In the previous lecture ...

Fields of a plane wave

$\bar{E}(z) = \bar{a}\, E_+\, e^{-jkz}$ $\quad(\bar{a} \perp \bar{a}_z)$ ← arbitrary unit vector

$\bar{H}(z) = \frac{1}{\eta}(+\bar{a}_z) \times \bar{E}(z)$

← for waves travelling along +z

$\bar{E}(z) = \bar{a}\, E_-\, e^{+jkz}$

$\bar{H}(z) = \frac{1}{\eta}(-\bar{a}_z) \times \bar{E}(z)$

← for waves travelling along −z

Poynting vector

$\bar{S}(\bar{R},t) = E(\bar{R},t) \times \bar{H}(R,t)$ ← for general fields

$\langle \bar{S} \rangle_t = \frac{1}{2} Re(\bar{E} \times \bar{H}^*)$ ← for time-harmonic fields

$\langle \bar{S} \rangle_t = \frac{1}{2\eta} |E_0|^2 \frac{\bar{k}}{|\bar{k}|}$ ← for time-harmonic plane waves in a lossless medium

For a lossy medium

$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}$ ← complex permittivity

$k_c = \omega\sqrt{\varepsilon_c \mu}$ ← complex wavenumber

$\alpha = -Im(k_c) > 0$ ← attenuation constant

$\beta = Re(k_c) > 0$ ← phase constant

$\delta = \frac{1}{\alpha}$ ← penetration depth

For good conductor (very lossy medium)

$$\frac{\sigma}{\omega} >> \varepsilon$$ ← ε_c becomes purely imaginary

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$ ← penetration depth (skin depth)

Boundary conditions for tangential fields

$$E_{1,tan} = E_{2,tan} \quad (at \; z=0)$$

$$H_{1,tan} = H_{2,tan} \quad (at \; z=0)$$

Reflection & transmission coefficients for normal incidence

$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

← complex-valued field ratios

Reflectance & transmitance for normal incidence

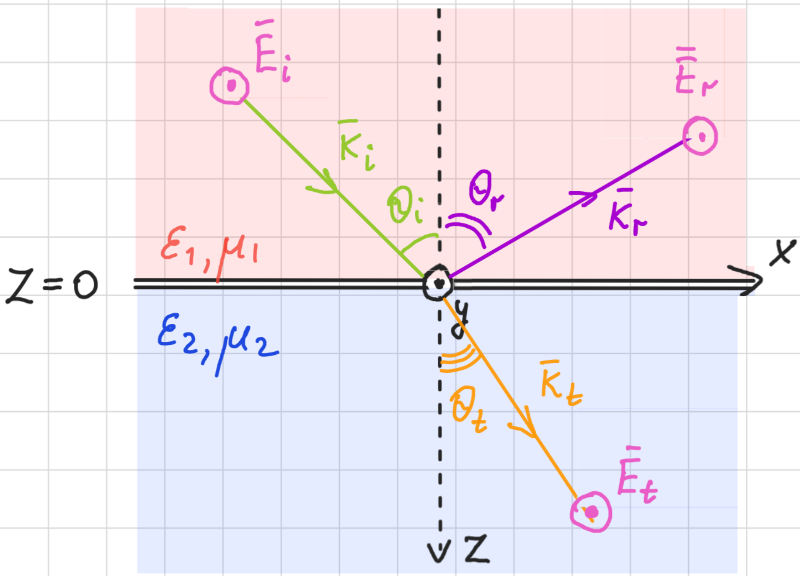
$$R = \frac{|\bar{S}_r|}{|\bar{S}_i|} = |\Gamma|^2$$

$$T = \frac{|\bar{S}_t|}{|\bar{S}_i|} = |\tau|^2 \frac{\eta_1}{\eta_2}$$

← real-valued energy ratios

$$R + T = 1$$ ← only for lossless case

Perpendicular polarization.



For $\bar{k} \parallel \bar{a}_z$: $\qquad \bar{E} = \bar{a} E_o e^{-jkz}$

For arbitrary $\bar{k}$: $\qquad \boxed{\bar{E} = \bar{a} E_o e^{-j\bar{k}\cdot\bar{R}} \quad (\bar{k} \perp \bar{a})}$

$$\bar{k}_i = k_1 \sin\theta_i \bar{a}_x + k_1 \cos\theta_i \bar{a}_z \qquad k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

$$\bar{k}_r = k_1 \sin\theta_r \bar{a}_x - k_1 \cos\theta_r \bar{a}_z$$

$$\bar{k}_t = k_2 \sin\theta_t \bar{a}_x + k_2 \cos\theta_t \bar{a}_z \qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$\bar{R} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$

$$\bar{E}_i = \bar{a}_y E_{io} e^{-jk_1 \sin\theta_i x} e^{-jk_1 \cos\theta_i z}$$

$$\bar{E}_r = \bar{a}_y E_{ro} e^{-jk_1 \sin\theta_r x} e^{+jk_1 \cos\theta_r z}$$

$$\bar{E}_t = \bar{a}_y E_{to} e^{-jk_2 \sin\theta_t x} e^{-jk_2 \cos\theta_t z}$$

$$(\bar{E}_i + \bar{E}_r)\cdot\bar{a}_y = \bar{E}_t \cdot \bar{a}_y \qquad (z=0)$$

