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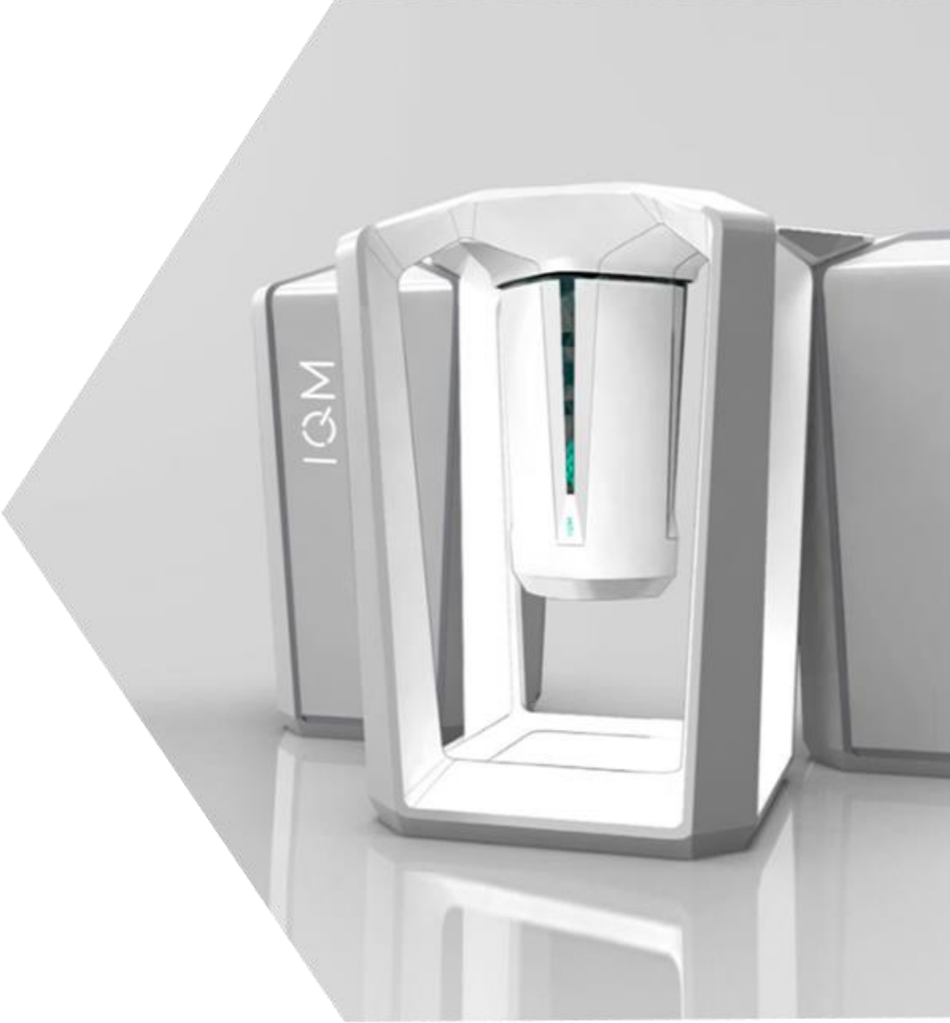


Hermanni Heimonen

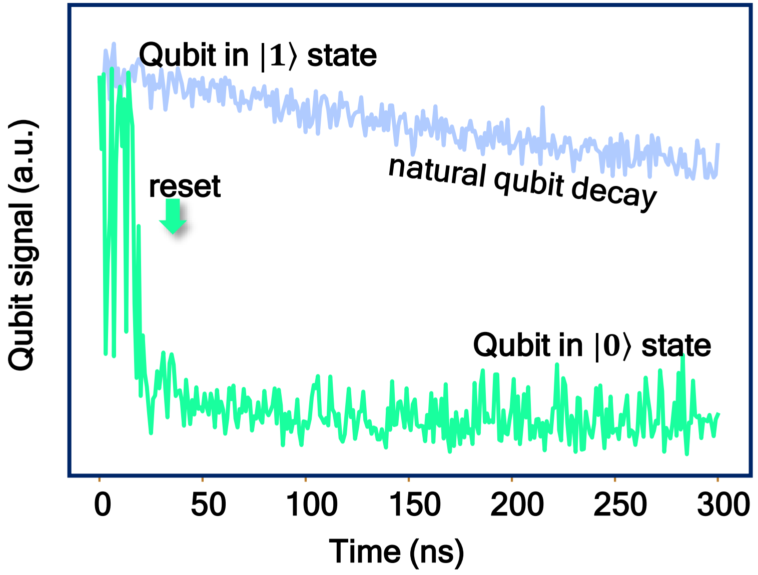
www.meetiqm.com

Lecture notes on PHYS-C0254 Quantum Circuits

Short recap from last week

9. Single-qubit operations:
   a. Initialization 2nd DiVincenzo criteria
   b. Readout 5th DiVincenzo criteria
   c. Control: T1, T2 measurements, Randomized benchmarking 3rd DiVincenzo criteria

a.



