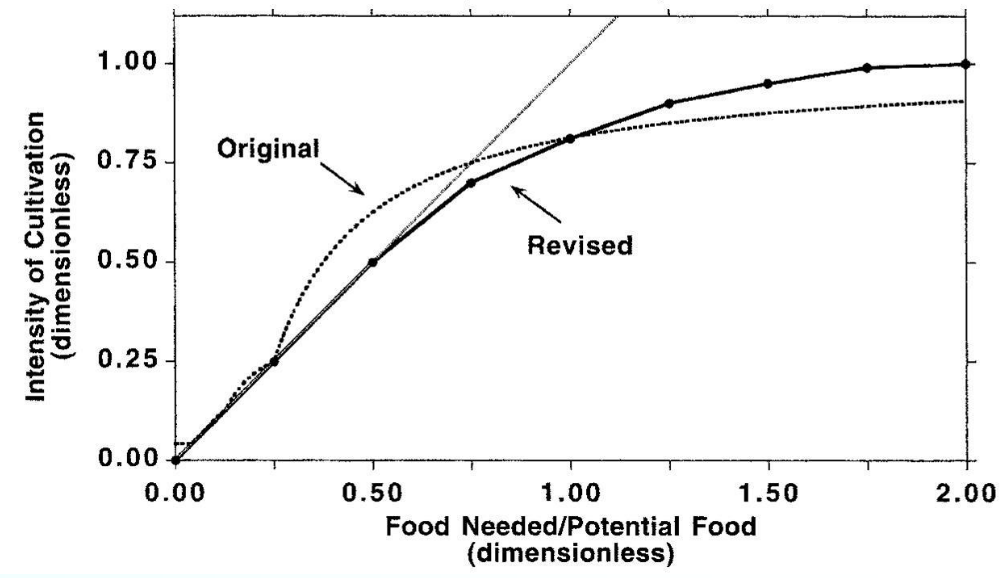


Base Case	Flat Case		Steep Case	
Range of CU	Open Loop	Closed Loop	Open Loop	Closed Loop
0.26	0.13	0.17	0.48	0.35
Change from Base	−50%	−35%	85%	35%

Inverting hyperbolic relationship

- If the reference policy is a hyperbola instead of 45^o line, this may cause problems with small values
- Inverting the input will produce a better model





Experts as source of information

- Sometimes numerical information is not available
- Group of experts and other first-hand users may be used
  - Don’t pressurizing experts to give a single consistent set of results
  - Don’t impose your own beliefs, maintain objectivity
- Three steps in collecting information
  - Positioning
  - Description
  - Discussion

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Experts as source of information

1. Positioning
   - Facilitator describes the model purpose, structure and the relationship to be estimated
   - Focus on a single nonlinearity at a time
   - Define the independent and dependent variables
   - Why are these important
   - Describe the overall system
   - Use examples so that everybody understand the problem correctly

Experts as source of information

2. Describing nonlinear relationship

- Four steps

  I. Visualize the process

  II. Record a description: Each experts creates a written ”walk through” of the target process

  III. Identify anchor points and reference policies

  IV. Graph the relationship:

  - First each expert plots the anchor points on a blank set of axes.
    Next they consider the shape of the relationnship between anchor points and use their reasoning to sketch their estimate of the relationship. Do not just connect anchor points!

Experts as source of information

3. Discussion
   - Purpose is to deepen understanding and improve information provided by individual experts
   - Present first individual experts’ views
   - Discuss why such shape for the relationship
   - Experts’ opinions may differ because they have different backgrounds and different mental models

Experts as source of information

EXTERNAL PRECEDENCE WORKSHEET

Upstream Development Activity: Product Definition

Downstream Development Activity: Design

Position held by author: Product Architect

PROCESS STORY NOTES:

1. Product "straw man" complete—can begin high-level design & acquisition of needed design info [information] (e.g., cells, tools).
2. Feedback incorporated into straw man, producing 1st-cut product def'n [definition].
3. Incremental product def'n [definition] refinement.
4. Handoff complete.