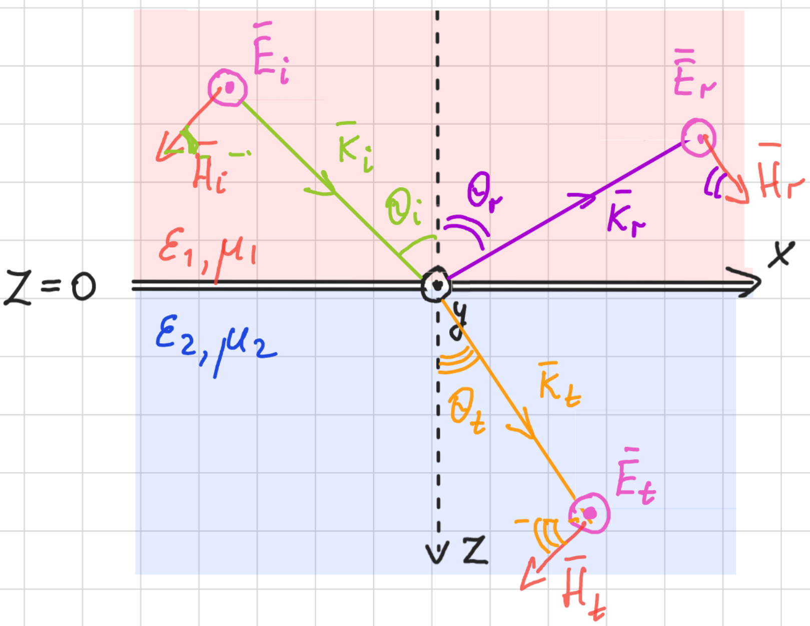
$$E_{io}\, e^{-jk_1 \sin\theta_i x} + E_{ro}\, e^{-jk_1 \sin\theta_r x} = E_{to}\, e^{-jk_2 \sin\theta_t x}$$

$$x\ \forall$$

$$\begin{cases} \boxed{E_{io} + E_{ro} = E_{to}} \quad (1) \\ e^{-jk_1 \sin\theta_i x} = e^{-jk_1 \sin\theta_r x} \rightarrow \boxed{\theta_i = \theta_r} \\ e^{-jk_1 \sin\theta_i x} = e^{-jk_2 \sin\theta_t x} \rightarrow k_1 \sin\theta_i = k_2 \sin\theta_t \end{cases}$$

$$\cancel{\frac{\omega}{c}}\, n_1 \sin\theta_i = \cancel{\frac{\omega}{c}}\, n_2 \sin\theta_t$$

Snell's law: $\boxed{\dfrac{\sin\theta_i}{\sin\theta_t} = \dfrac{n_2}{n_1}}$



1) $\bar{E} \times \bar{H} \sim \bar{k}$

2) $|\bar{E}| = \eta_{1,2}\, |\bar{H}|$

$$\frac{|\bar{E}|}{\eta_{1,2}} = |\bar{H}|$$

$$\bar{H}_i = \left(-\cos\theta_i\, \bar{a}_x + \sin\theta_i\, \bar{a}_z\right) \frac{E_{io}}{\eta_1}\, e^{-jk_1 \sin\theta_i x}\, e^{-jk_1 \cos\theta_i z}$$

$$\bar{H}_r = \left(+\cos\theta_r\,\bar{a}_x + \sin\theta_r\,\bar{a}_z\right)\frac{E_{ro}}{\eta_1}\,e^{-jk_1\sin\theta_r x}\,e^{+jk_1\cos\theta_r z}$$

$$\bar{H}_t = \left(-\cos\theta_t\,\bar{a}_x + \sin\theta_t\,\bar{a}_z\right)\frac{E_{to}}{\eta_2}\,e^{-jk_2\sin\theta_t x}\,e^{-jk_2\cos\theta_t z}$$

$$H_{ix} + H_{rx} = H_{tx} \quad (z=0)$$

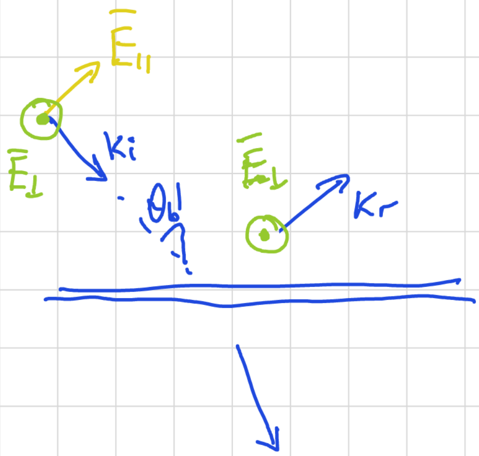
$$\boxed{-\cos\theta_i\,\frac{E_{io}}{\eta_1} + \cos\theta_i\,\frac{E_{ro}}{\eta_1} = -\cos\theta_t\,\frac{E_{to}}{\eta_2}} \qquad (2)$$

$$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\Gamma_\parallel = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

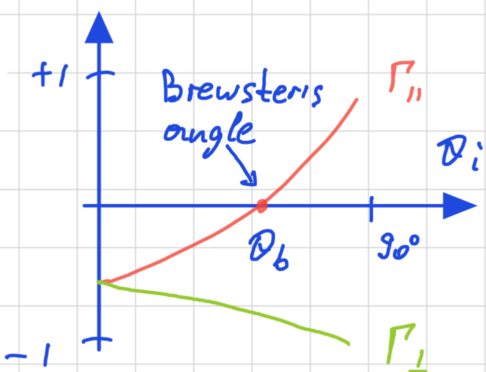
$$\tau_\parallel = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$



$\theta_b = 53°$ (air–water)

When $\varepsilon_{r1} = \mu_{r1} = \mu_{r2} = 1$: $\quad \sqrt{\varepsilon_{r2}} = n$

$$\Gamma_\parallel = \frac{\cos\theta_t - n\cos\theta_i}{\cos\theta_t + n\cos\theta_i} \qquad \theta_b = \operatorname{atan}(n)$$



