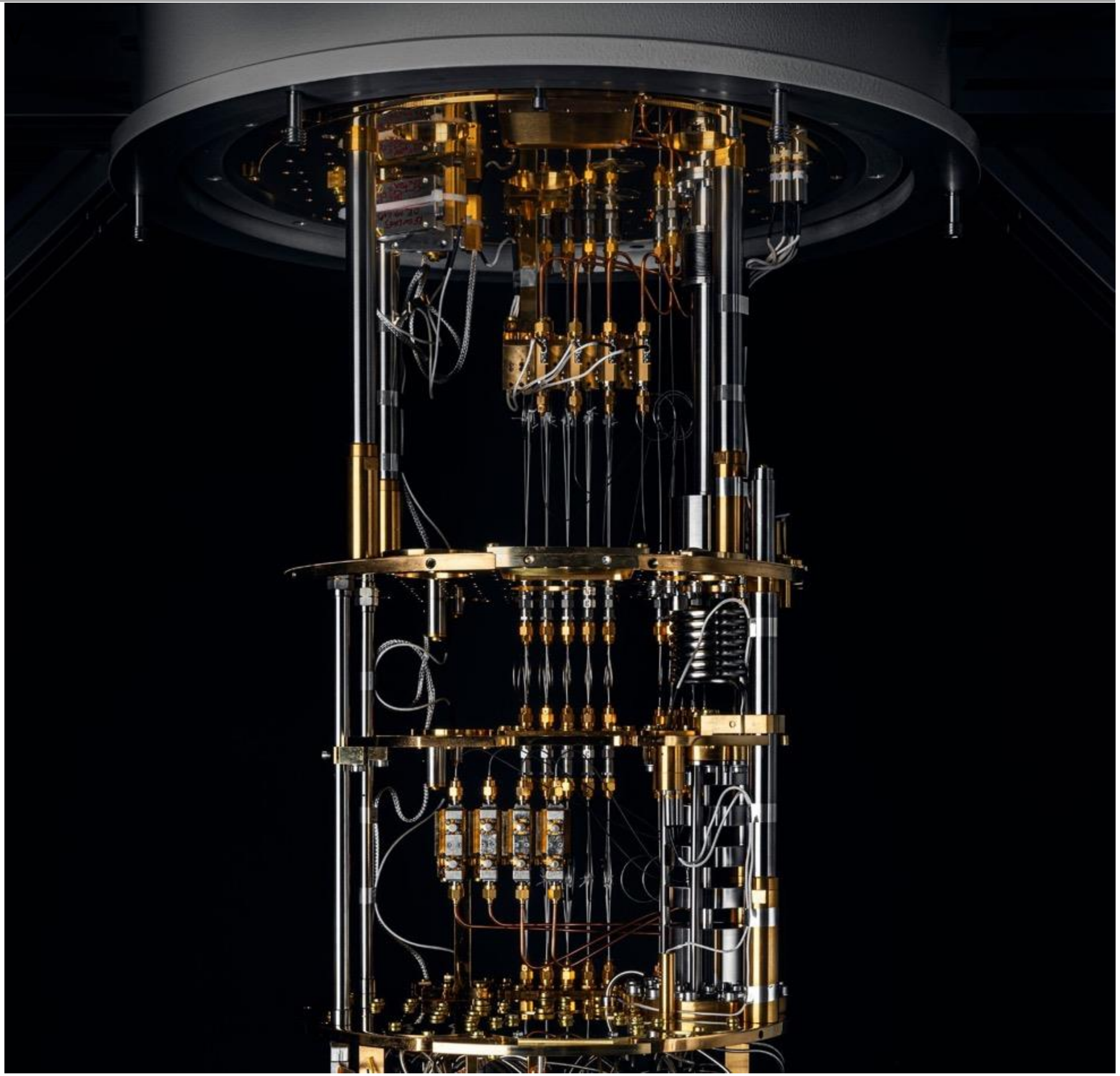
IQM

WE BUILD QUANTUM COMPUTERS

Basics of superconducting qubits



Dr. Vladimir Milchakov

Engineer of Quantum Processors

www.meetiqm.com

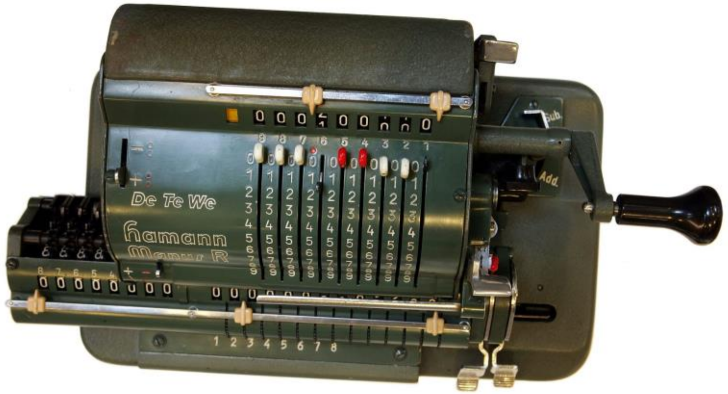
How to make a quantum computer?

How to make a ~~quantum~~ computer?

- Memory register
- Way to read it and modify

How to make a bit

- Device with two separate state
- Accessible(w/r), stable, compact...

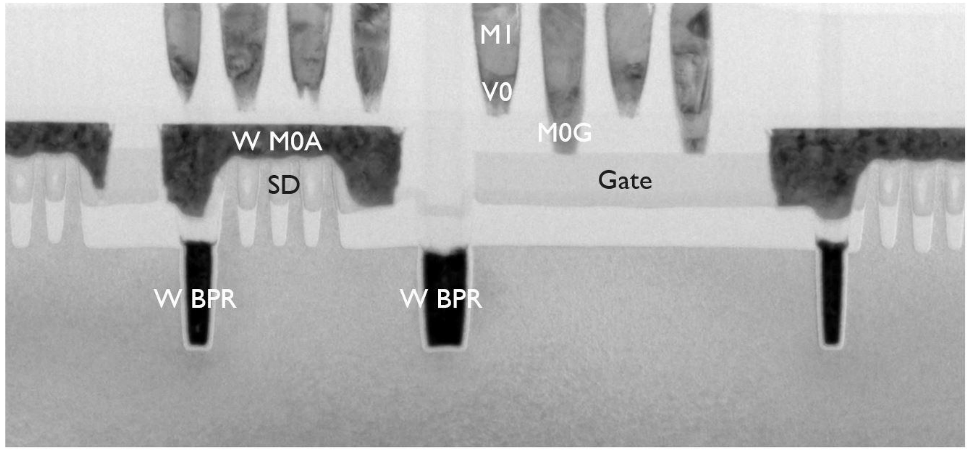


Mechanical computer

Vacuum tube

Semiconductor transistor

CMOS micro-transistor



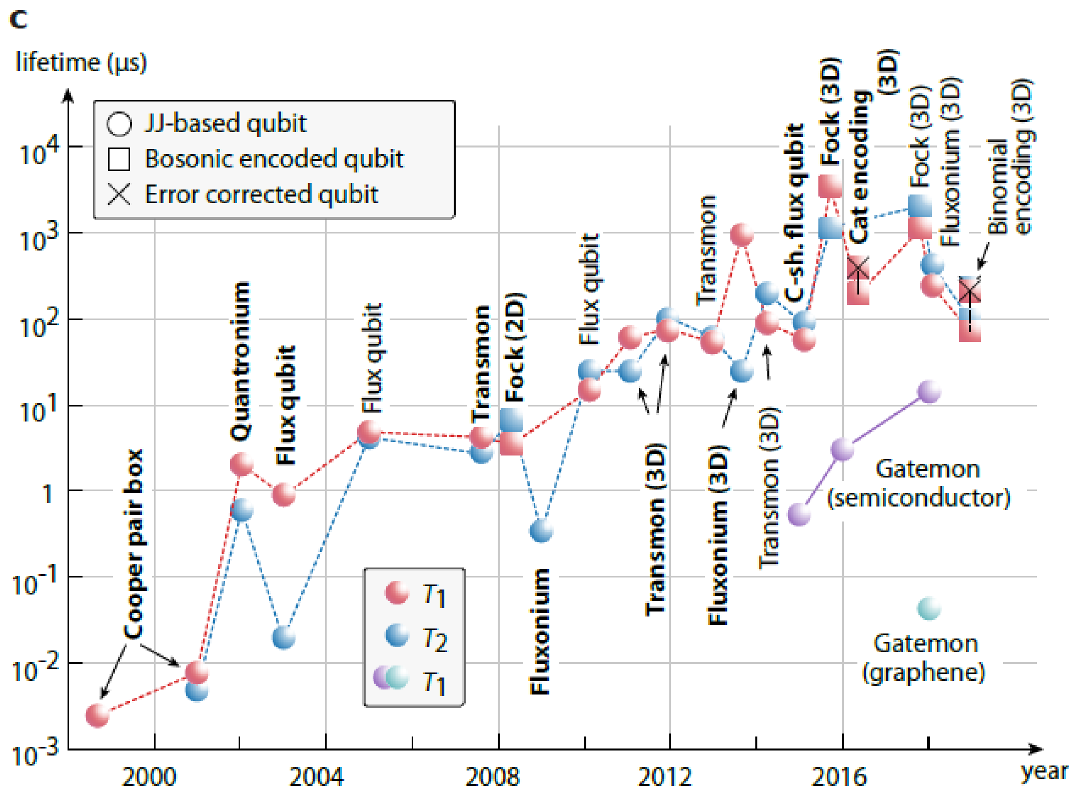
Qubits

- Quantum system with two states (energy levels), which are addressable(r/w)

Many physical implementations...

And we now focus on superconducting circuits

(there also a lot)



Quantum harmonic oscillator

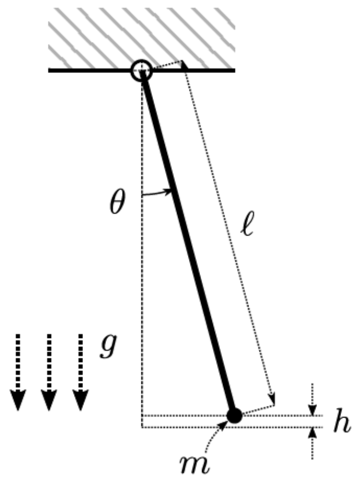
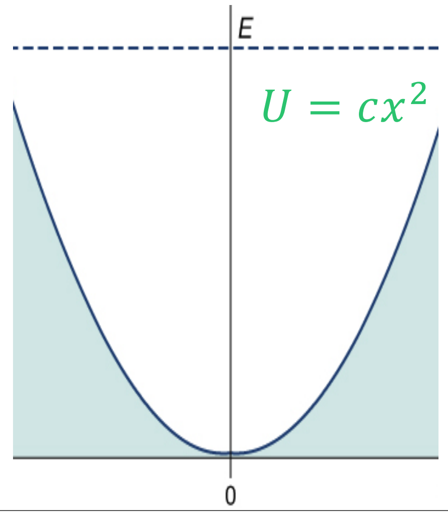


Figure 1: Classical pendulum.