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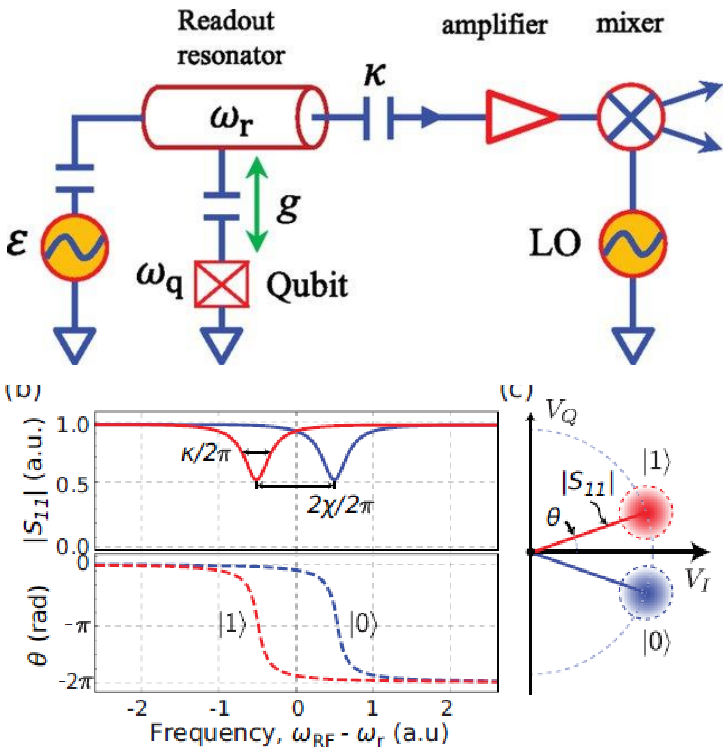

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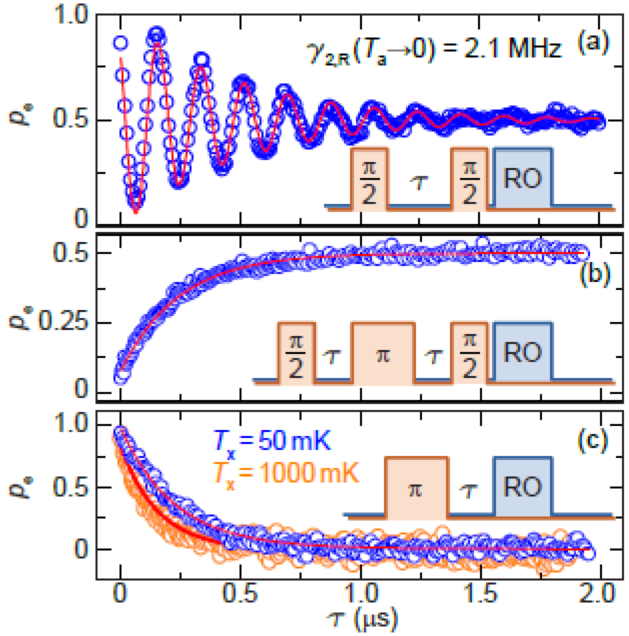
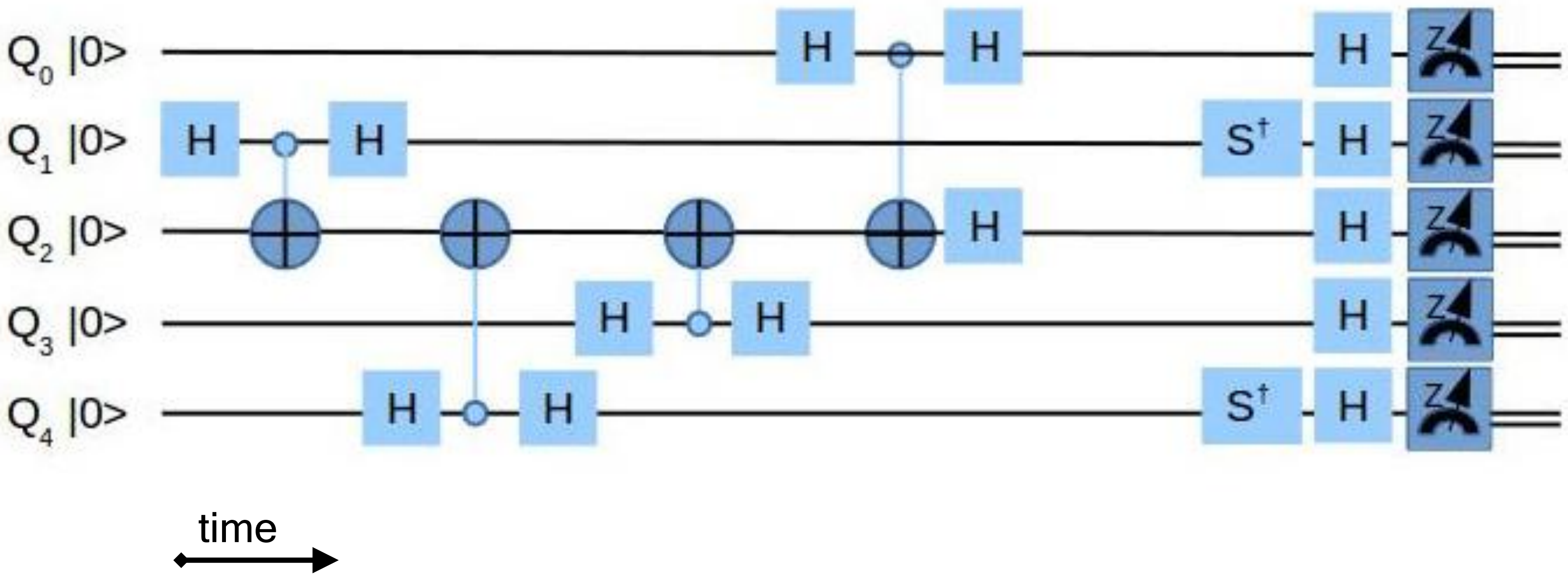