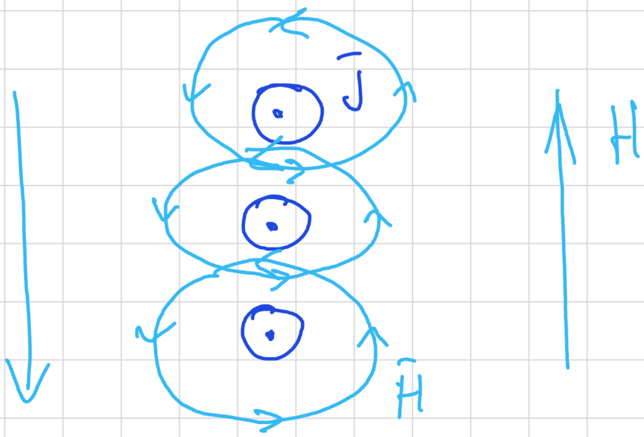
Show slides

Topics for this week:

Hertzian dipole and its radiation

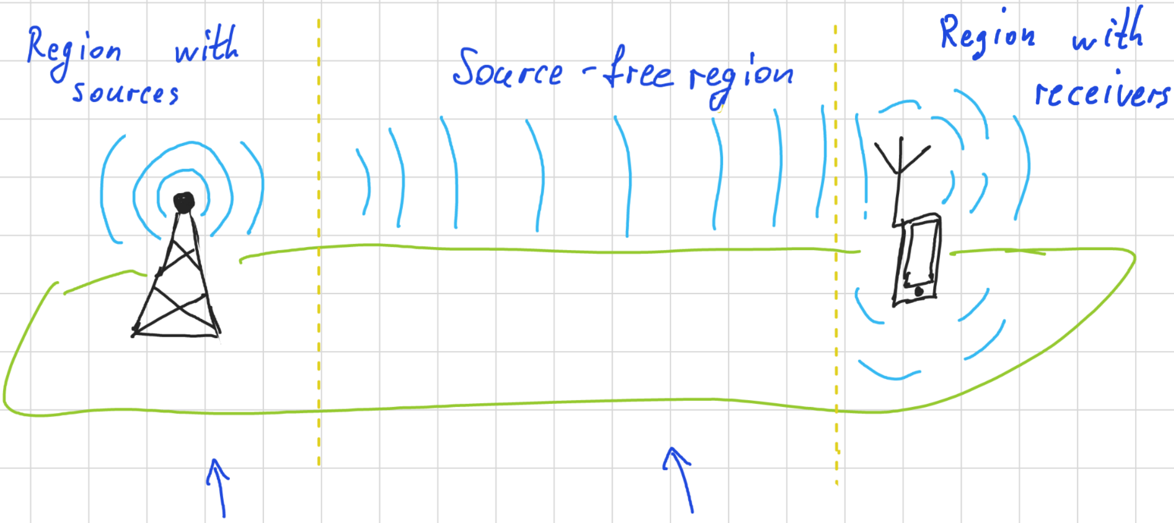
Friis transmission formula

top view



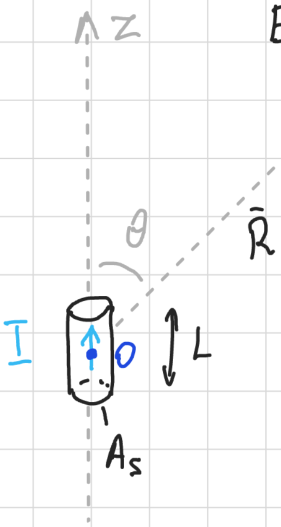
Hertzian dipole and its radiation

Illustration of plane wave generation & propagation



Region with sources	Source-free region
Find $V(\bar{R},t)$, $\bar{A}(\bar{R},t)$	Directly find $\bar{E}(\bar{R},t)$
Find $\bar{E}(\bar{R},t)$, $\bar{H}(\bar{R},t)$	$\bar{H}(\bar{R},t)$

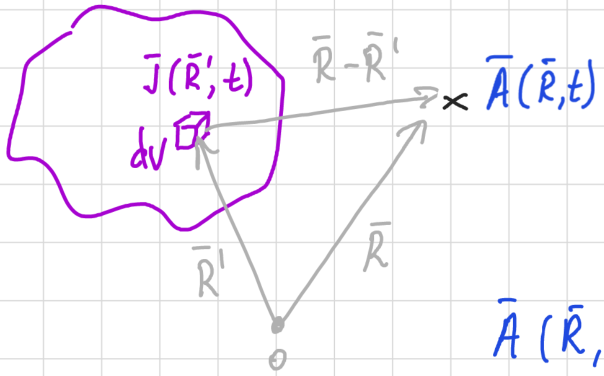
$\bar{E}(\bar{R},t)$, $\bar{H}(\bar{R},t)$ – ?