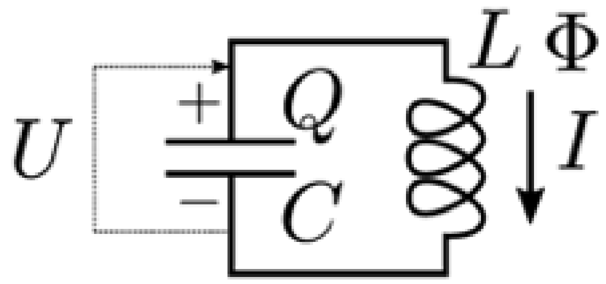
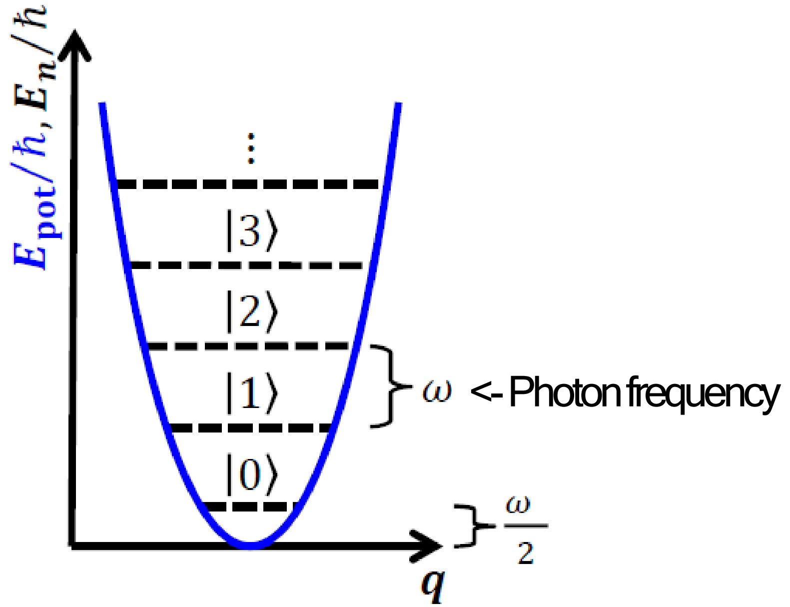


Figure 2: Superconducting LC oscillator.



To memorize:

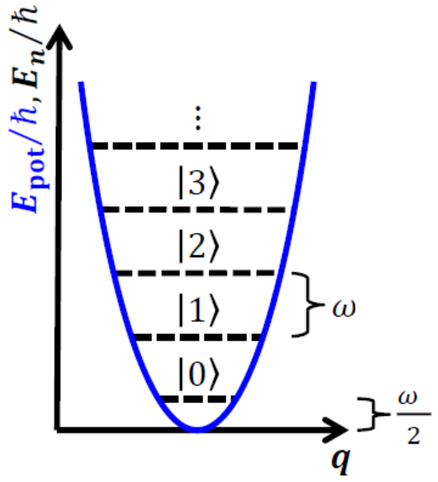
1. What is harmonic oscillator
2. Quantized energy levels
3. *non-zero lowest energy

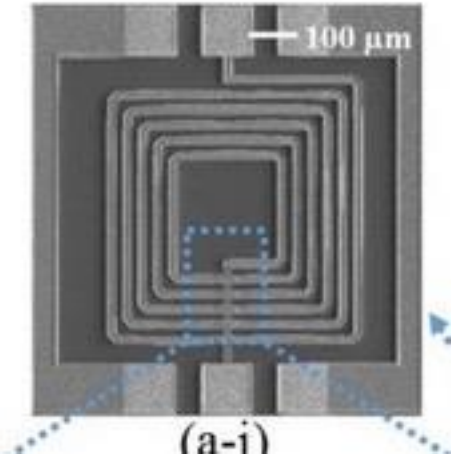
Takeaways:

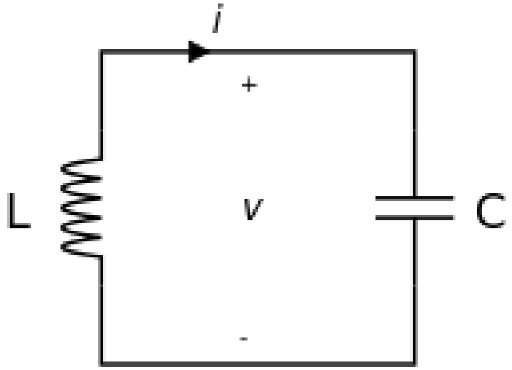
- Total energy of system derived from Lagrangian
- Eigenfrequency of system derived from energy considerations

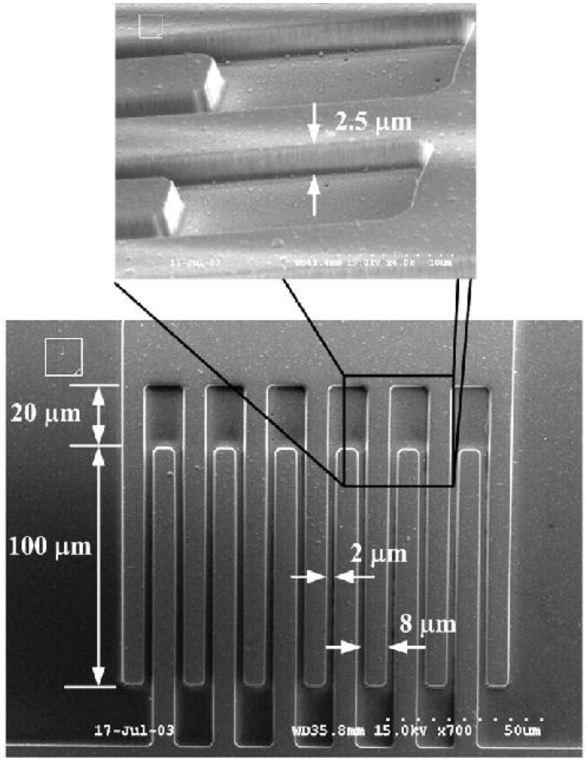
$$E_{total} = Frequency_{photons} \cdot (N_{photons} + 0.5)$$

How to make LC-oscillator on chip?





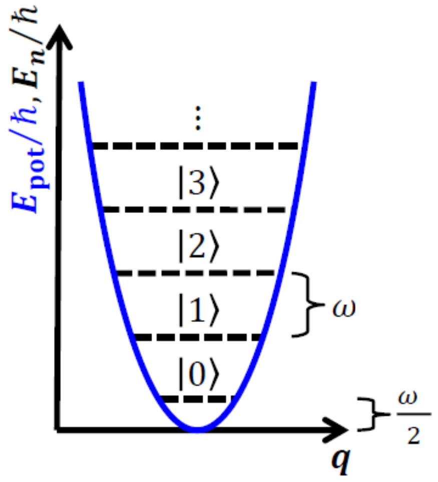


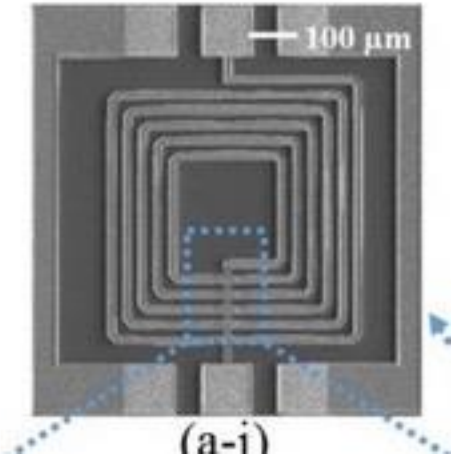


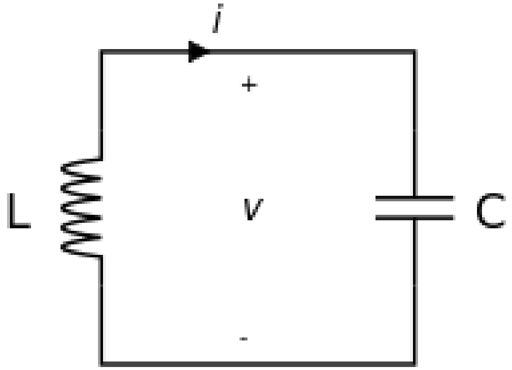
$$E_c = e^2/2C$$

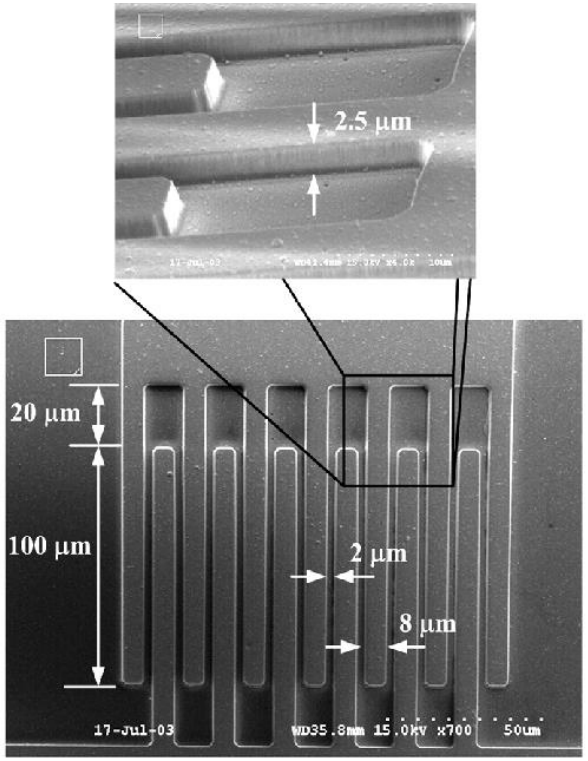
Kenney J, Yoon Y et al (2023)

Is it a qubit?









$$E_c = e^2/2C$$

Kenney J, Yoon Y et al (2023)

Josephson junction

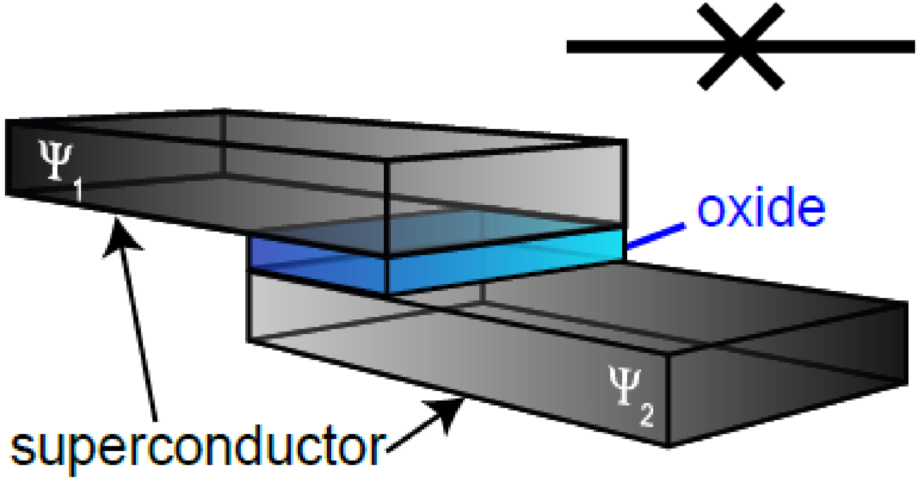


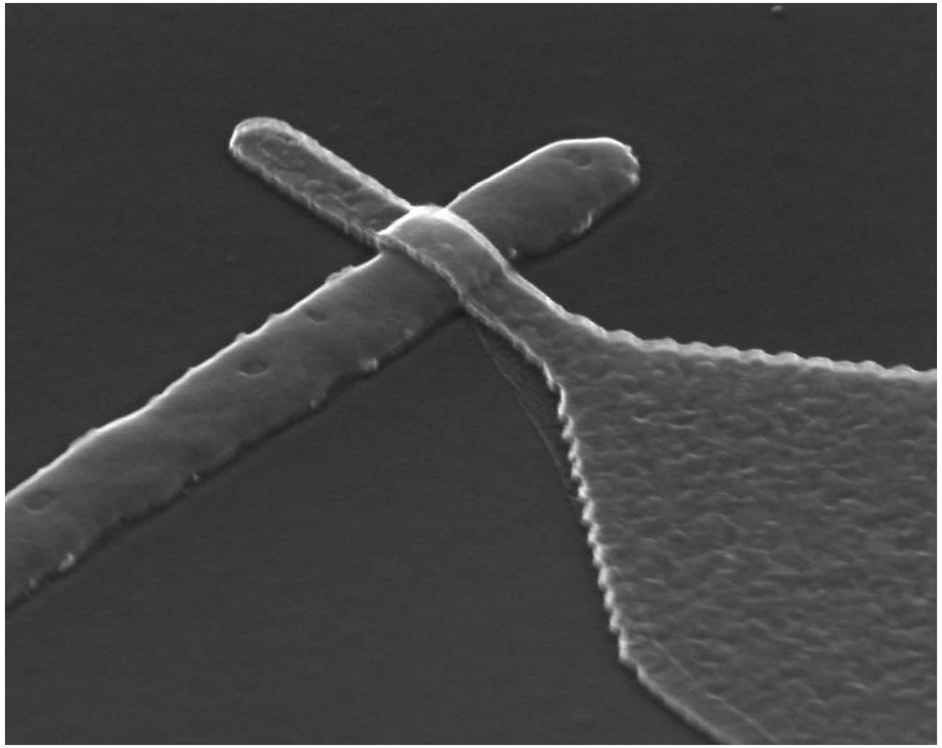
Figure 2.4: Sketch of a Josephson junction.