Agenda for lectures 7-12

7. Quantization of electrical networks
   a. Harmonic oscillator: Lagrangian, eigenfrequency
   b. Transfer step: LC oscillator, Legendre transform to Hamiltonian
   d. Quantization of oscillators
8. Superconducting quantum circuits
   a. Qubits: Transmon qubit, Charge qubit, Flux qubit 1st DiVincenzo criteria
   b. Circuit-QED: Rabi model
   c. Rotating Wave approximation: Jaynes-Cummings model
9. Single-qubit operations:
   a. Initialization 2nd DiVincenzo criteria
   b. Readout 5th DiVincenzo criteria
   c. Control:T1, T2 measurements, Randomized benchmarking 3rd DiVincenzo criteria
10. Two-qubit operations: Architectures for 2-qubit gates 4th DiVincenzo criteria
    a. iSWAP
    b. cPhase
    c. cNot
11. Quantum algorithms
    a. Deutsch-Josza Algorithm
    b. Parameterised circuits and VQE
12. Challenges in quantum computing
    a. Scaling
    b. SW-HW gap
    c. Error-correction

Short intro: The basic elements of quantum algorithms



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General approach: Two-qubit gates

- The 4th DiVicenzo criterion is being able to generate entanglement via two-qubit gates (operations)

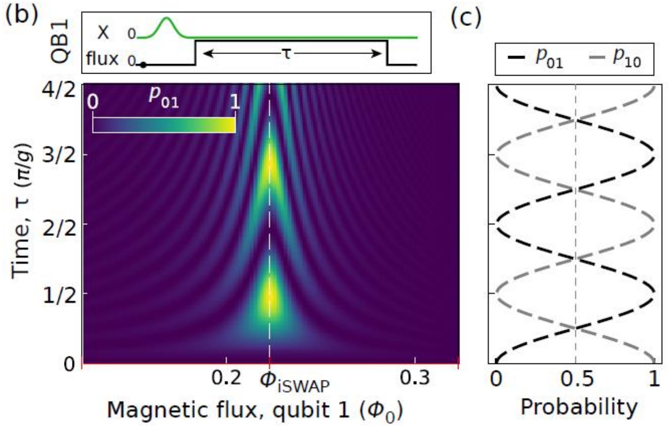
- Three favorite gates with transmon qubits are:
  - iSWAP & sqrt(iSWAP)
  - CPhase
  - CNOT

- They are realized by creating a mutual interaction between the qubits, either directly or through a coupling element.

Agenda for today

10. Two-qubit operations: Architectures for 2-qubit gates 4th DiVincenzo criteria
    a. iSWAP
    b. CPhase (CZ)
    c. CNOT

a.