Hertzian dipole: $i(t) = Re(I e^{j\omega t})$

$$\bar{A}(\bar{R},t) \rightarrow \bar{B}(\bar{R},t) \rightarrow \bar{E}(\bar{R},t)$$

$$\bar{B} = \nabla \times \bar{A} \qquad \nabla \times \bar{H} = j\omega \bar{D} = j\omega \varepsilon_0 \bar{E}$$



$$\bar{A}(\bar{R},t) = \int_V \frac{\mu_0 \, \bar{J}\left[\bar{R}', t - \frac{|\bar{R}-\bar{R}'|}{v}\right]}{4\pi |\bar{R}-\bar{R}'|} dV'$$

$$Re\left(\bar{A}(R)\, e^{j\omega t}\right) = \frac{\mu_0}{4\pi} \int_V \frac{Re\left(\bar{J}(\bar{R}')\, e^{j\omega\left[t - \frac{\bar{R}-\bar{R}'}{c}\right]}\right.}{|\bar{R}-\bar{R}'|} dV'$$

$$\bar{A}(\bar{R}) = \frac{\mu_0}{4\pi} \int_V \frac{\bar{J}(\bar{R}')\, e^{-jk_0|\bar{R}-\bar{R}'|}}{|\bar{R}-\bar{R}'|} dV'$$

$\bar{R}' = 0$

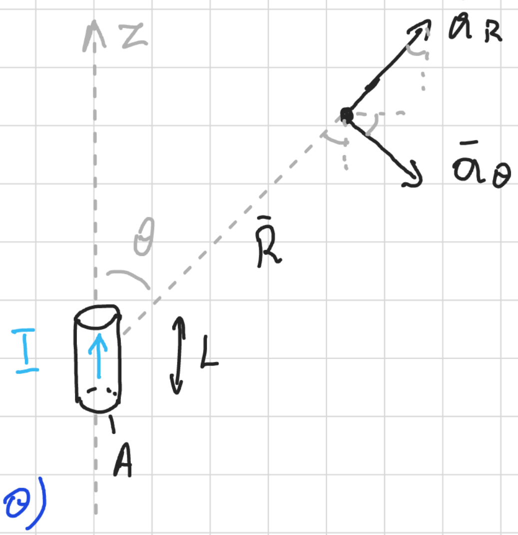
$$\bar{A}(\bar{R}) = \frac{\mu_0}{4\pi} \int_V \frac{\bar{J}(0)\, e^{-jk_0 R}}{R} dV$$

$$J = \frac{dI}{dS}$$

$$\bar{J}(0)\, dv = \underbrace{\bar{J}(0)\, S_A}\, L = I L\, \bar{a}_z$$

$$\bar{A}(\bar{R}) = \frac{\mu_0}{4\pi} \frac{I L\, e^{-jk_0 R}}{R} \bar{a}_z \leftarrow \text{Cartesian CS}$$

$$\bar{a}_z = \bar{a}_R \cos\theta - \bar{a}_\theta \sin\theta$$



$$\bar{A}(\bar{R}) = \frac{\mu_0}{4\pi} \frac{I L e^{-jk_0 R}}{R} (\bar{a}_R \cos\theta - \bar{a}_\theta \sin\theta)$$

$$\boxed{\begin{aligned} \bar{H}(R,\theta) &= \frac{\nabla \times \bar{A}(R,\theta)}{\mu_0} = \dots \\ &= \bar{a}_\varphi \; j k_0 I L \frac{e^{-jk_0 R}}{4\pi R} \sin\theta \left(1 + \frac{1}{jk_0 R}\right) \end{aligned}}$$

$$\nabla \times \bar{H} = j\omega \bar{D} + \cancel{\bar{J}}$$

$$\nabla \times \bar{H} = j\omega\varepsilon_0 \bar{E}$$

$$\boxed{\begin{aligned} \bar{E}(R,\theta) &= \frac{\nabla \times \bar{H}}{j\omega\varepsilon_0} = j\omega\mu_0 I L \frac{e^{-jk_0 R}}{4\pi R} \\ &\cdot \left[\bar{a}_R \, 2\cos\theta \left(\frac{1}{jk_0 R} + \frac{1}{(jk_0 R)^2}\right) + \bar{a}_\theta \sin\theta \left(1 + \frac{1}{jk_0 R} + \frac{1}{(jk_0 R)^2}\right)\right] \end{aligned}}$$

Animation (slide 21)

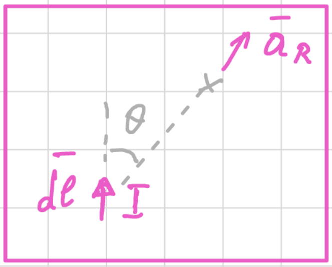
Near field $R << \lambda_0$

$$k_0 R = \frac{2\pi}{\lambda} R << 1 \qquad e^{-jk_0 R} \approx 1$$

$$\bar{H} = \bar{a}_\varphi \, IL \, \frac{1}{4\pi R^2} \sin\theta \leftarrow \text{phasor}$$

From past lecture (Magnetostatics):

$$\bar{B} = \frac{\mu_0 I \, \bar{dl} \times \bar{a}_R}{4\pi R^2} \rightarrow \bar{H} = \frac{I \, dl \sin\theta}{4\pi R^2} \bar{a}_\varphi$$



real-valued field ($\bar{H} \neq \bar{H}(t)$)

$$\bar{E}(R,\theta) = j\omega\mu_0 IL \frac{1}{4\pi R}$$

$$\cdot \left[\bar{a}_R \, 2\cos\theta \left(\cancel{\frac{1}{jk_0 R}} + \frac{1}{(jk_0R)^2} \right) + \bar{a}_\theta \sin\theta \left(\cancel{1} + \cancel{\frac{1}{jk_0R}} + \frac{1}{(jk_0R)^2} \right) \right]$$

$$= j\omega\mu_0 IL \frac{1}{4\pi R} \left[\bar{a}_R \, 2\cos\theta \frac{1}{(jk_0R)^2} + \bar{a}_\theta \sin\theta \frac{1}{(jk_0R)^2} \right]$$

$$= -j \frac{IL}{4\pi \, \omega \, \varepsilon_0 \, R^3} \left[\bar{a}_R \cdot 2\cos\theta + \bar{a}_\theta \sin\theta \right]$$

$$= \frac{IL}{j\omega} \frac{1}{4\pi\varepsilon_0 R^3} \left[\bar{a}_R \cdot 2\cos\theta + \bar{a}_\theta \sin\theta \right]$$