$$I(t) = I_c \sin(\varphi(t))$$

$$\frac{\partial \varphi}{\partial t} = \frac{2eV(t)}{\hbar}$$

$$E_{\mathrm{J}} = \frac{\Phi_0 I_{\mathrm{c}}}{2\pi}(1 - \cos\phi_{\mathrm{J}})$$

Non-linear inductive element



Josephson junction (IMEC)

Josephson junction

Where to encode the states?

$$[\delta, Q] = i2e.$$

Phase and Charge **commutate**
(like coordinate and momentum)

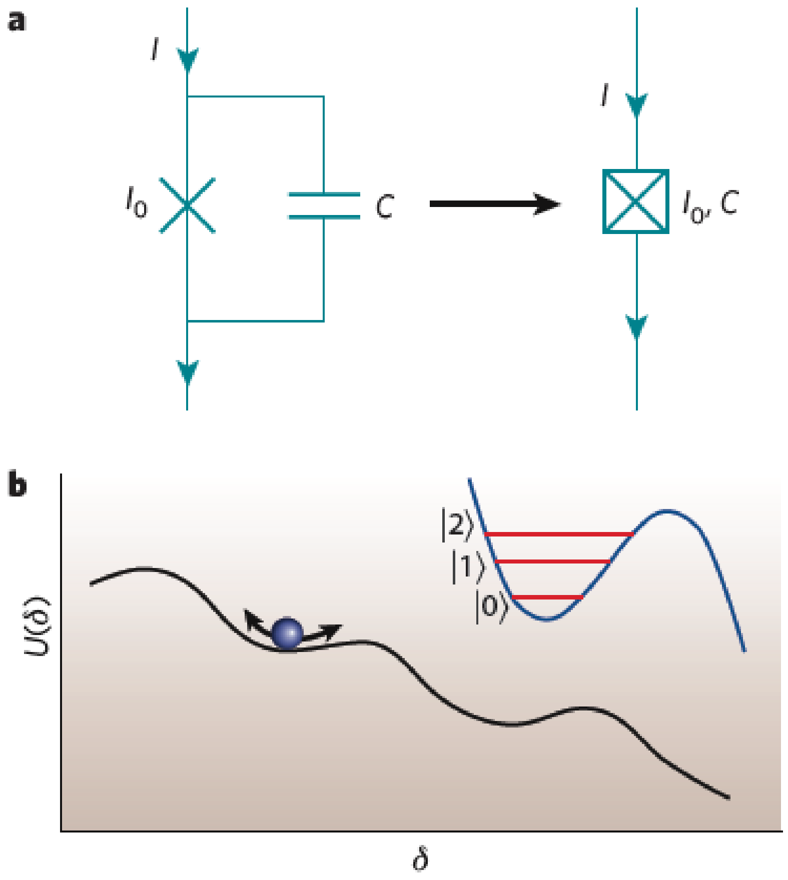
- If $E_J >> E_C$ then: δ - well defined, Q - fluctuates
- If $E_J << E_C$ then: δ - fluctuates, Q - defined well

Recommendation: doi:10.1038 / nature07128

NATURE|Vol 453|19 June 2008|doi:10.1038/nature07128

Superconducting quantum bits

John Clarke[1,2] & Frank K. Wilhelm[3]



Flux qubit

Flux is also quantized by the way!

$E_J >> E_C$ (50 times)

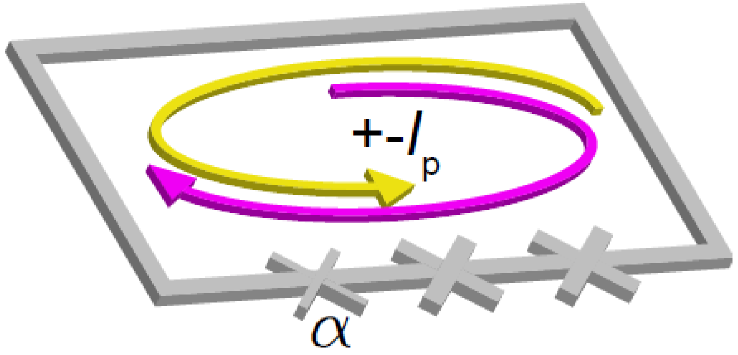
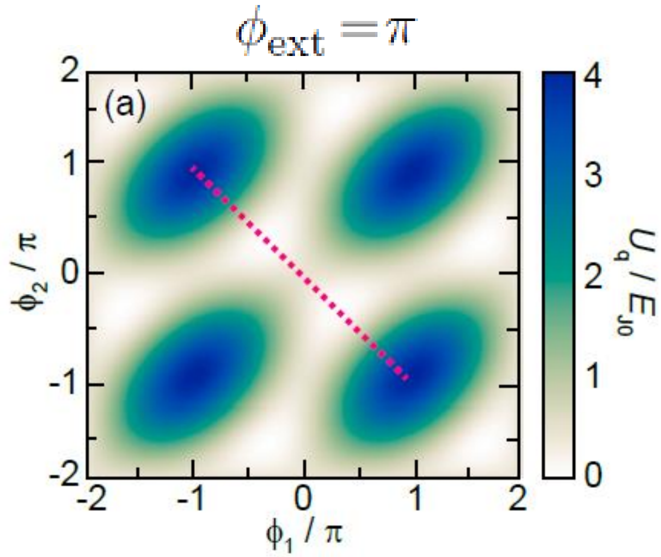
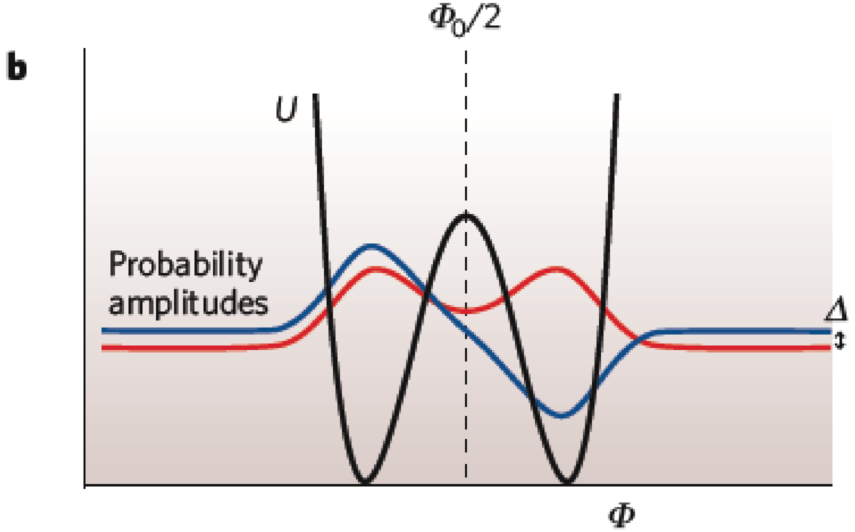


Figure 2.7: Sketch of a three junction flux qubit and the superposition of current states.

$$\mathcal{H}_q = \frac{\hbar\Delta}{2}\hat{\sigma}_x + \frac{\hbar\varepsilon}{2}\hat{\sigma}_z = \frac{\hbar}{2}\begin{pmatrix} \varepsilon & \Delta \\ \Delta & -\varepsilon \end{pmatrix}$$

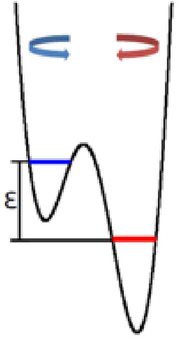




$$U_q = E_{J0}\left[2 + \alpha - \cos(\phi_1) - \cos(\phi_2) - \alpha\cos\left(\phi_{ext} + \phi_1 - \phi_2\right)\right]$$

$$, \phi_{ext} = 2\pi\Phi_{ext}/\Phi_0$$

$$\omega_q = \sqrt{\Delta^2 + \varepsilon^2}.$$



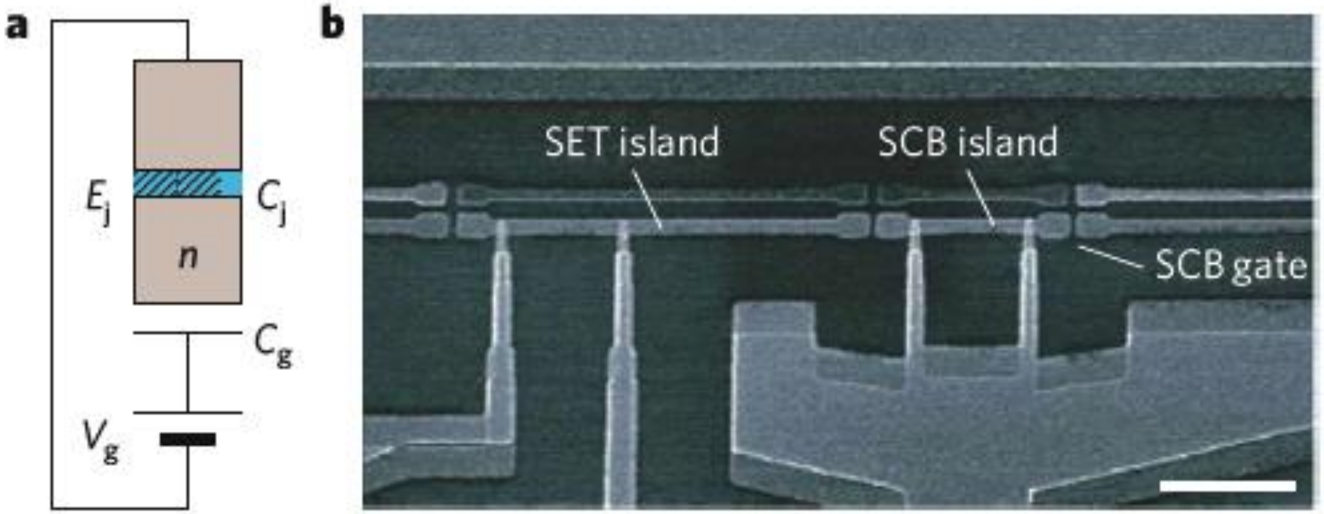
A change in magnetic flux bias tilts the potential leading to an additional energy ε.