ANCHOR POINTS IN TABLE:

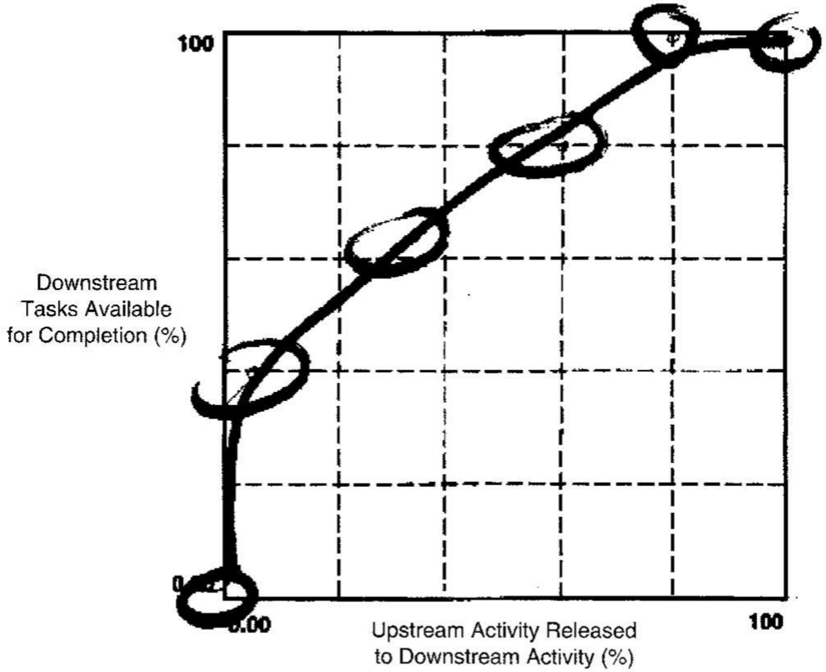
Upstream Tasks Released (%)	Downstream Tasks Available (%)	Notes
10	40	1. [see above]
35	65	2. [see above]
60	85	3. [see above]
80	100	

EXTERNAL PRECEDENCE WORKSHEET

Upstream Development Activity: [Product Definition]

Downstream Development Activity: [Design]

Position held by Author: [Product Architect]



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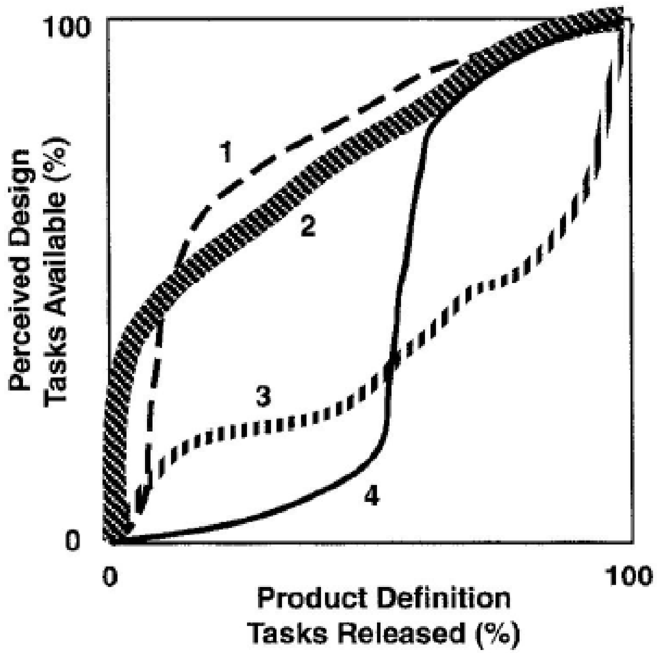
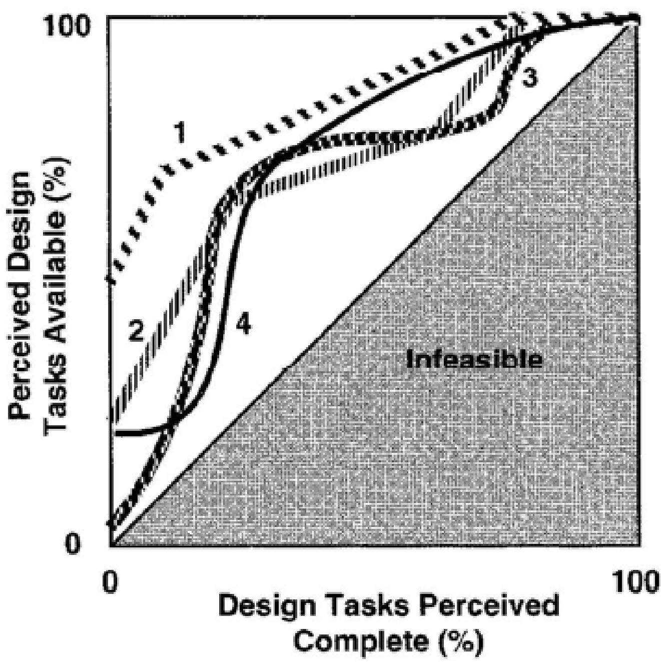


1, Strategic marketing representative; 2, product architect; 3, design manager; 4, design engineer.



1, Design manager; 2, design engineer A; 3, design engineer B; 4, revised description after discussion phase.