$$\frac{IL}{j\omega} = p$$

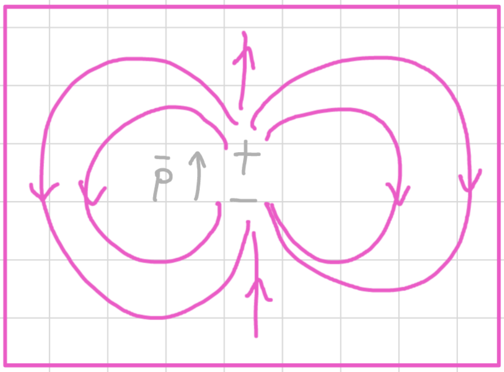
$$p = QL \qquad \frac{dp}{dt} = IL \qquad \frac{dp}{dt} = j\omega p = IL$$

$$p = IL/j\omega$$

$$\bar{E}(\bar{R},\theta) = \frac{p}{4\pi\epsilon_0 R^3}\left[\bar{a}_R \cdot 2\cos\theta + \bar{a}_\theta \sin\theta\right]$$

From past lecture (Electrostatics):

$$\bar{E}_d(R,\theta) = \frac{p}{4\pi\epsilon R^3}\left(\bar{a}_R 2\cos\theta + \bar{a}_\theta \sin\theta\right) \leftarrow \text{static field}$$



$\bar{H}(\bar{R},\theta)$ and $\bar{E}(\bar{R},\theta)$ have $\pi/2$ phase difference

$$\bar{S} = \frac{1}{2}\mathrm{Re}\left(\underbrace{\bar{E}\times\bar{H}\, e^{2j\omega t}}\right) + \frac{1}{2}\mathrm{Re}\left(\bar{E}\times\bar{H}^*\right) \quad {=0}$$

$$\mathrm{Re}\left(\bar{E}\cdot\alpha\bar{E}^*(-j)\right)$$

$$\mathrm{Re}\left(-|\bar{E}|^2 j\alpha\right)$$

Animation (slide 22)

Far field $R >> \lambda_0$

$k_0 R >> 1 \qquad \frac{1}{|jk_0R|} << 1$

$$\bar{H}(\bar{R},\theta) = \bar{a}_\varphi \; jk_0 I L \frac{e^{-jk_0R}}{4\pi R} \sin\theta$$

$$\bar{E}(R,\theta) = \frac{\nabla \times \bar{H}}{j\omega\varepsilon_0} = j\omega\mu_0 I L \frac{e^{-jk_0R}}{4\pi R}$$

$$\cdot \left[\bar{a}_R \, 2\cos\theta \left(\cancel{\frac{1}{jk_0R}} + \cancel{\frac{1}{(jk_0R)^2}} \right) + \bar{a}_\theta \sin\theta \left(1 + \cancel{\frac{1}{jk_0R}} + \cancel{\frac{1}{(jk_0R)^2}} \right) \right]$$

$$\boxed{\begin{aligned} \bar{E}(R,\theta) &= j\omega\mu_0 I L \frac{e^{-jk_0R}}{4\pi R} \sin\theta \; \bar{a}_\theta \\ \bar{H}(\bar{R},\theta) &= jk_0 I L \frac{e^{-jk_0R}}{4\pi R} \sin\theta \; \bar{a}_\varphi \end{aligned}}$$

$$E/H = \frac{\omega\mu_0}{k_0} = \frac{\omega\mu_0}{\omega\sqrt{\varepsilon_0\mu_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0$$

Animation (slides 22-23)

$$\bar{S} = \frac{1}{2} Re\left(\bar{E} \times \bar{H} \, e^{2j\omega t} \right) + \frac{1}{2} Re\left(\bar{E} \times \bar{H}^* \right)$$

nonzero

nonzero

In the previous lecture...

Fields of a plane wave propagating at arbitrary $\bar{a}_k$

$$\bar{E}(\bar{R},t) = \bar{a}\, E_0\, e^{-j\bar{k}\cdot\bar{R}} \quad (\bar{k} = \bar{a}_k k \perp \bar{a})$$

$$\bar{H}(\bar{R},t) = \frac{\bar{k}\times\bar{E}(\bar{R},t)}{\omega\mu}$$ see derivation in eq. (7-25) in the coursebook

Relations for angles at an interface of 2 dielectrics

$$\theta_r = \theta_i$$ reflection law

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1}$$ Snell's law

$$\theta_b = \operatorname{atan}\left(\frac{n_2}{n_1}\right)$$ Brewster's angle

Reflection & transmission coefficients for oblique incidence

$$\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$ s-polarization

$$\tau_\perp = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$ s-polarization

$$\Gamma_\parallel = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$ p-polarization

$$\tau_\parallel = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$ p-polarization

Retarded magnetic potential for time-harmonic sources

$$\bar{A}(\bar{R}) = \frac{\mu_0}{4\pi} \int_V \frac{\bar{J}(\bar{R}')\, e^{-jk_0|\bar{R}-\bar{R}'|}}{|\bar{R}-\bar{R}'|}\, dV'$$ phasor form

Fields radiated by a Hertzian dipole

$$\bar{E}(R,\theta) = j\omega\mu_0 I L \frac{e^{-jk_0R}}{4\pi R} \cdot \left[\bar{a}_R\, 2\cos\theta \left(\frac{1}{jk_0R} + \frac{1}{(jk_0R)^2}\right) + \bar{a}_\theta \sin\theta \left(1 + \frac{1}{jk_0R} + \frac{1}{(jk_0R)^2}\right)\right]$$

$$\bar{H}(\bar{R}) = \bar{a}_\varphi\, jk_0 I L \frac{e^{-jk_0R}}{4\pi R} \sin\theta \left(1 + \frac{1}{jk_0R}\right)$$