Charge qubit (Cooper-pair box)

Count every cooper pair

$E_J << E_C$

C < 1fF (for E_C to be >> $k_B T$) [T=1K]



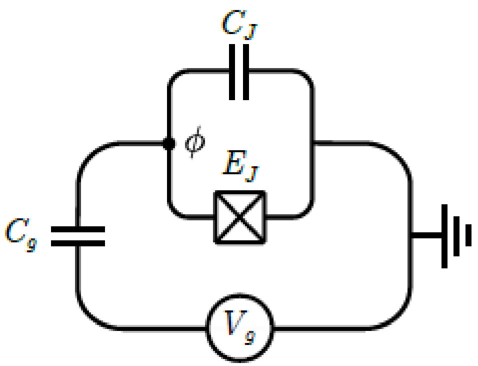
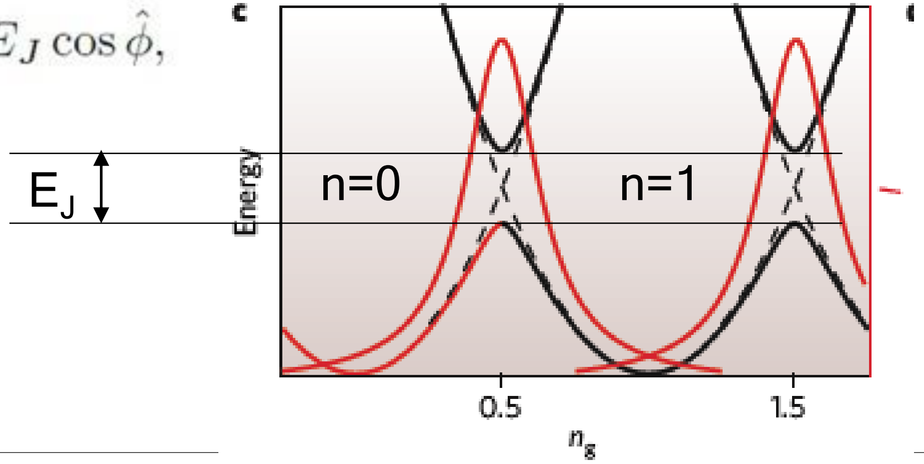


Figure 12. Circuit diagram of the single Cooper pair box, consisting of a Josephson junction, with energy E_J and parasitic capacitance C_J, in series with a capacitor with capacitance C_g. The gate voltage is denoted V_g and the system is connected to the ground in the right corner. There is only one active node, denoted with a dot.

$$\hat{\mathcal{H}} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\phi},$$

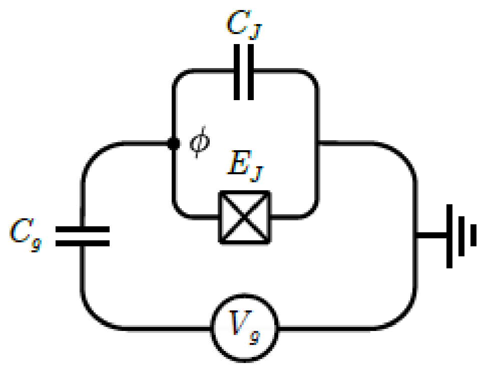


arXiv:2103.01225v1

doi:10.1038 / nature07128

Charge qubit (Cooper-pair box)

Charge noise…



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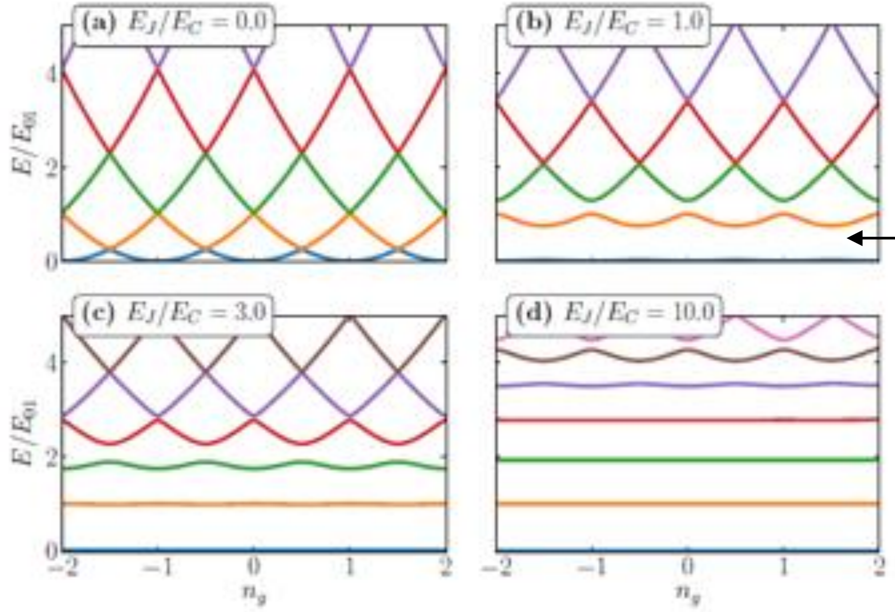


$$\mathcal{H}_\mathrm{q} = \frac{\omega_\mathrm{q}}{2}\hat{\sigma}_\mathrm{z} \, .$$

Figure 13. The energies of the lowest lying states of the single Cooper pair box/transmon qubit as a function of the bias charge n_g. The difference between the two lowest bands are approximately equal to E_J at the avoided crossing.

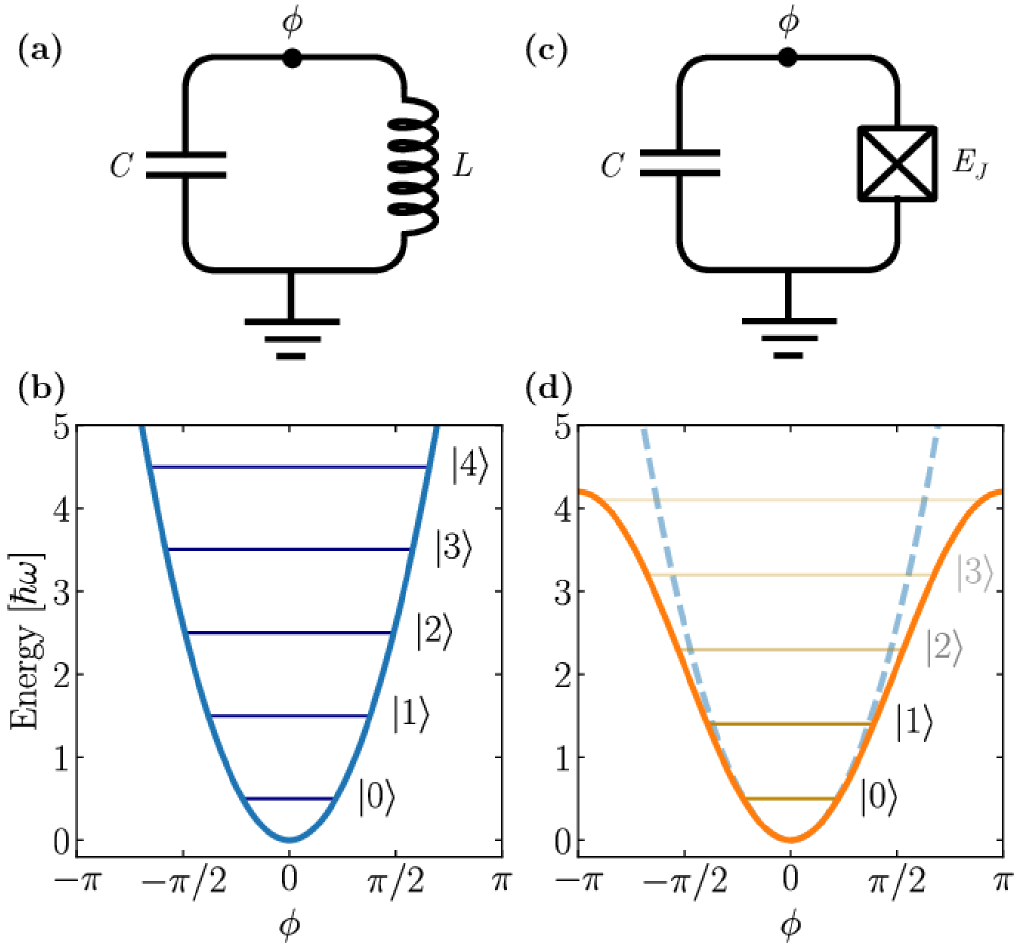
arXiv:2103.01225v1

doi:10.1038 / nature07128

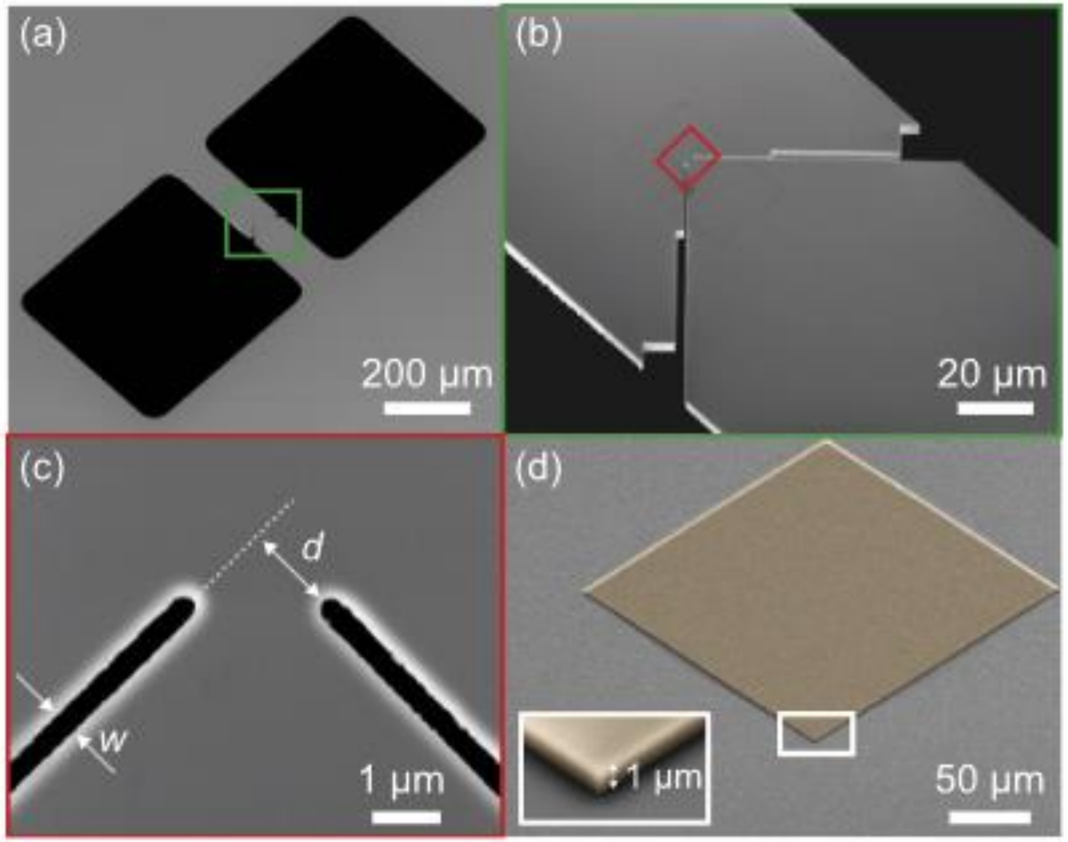
Transmon

Non-harmonic oscillator
Or
Capacitive shunted Josephson junction

arXiv:2103.01225v1
arXiv:1904.06560v3



Rasmussen, S., Christensen, et al (2021)