Fields in the far zone $(R \gg \lambda_0)$

$$\bar{E}(R,\theta) = j\omega\mu_0 I L \frac{e^{-jk_0R}}{4\pi R} \sin\theta\ \bar{a}_\theta$$

$$\bar{H}(R,\theta) = jk_0 I L \frac{e^{-jk_0R}}{4\pi R} \sin\theta\ \bar{a}_\varphi$$ $|\bar{H}(R,\theta)| = \frac{1}{\eta}|\bar{E}(R,\theta)|$

Slides 23-24

Poynting's vector for Hertzian dipole (in the far zone)

$$\langle S\rangle_t(\bar{R}) = \frac{1}{2}\,\mathrm{Re}\left[\bar{E}(\bar{R})\times\bar{H}^*(\bar{R})\right]$$

$$= \frac{1}{2}\,\mathrm{Re}\left[\cancel{j}\,\overset{k_0 c}{\overbrace{\omega\mu_0}}\,IL\,\frac{\cancel{e^{-jk_0R}}}{4\pi R}\sin\theta\;\bar{a}_\theta\times\bar{a}_\varphi\,(-\cancel{j})\,k_0\,\bar{I}L\,\frac{\cancel{e^{+jk_0R}}}{4\pi R}\sin\theta\right]$$

$$= \frac{1}{2}\,k_0^2\,\underbrace{c\,\mu_0}\,(IL)^2\sin^2\theta\,\frac{1}{(4\pi R)^2}\,\bar{a}_R = \frac{\eta_0}{2}\left(\frac{k_0 IL}{4\pi R}\right)^2\sin^2\theta\;\bar{a}_R$$

$$c\mu_0 = \frac{\mu_0}{\sqrt{\varepsilon_0\mu_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0$$

1) Poynting vector has only radial component (spherical-like wave)

2) The wave has polar-nonuniform intensity

3) $S \sim \frac{1}{R^2}$. Slide 25

Total radiated power by a Hertzian dipole

$$P_{rad} = \oint_{S_R} \langle\bar{S}\rangle_t(R,\theta)\cdot \underbrace{d\bar{S}_R}$$

$$= \frac{\eta_0}{2}\left(\frac{k_0 IL}{4\pi \cancel{R}}\right)^2 \int_0^{\pi}\int_0^{2\pi} \sin^2\theta\;\cancel{R^2}\sin\theta\,d\theta\,d\varphi\,\overset{1}{\overbrace{(\bar{a}_R\cdot\bar{a}_R)}}$$

Slide 26

$$= \frac{\eta_0}{2}\left(\frac{k_0 \hat{I} L}{4\pi}\right)^2 \int_0^{2\pi} d\varphi \int_0^{\pi} \sin^3\theta \, d\theta$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = \int_0^{\pi} \sin\theta \, d\theta \cdot \sin^2\theta = -\int_0^{\pi} (1-\cos^2\theta) \, d(\cos\theta)$$

$$= -\left[\cos\theta - \frac{\cos^3\theta}{3}\right]\Big|_0^{\pi} = -\left[-1 + \frac{1}{3} - 1 + \frac{1}{3}\right] =$$

$$= +\frac{4}{3}$$

$$P_{rad} = \frac{\eta_0}{2} \frac{(k_0 \hat{I} L)^2}{16\pi^2} \cdot \frac{2\pi \cdot 4}{3} = \frac{\eta_0}{12\pi} (k_0 \hat{I} L)^2$$

Independent of R

Exercise:

$$\oint_S d\Omega = ? \qquad d\Omega = \sin\theta \, d\theta \, d\varphi$$

$$\int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi = -\cos\theta\Big|_0^{\pi} \cdot 2\pi = 4\pi$$

$$[\Omega] = sr \qquad (\text{steradian})$$

Characteristics of an arbitrary antenna

Definition of an antenna.

In the far zone, what is general form of $\langle \bar{S} \rangle_t$?

$$\langle \bar{S} \rangle_t (\bar{R}) = \frac{U(\theta,\varphi)}{R^2}\, \bar{a}_R$$ ← energy conservation

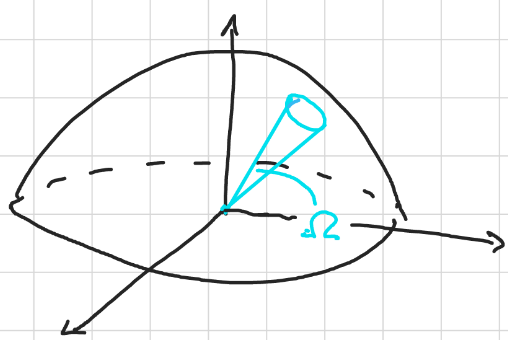
$U(\theta,\varphi)$ ← radiation intensity

$$U(\theta,\varphi) = \frac{\eta_0}{2}\left(\frac{k_0 I L}{4\pi}\right)^2 \sin^2\theta$$ ← for Hertzian dipole

$$P_{rad} = \oint_{S_R} \langle \bar{S} \rangle_t (\bar{R}) \cdot \bar{a}_R \, R^2 \, d\Omega = \oint_{S_R} U(\theta,\varphi)\, d\Omega$$

$$\boxed{P_{rad} = \oint_{S_R} U(\theta,\varphi)\, d\Omega}$$

$[U] = \frac{W}{sr}$ ← angular density



$$\frac{U(\theta,\varphi)}{U_{max}} = |F(\theta,\varphi)|^2 \leftarrow \text{unitless}$$
← power pattern

$$|F(\theta,\varphi)| = \frac{|\bar{E}(R,\theta,\varphi)|}{|\bar{E}_{max}|}$$
← radiation pattern

$$|F(\theta,\varphi)| = \sin\theta$$
$$|F(\theta,\varphi)|^2 = \sin^2\theta$$
← for Hertzian dipole

We want often directive antennas

Slides 27-28

$$D(\theta,\varphi) = \frac{U(\theta,\varphi)}{\langle U \rangle}$$
← directivity

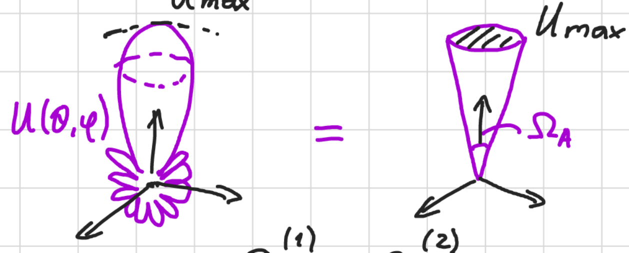
$$\langle U \rangle = \frac{\oint U(\theta,\varphi)\, d\Omega}{\oint d\Omega = 4\pi}$$

$$D(\theta,\varphi) = \frac{U(\theta,\varphi)}{\frac{1}{4\pi}\oint U(\theta,\varphi)\, d\Omega} = \frac{4\pi \cancel{U_{max}} |F(\theta,\varphi)|^2}{\oint \cancel{U_{max}} |F(\theta,\varphi)|^2 d\Omega}$$

$$\oint |F(\theta,\varphi)|^2 d\Omega = \Omega_A$$
← beam solid angle

$$\oint \frac{U(\theta,\varphi)}{U_{max}} d\Omega = \Omega_A$$

$$\oint U(\theta,\varphi)\, d\Omega = U_{max}\, \Omega_A$$



$$P_{rad}^{(1)} = P_{rad}^{(2)}$$

$$\mathcal{D}(\theta,\varphi) = \frac{4\pi \; |F(\theta,\varphi)|^2}{\Omega_A}$$

$$\mathcal{D}_{max} = \frac{4\pi}{\Omega_A}$$ ← maximal directivity

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$$\Omega_A = \oint \sin^2\theta \, d\Omega = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin^3\theta \, d\theta = \frac{8\pi}{3}$$

← for Hertzian dipole

$$\mathcal{D}_{max} = \frac{4\pi \cdot 3}{8\pi} = 1{,}5$$

What is $\mathcal{D}_{max}$ for an ideal isotropic antenna?

$$\mathcal{D}^{(iso)}_{max} = 1$$

Decibel

10^{12}

	Linear	→	Logarithmic Bels	dB
	$10:1$		1 bel	10
	$100:1$		2 bel	20
	$1000.000:1$		6 bel	60
	⋮		⋮	⋮

power ratio ↓

$$\text{Bel} = \log_{10}(P/P_{ref})$$

$1\,dB = 0{,}1\,\text{Bel}$ $\quad (1\,dm) = 0{,}1\,m$

$dB = 10\log_{10}(P/P_{ref})$ ← power ratios

Slide 29

What is value in dB for $P/P_{ref} = 0{,}001$?

$$P/P_{ref} = (E/E_{ref})^2$$

$$dB = 10\log_{10}(P/P_{ref}) = 10\log_{10}(E^2/E_{ref}^2) = 20\log_{10}(E/E_{ref})$$

$$\boxed{dB = 20\log_{10} E/E_{ref}}$$

For Hertzian dipole:

$$D_{max}/D^{(iso)} = 1{,}5/1$$

$$D_{max}(dB) = 10\log_{10}(1{,}5/1) = 1{,}76\,dB$$

Linear	Log
$A \cdot B$	$A + B$
A/B	$A - B$
$2 \cdot A$	$10\log_{10}(2) + 10\log_{10}(A) = 3dB + 10\log_{10}A$

Gain

$$D_{max} = \frac{U_{max}}{\langle U \rangle} = \frac{U_{max}}{\frac{1}{4\pi}\underbrace{\oint U(\theta,\varphi)\, d\Omega}_{P_{rad}}} = \frac{4\pi U_{max}}{P_{rad}}$$

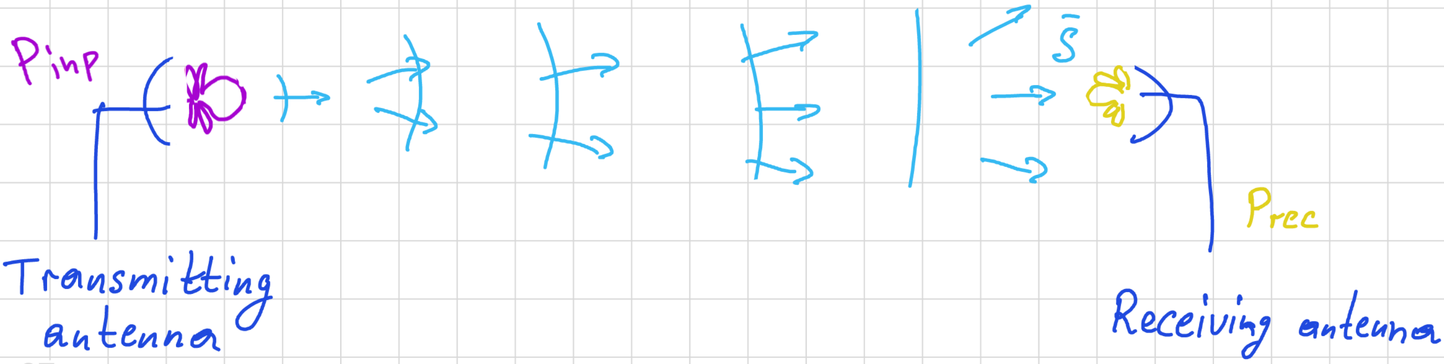
$$\frac{P_{rad}}{P_{inp}} = g_r \sim 80\% - 100\%$$

$$G_{max} = \frac{4\pi U_{max}}{P_{inp}}$$ ← maximal gain

$$\boxed{G_{max} = D_{max} \cdot g_r}$$

Receiving regime

Radiation patterns in the transmitting and receiving regimes are equal for most antennas.