Tsioutsios I., Serniak K., et al (2019)

$$\hbar\,\omega_0 = \sqrt{8E_cE_J} \qquad \delta = -E_c$$

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Good to read

A Quantum Engineer's Guide to Superconducting Qubits

P. Krantz[1,2,†], M. Kjaergaard[1], F. Yan[1], T.P. Orlando[1], S. Gustavsson[1], and W. D. Oliver[1,3,‡]
[1] *Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*
[2] *Wallenberg Centre for Quantum Technology (WACQT), Chalmers University of Technology, Gothenburg, SE-41296, Sweden and*
[3] *MIT Lincoln Laboratory, 244 Wood Street, Lexington, MA 02420, USA*

(Dated: 9 July 2021)

arXiv:1904.06560

NATURE|Vol 453|19 June 2008|doi:10.1038/nature07128

doi:10.1038 / nature07128

Superconducting quantum bits

John Clarke[1,2] & Frank K. Wilhelm[3]

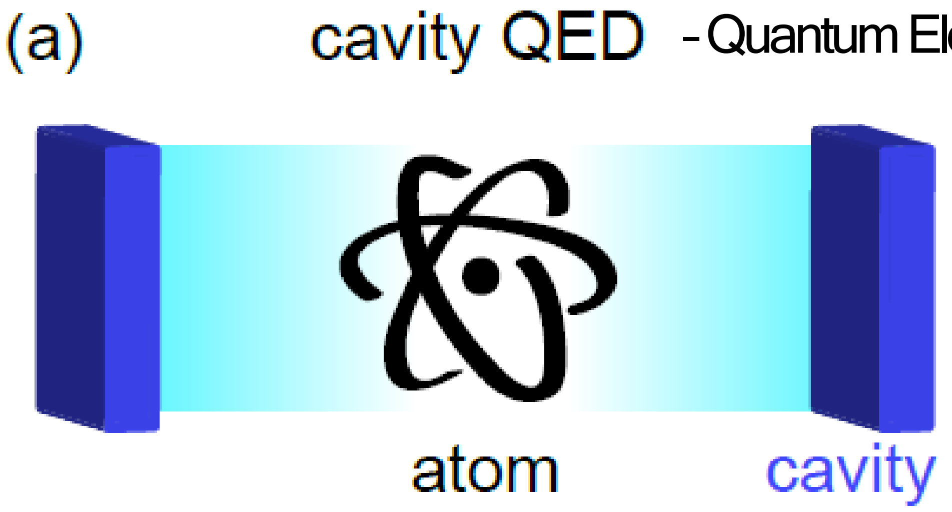
Break

- Questions? Discussion?

Rabi model, light-matter interaction

Interaction between single atom and electromagnetic field (light)

(Atom has a dipole moment and involved in dipole-dipole interaction) -> excitation, ionization

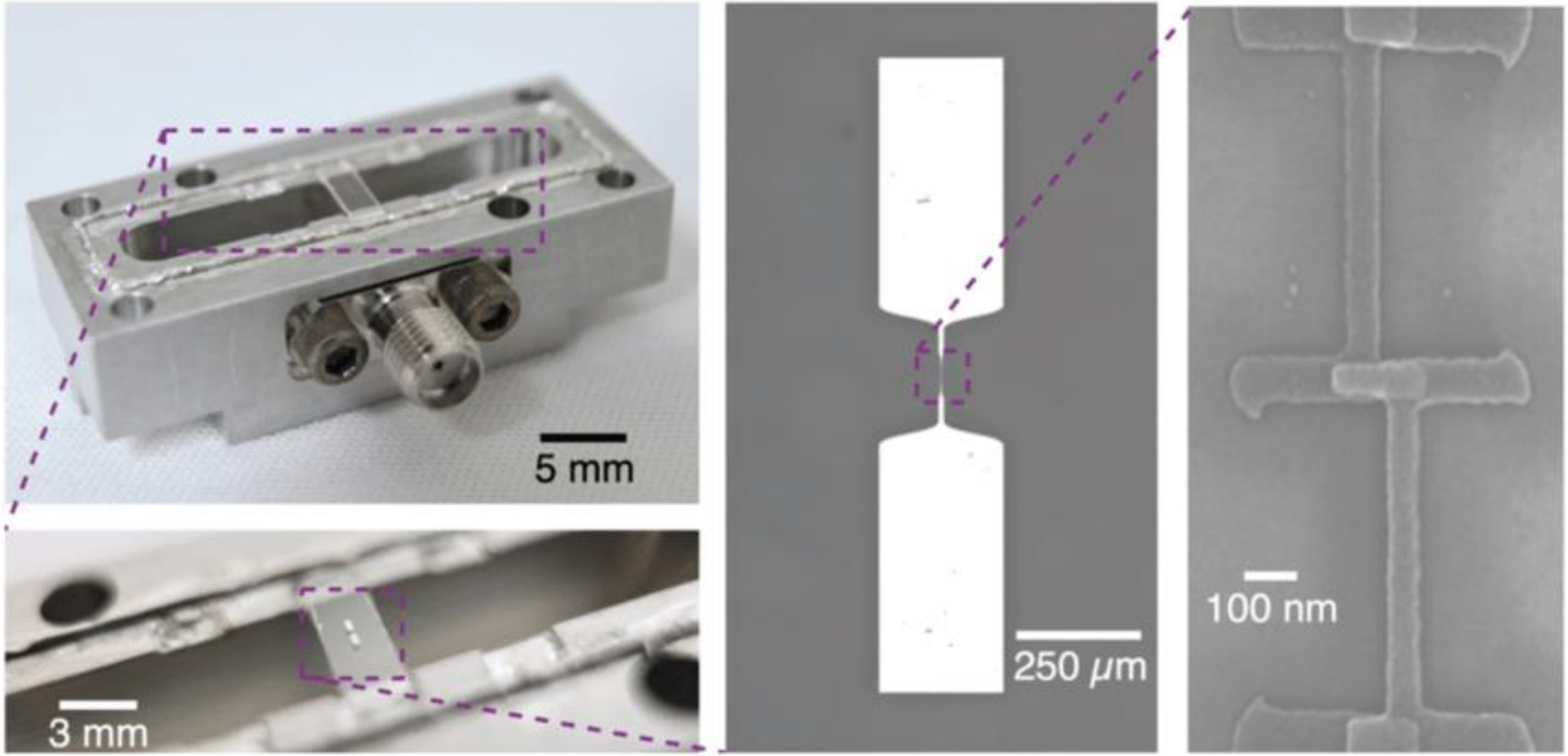


- Quantum Electro Dynamics

Rabi model, light-matter interaction

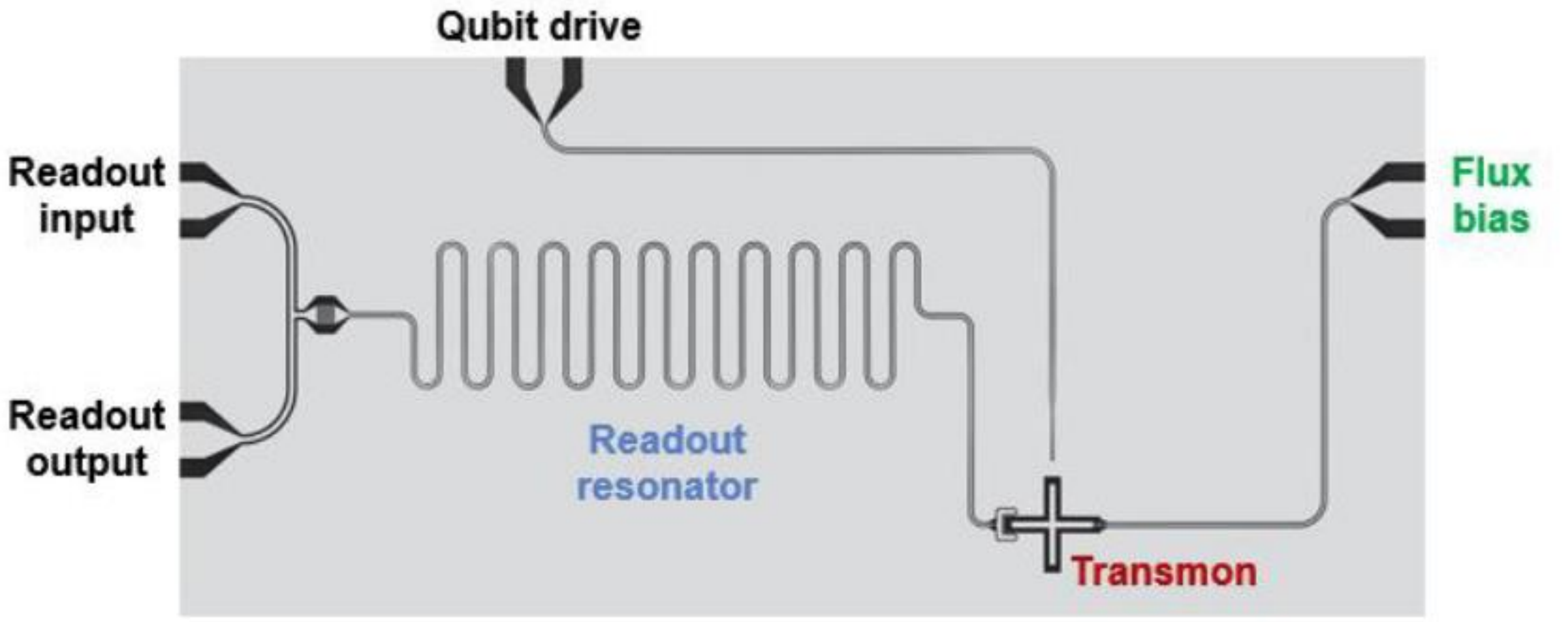
- Qubit can play a role of single atom
- Make qubit's dipole moment interact with microwave radiation -> useful for science
- (spoiler) this way we will also readout the qubit state

Transmon in resonator



Topel S., Serniak.K et al (2022)

Transmon coupled to CPW resonator



arXiv:2106.11352

Generalized light-matter interaction: The quantum Rabi model

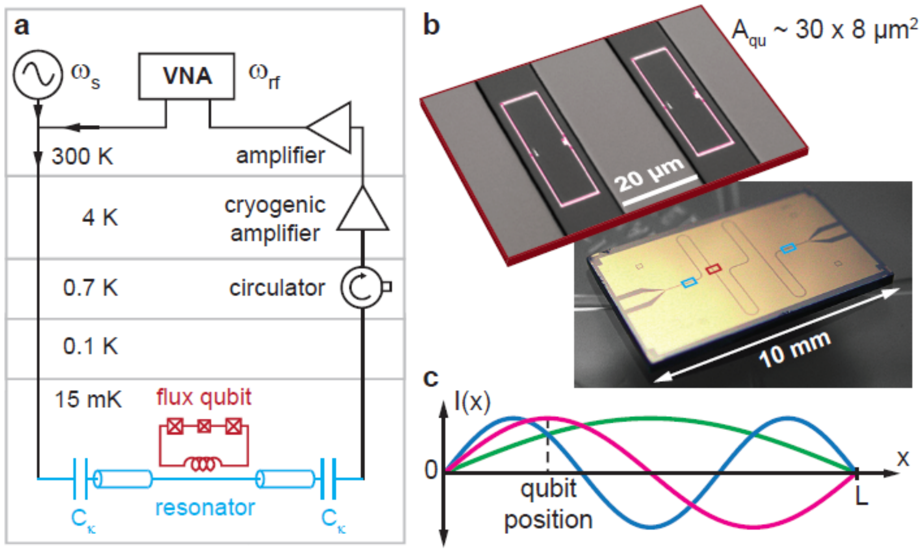


Figure 5.6: Measurement setup, images of the quantum circuit and sketch of the resonator's current distribution. (a) The amplified cavity transmission at ω_{rf} is probed using a vector network analyzer. For spectroscopy, a second tone ω_s can be applied to the cavity (light blue). For clarity, only one of the two qubits (dark red; crossed boxes represent Josephson junctions) is sketched. The microwave components are explained in the caption of Fig. 5.3. (b) Optical and false-color scanning electron images of the quantum circuit. The position of the flux qubits (magenta) is indicated by the red box and the light blue boxes mark the position of the coupling capacitors. (c) Sketch of the current distribution $I(x)$ of the first three resonator modes. Their resonance frequencies are: $\omega_1/2\pi = 2.624\,\mathrm{GHz}$ ($\lambda/2$-mode, green), $\omega_2/2\pi = 5.244\,\mathrm{GHz}$ (λ-mode, magenta) and $\omega_3/2\pi = 7.860\,\mathrm{GHz}$ ($3\lambda/2$-mode, blue). The cavity has a length $L = 23\,\mathrm{mm}$ and with $C_\kappa \sim 6\,\mathrm{fF}$, all quality factors $Q_n > 15 \cdot 10^3$.

We recall the following physical properties and their quantum mechanical description

Physical system / parameter	Effective Description
Qubit	$\mathcal{H}_q = \frac{\omega_q}{2}\hat{\sigma}_z\,.$
Resonator	$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)$
Magnetic field	$LI_{\mathrm{vac}}(\hat{a} + \hat{a}^\dagger)$
Qubit energy bias	$\varepsilon\left(\hat{\sigma}_+ + \hat{\sigma}_-\right)$

If we bring qubit and resonator in close vicinity, we create a mutual inductance leading to a coupling term (interaction Hamiltonian)

$$\hbar g\left(\hat{\sigma}_+ + \hat{\sigma}_-\right)\left(\hat{a} + \hat{a}^\dagger\right)$$

Here, we are hiding all physical properties in the coupling constant g

https://www.wmi.badw.de/publications/theses/Niemczyk_Doktorarbeit_2011.pdf

Generalized light-matter interaction: The quantum Rabi model

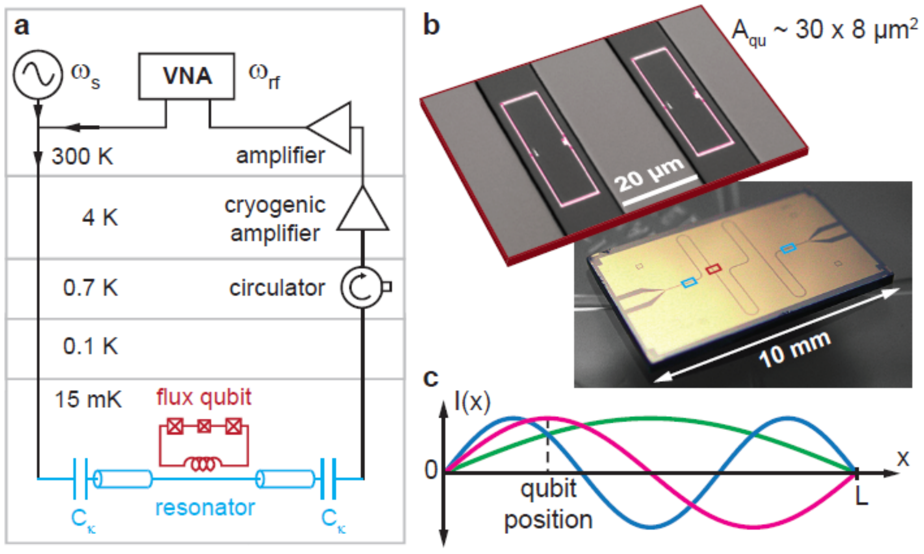
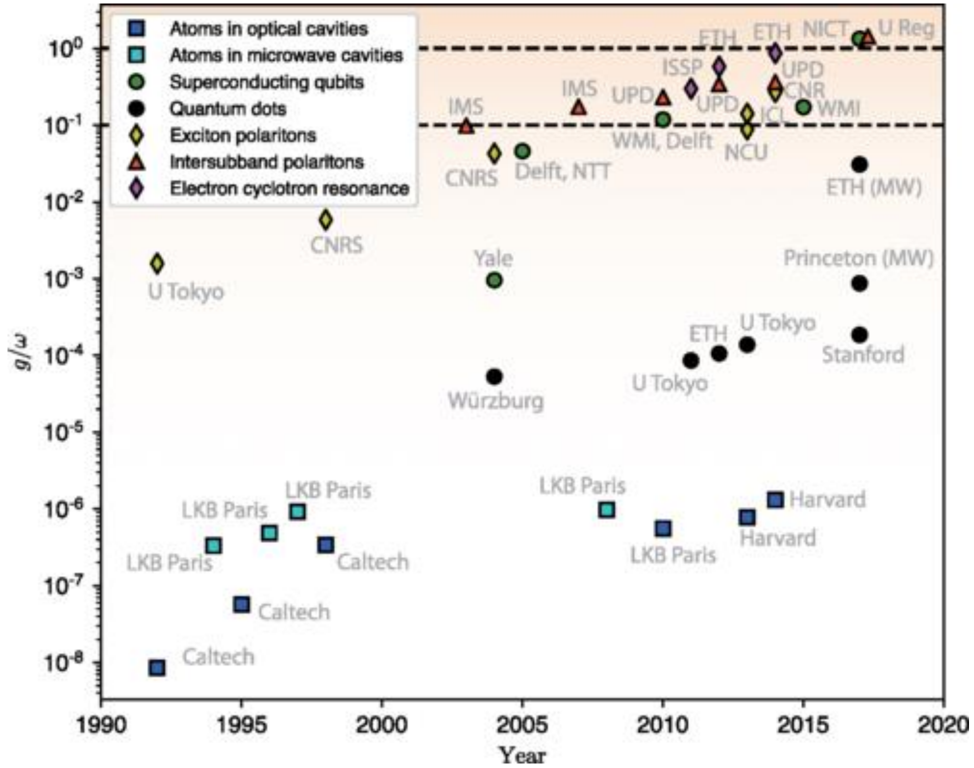


Figure 5.6: Measurement setup, images of the quantum circuit and sketch of the resonator's current distribution. (a) The amplified cavity transmission at $\omega_{\rm rf}$ is probed using a vector network analyzer. For spectroscopy, a second tone ω_s can be applied to the cavity (light blue). For clarity, only one of the two qubits (dark red; crossed boxes represent Josephson junctions) is sketched. The microwave components are explained in the caption of Fig. 5.3. (b) Optical and false-color scanning electron images of the quantum circuit. The position of the flux qubits (magenta) is indicated by the red box and the light blue boxes mark the position of the coupling capacitors. (c) Sketch of the current distribution $I(x)$ of the first three resonator modes. Their resonance frequencies are: $\omega_1/2\pi = 2.624\,\mathrm{GHz}$ ($\lambda/2$-mode, green), $\omega_2/2\pi = 5.244\,\mathrm{GHz}$ (λ-mode, magenta) and $\omega_3/2\pi = 7.860\,\mathrm{GHz}$ ($3\lambda/2$-mode, blue). The cavity has a length $L = 23\,\mathrm{mm}$ and with $C_\kappa \sim 6\,\mathrm{fF}$, all quality factors $Q_n > 15\cdot 10^3$.

Adding the individual terms for qubit and resonator results in the system Hamiltonian

$$\mathcal{H}_{\rm QR} = \frac{\hbar\omega_{\rm q}}{2}\hat{\sigma}_z + \hbar\omega_{\rm r}\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \underbrace{\hbar g\left(\hat{\sigma}_+ + \hat{\sigma}_-\right)\left(\hat{a} + \hat{a}^\dagger\right)}_{\mathcal{H}_{\rm int}}$$

The above Hamiltonian is valid in all regimes for g



https://www.wmi.badw.de/publications/theses/Niemczyk_Doktorarbeit_2011.pdf

General note: Circuit QED

- Qubits can be seen as artificial atoms and resonators as microwave light.

- When we bring them close to each other we create “light-matter” coupling that is treated in the same way as quantum optics.

- Superconducting circuits allow on-chip study of quantum optics in regimes that cannot be reached in nature.

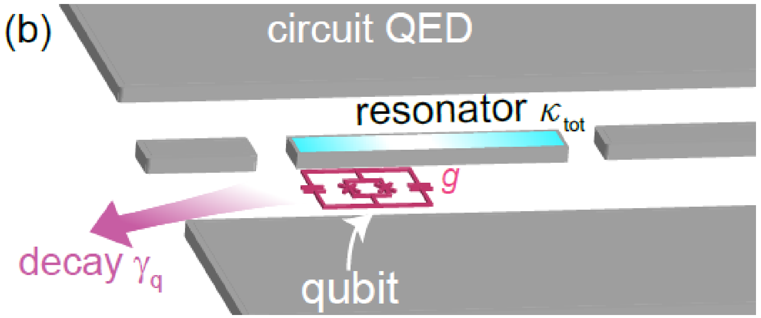
General note: Jaynes-Cummings model

- Usually one operates quantum circuits in a “practical” parameter regime, called strong coupling limit.

- In this limit, the coupling between qubit and electromagnetic field is much stronger as their loss rates but smaller than their eigenfrequencies.

- In this regime, the eigenstates experience a qubit state-dependent energy shift. Detecting this shift is used for qubit readout.

Rotating Wave approximation: Jaynes-Cummings model



We consider a transmon qubit that is capacitively coupled to a transmission line resonator. We operate in the strong coupling regime, where $g << \omega_q$, ω_r.

This allows us to move into the interaction picture (a.k.a. rotating frame) defined as

$$\hat{H}_{\text{int}}(t) = \frac{\hbar g}{2}\left(\hat{a}\hat{\sigma}_- e^{-i(\omega_r+\omega_q)t} + \hat{a}^\dagger\hat{\sigma}_+ e^{i(\omega_r+\omega_q)t} + \hat{a}\hat{\sigma}_+ e^{i(-\omega_r+\omega_q)t} + \hat{a}^\dagger\hat{\sigma}_- e^{-i(-\omega_r+\omega_q)t}\right).$$

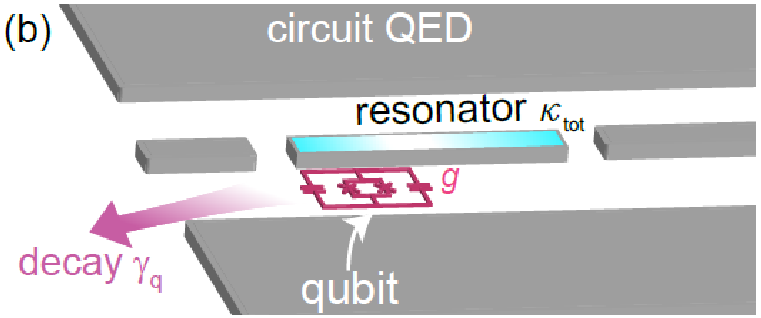
This Hamiltonian contains both quickly and slowly oscillating components

$$\omega_r + \omega_q \qquad\qquad \omega_r - \omega_q$$

To get a solvable model, the quickly oscillating "counter-rotating" terms, are ignored. This is referred to as the rotating wave approximation, and it is valid since the fast oscillating term couples states of comparatively large energy difference.