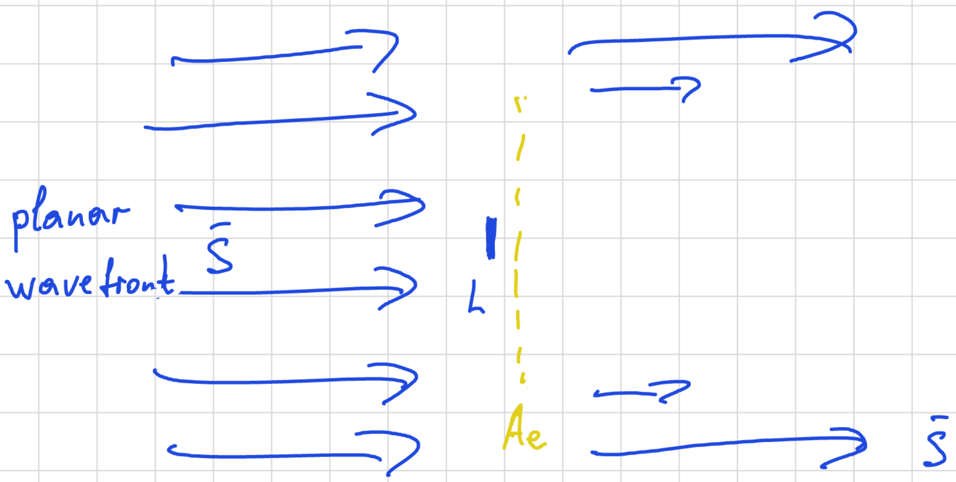


$P_{rec} \sim |\bar{S}(\bar{R}_{rec})|$ $\qquad$ $P_{rec} = A_e\, |\bar{S}(\bar{R}_{rec})|$

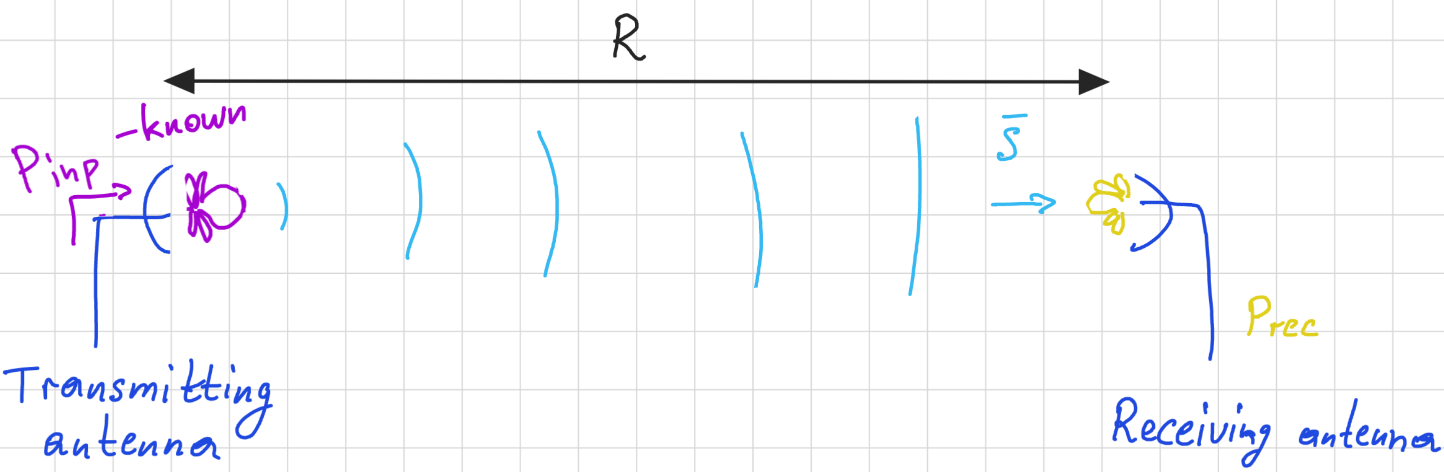
$[A_e] = W/(W/m^2) = m^2$ $\quad \leftarrow$ effective area (aperture)

$$A_e(\theta,\varphi) = \frac{\lambda^2}{4\pi}\, G(\theta,\varphi)$$

Derivation for arbitrary antenna can be found in Kraus, "Antennas", 1988 Sec. 2-22.



Friis transmission formula



P_{rec} – ?

Poynting vector at the receiver:

$$S(R) = \frac{U_{max}}{R^2} = \frac{P_{inp}\, G^{(tr)}_{max}}{4\pi R^2}$$

$$P_{rec} = A_e\, S(R) = \frac{P_{inp}\, G^{(tr)}_{max}}{4\pi R^2}\, \frac{\lambda^2}{4\pi}\, G^{(rec)}_{max}$$

$$\boxed{P_{rec} = \left(\frac{\lambda}{4\pi R}\right)^2 G^{(tr)}_{max}\, G^{(rec)}_{max}\, P_{inp}} \leftarrow \text{Friis transmission formula}$$