https://mediatum.ub.tum.de/1326240

Rotating Wave approximation: Jaynes-Cummings model



Transforming back into the Schrödinger picture the Jaynes-Cummings Hamiltonian is thus written as

$$\mathcal{H}_{\mathrm{JC}} = \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_{\mathrm{z}} + \hbar\omega_{\mathrm{r}}\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right) + \underbrace{\hbar g(\hat{\sigma}_{+}\hat{a} + \hat{\sigma}_{-}\hat{a}^{\dagger})}_{\mathcal{H}_{\mathrm{int}}} .$$

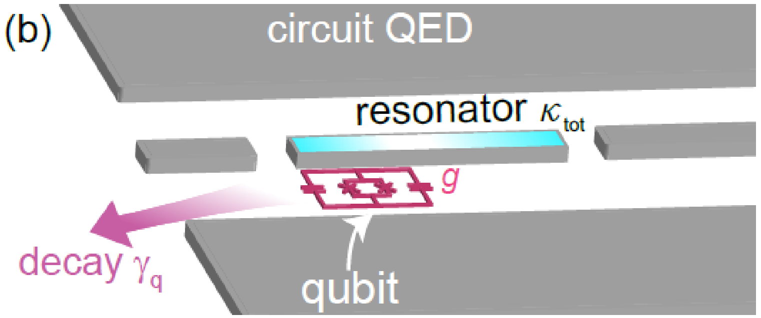
In the Jaynes-Cummings Hamiltonian, we can either excite the qubit by absorbing a photon $(\hat{\sigma}_{+}\hat{a})$ or take one excitation from the qubit and generate a photon $(\hat{\sigma}_{-}\hat{a}^{\dagger})$

In the basis of uncoupled resonator excitation number (n_r) and qubit eigenstates, the Hamiltonian is transformed to

$$\mathcal{H}_{\mathrm{JC},n} = \frac{\hbar}{2}\begin{pmatrix} 2n_{\mathrm{r}}\omega_{\mathrm{r}} + \omega_{\mathrm{q}} & g\sqrt{n_{\mathrm{r}}+1} \\ g\sqrt{n_{\mathrm{r}}+1} & (n_{\mathrm{r}}+1)\omega_{\mathrm{r}} - \omega_{\mathrm{q}} \end{pmatrix} .$$

We can diagonalize this Hamiltonian and discuss two parameter regimes: Resonant, i.e. no detuning between qubit and resonator, and off-resonant, i.e. large detuning.

Rotating Wave approximation: Jaynes-Cummings model



The eigenfrequencies of the Jaynes-Cummings Hamiltonian are given as

$$\omega_{\pm,n} = (n_r + 1/2)\omega_r \pm 1/2\sqrt{\delta^2 + 4g^2(n_r+1)}$$

Here, we have defined the detuning $\delta \equiv \omega_q - \omega_r$ and the ground state is $\omega_{-,0} = -\delta/2$.

The new dressed eigenstates of the system are the superposition states

$$|+,n_r\rangle = \cos\Theta_{n_r}|e,n_r\rangle + \sin\Theta_{n_r}|g,n_r\rangle$$

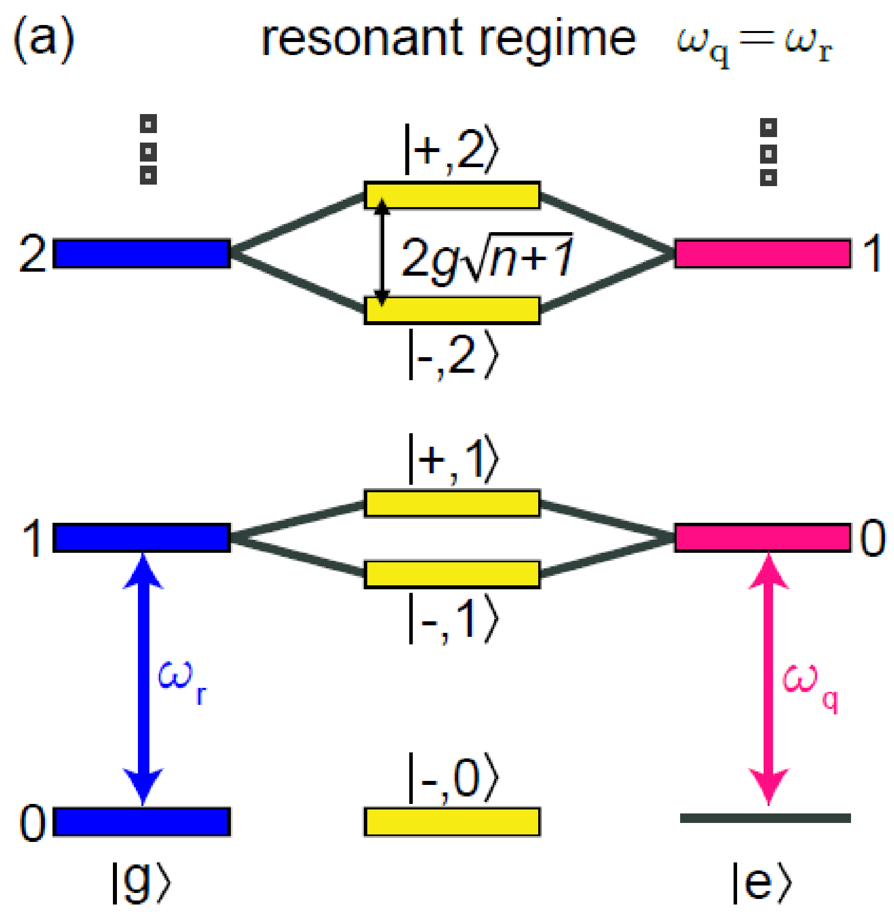
$$|-,n_r\rangle = \cos\Theta_{n_r}|g,n_r+1\rangle - \sin\Theta_{n_r}|e,n_r\rangle$$

Here, the mixing angle is a measure for the degree of entanglement between qubit and resonator states:

$$\Theta_n = \tan^{-1}(2g\sqrt{n_r+1}/\delta)/2$$

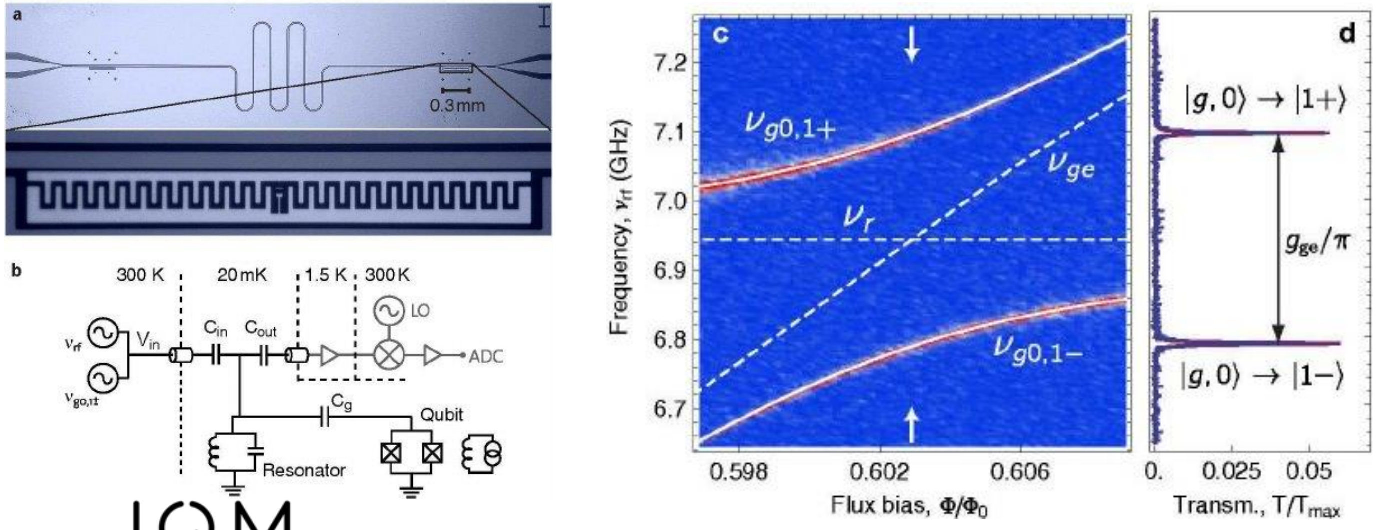
https://mediatum.ub.tum.de/1326240

Rotating Wave approximation: Jaynes-Cummings model



When qubit and light mode are on resonance, i.e., $\delta \simeq 0$ the mixing angle $\Theta_n = \pi/4$is maximum and consequently there is strong entanglement.

In this regime, a coherent exchange of excitations between qubit and resonator occurs with the vacuum Rabi frequency 2*g*. This interaction lifts the degeneracy of the corresponding eigenenergies By $2g\sqrt{n_r+1}$ to new doublet eigenstates.



https://arxiv.org/abs/0902.1827

https://mediatum.ub.tum.de/1326240

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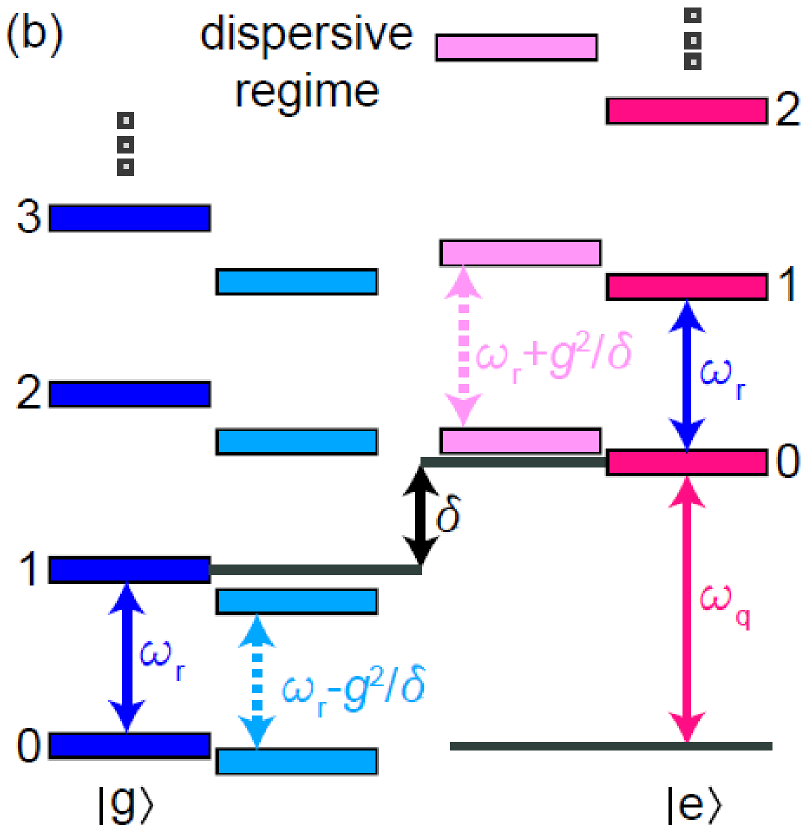


https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.94.123602

In the dispersive regime, the detuning between qubit and resonator frequency is much larger than the coupling, i.e., $\delta \gg g$.

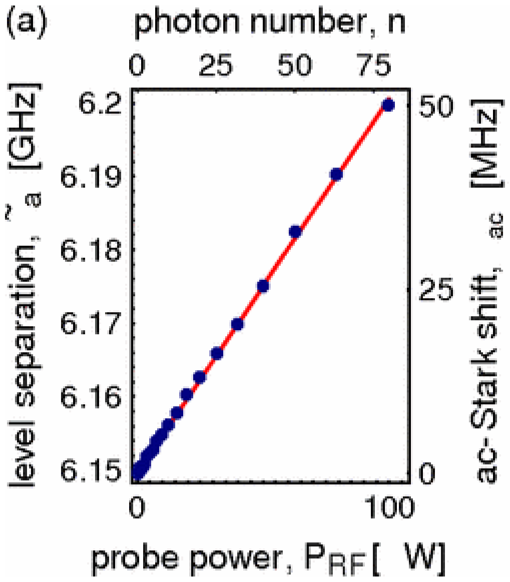
In this regime, there is no exchange of excitations anymore but virtual photons mediate a dispersive interaction between qubit and light field. This interaction leads to frequency shifts of the qubit and resonator eigenfrequencies. The dressed states are either more photon-like or more atom-like.

In the atom-like case (close to qubit states), the Hamiltonian can be derived as

$$\mathcal{H}_{\mathrm{disp}} \approx \hbar\omega_{\mathrm{r}} \left(\hat{a}^\dagger\hat{a} + 1/2\right) + \hbar/2 \left(\omega_{\mathrm{q}} + 2\chi\hat{a}^\dagger\hat{a} + \chi\right)\hat{\sigma}_z\,.$$

AC-Stark shift

Lamb shift



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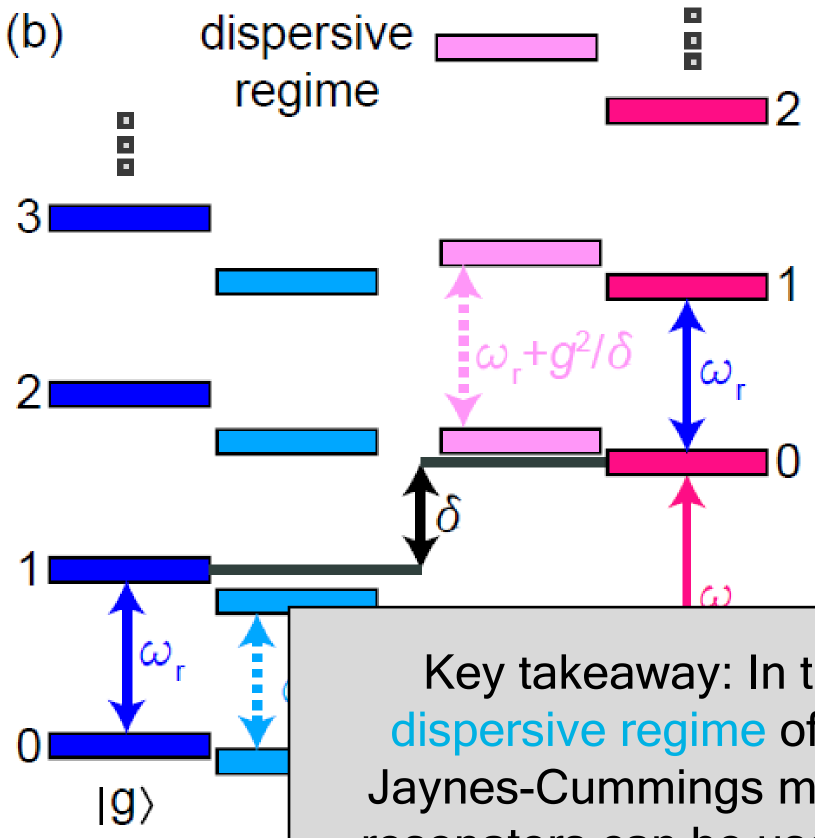


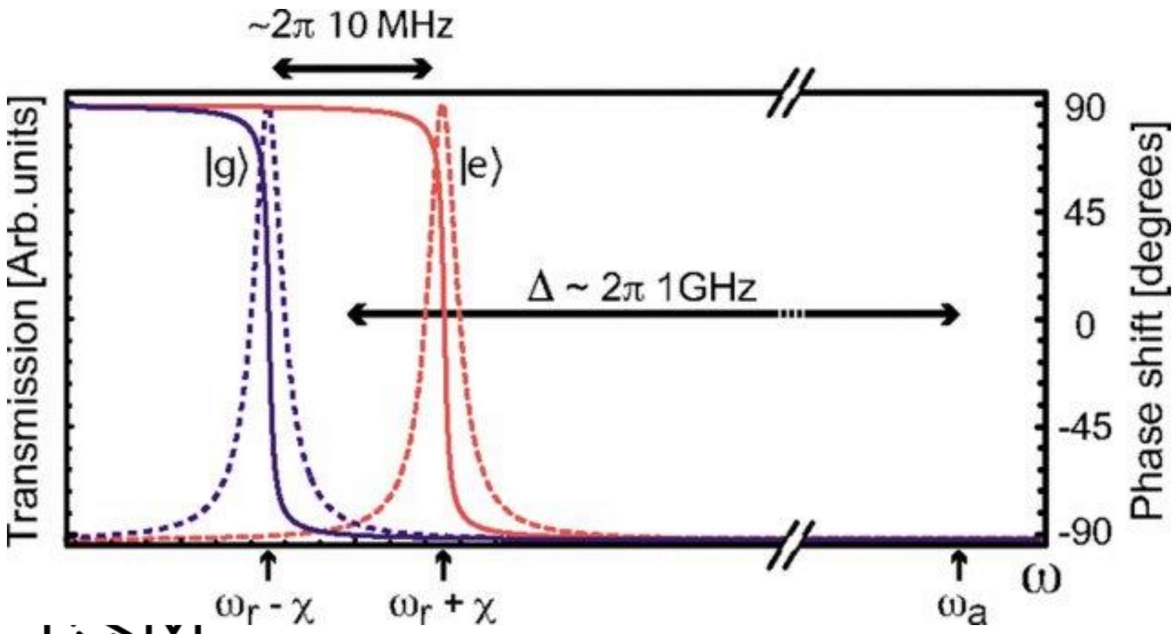
Key takeaway: In the dispersive regime of the Jaynes-Cummings model, resonators can be used for qubit readout.

https://journals.aps.org/pra/abstract/10.1103/PhysRevA.75.032329

In the photon-like case (close to resonator states), the Hamiltonian can be derived as

$$\mathcal{H}_{\mathrm{disp,r}} \approx \hbar\omega_{\mathrm{q}}\hat{\sigma}_{\mathrm{z}}/2 + \hbar\left(\omega_{\mathrm{r}} + \chi\hat{\sigma}_{\mathrm{z}}\right)\left(\hat{a}^{\dagger}\hat{a} + {}^{1}/_{2}\right)$$

describing the qubit state-dependent resonator frequency, which we use for readout purposes.



https://mediatum.ub.tum.de/1326240

General note: Jaynes-Cummings model

- Usually one operates quantum circuits in a “practical” parameter regime, called strong coupling limit.

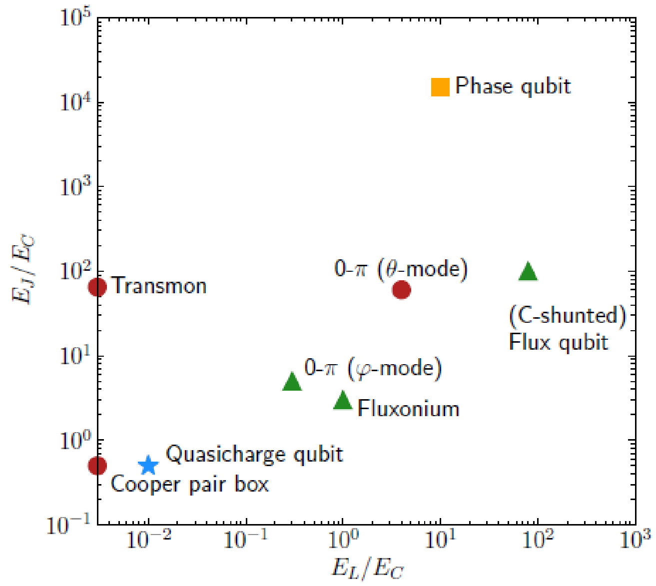
- In this limit, the coupling between qubit and electromagnetic field is much stronger as their loss rates but smaller than their eigenfrequencies.

- In this regime, the eigenstates experience a qubit state-dependent energy shift. Detecting this shift is used for qubit readout.

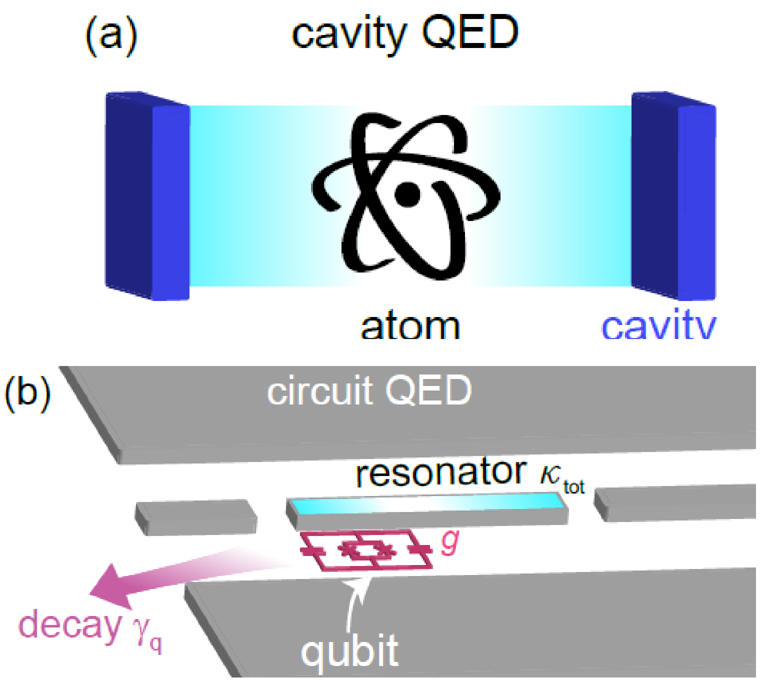
Agenda for today

8. Superconducting quantum circuits
   a. Qubits: Transmon qubit, Charge qubit, Flux qubit 1st DiVincenzo criteria
   b. Circuit-QED: Rabi model
   c. Rotating Wave approximation: Jaynes-Cummings model

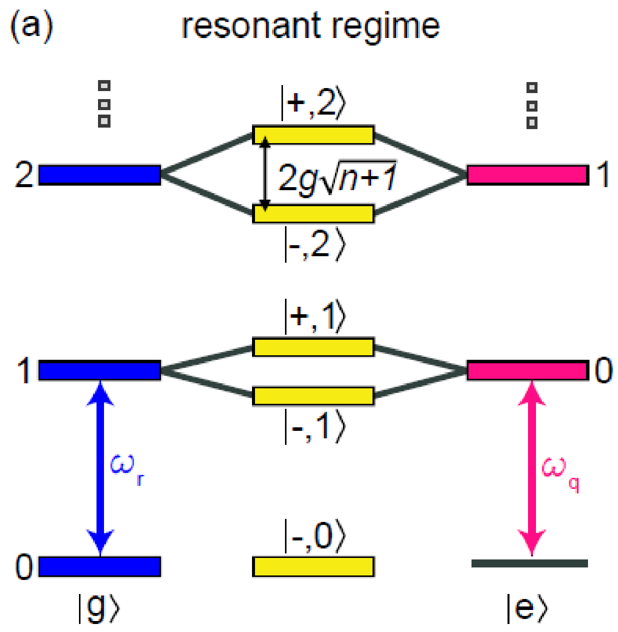
a.



b.



c.



IQM

Thank you