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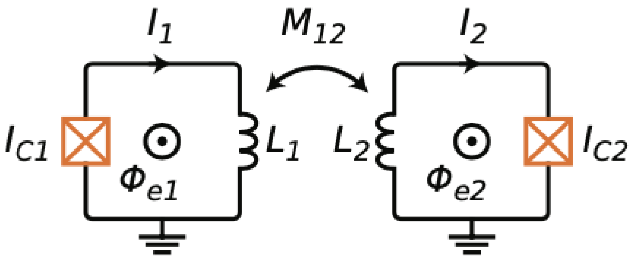

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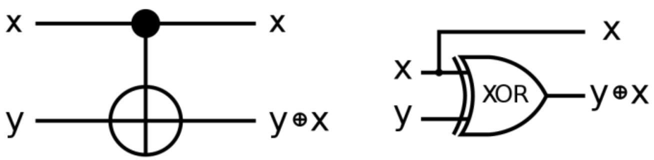

input x	input y	output x	output y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

input x	input y	output x	output y+x
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

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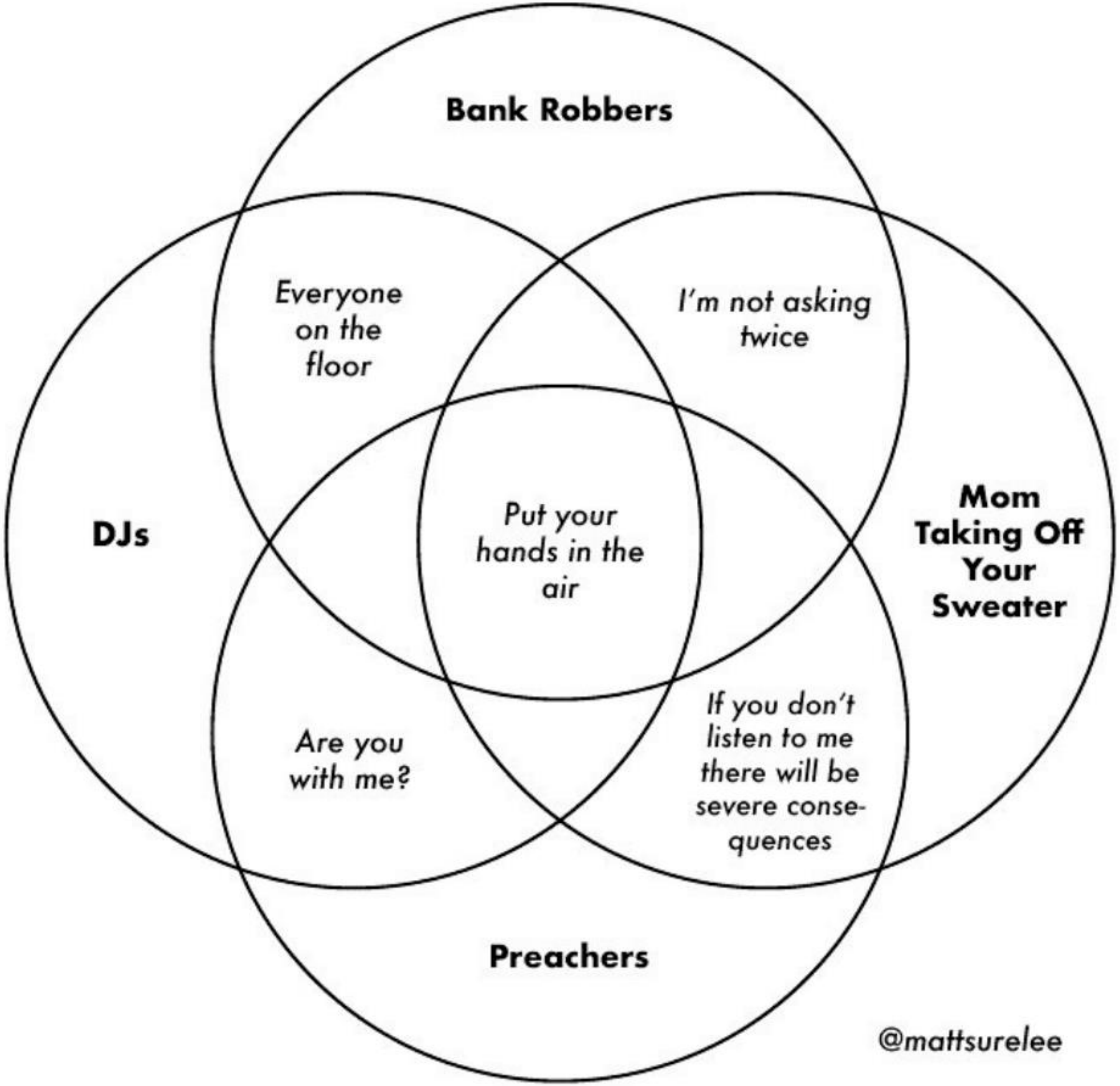

Bank Robbers
DJs
Mom Taking Off Your Sweater
Preachers
Everyone on the floor
I'm not asking twice
Put your hands in the air
Are you with me?
If you don't listen to me there will be severe conse-quences
@mattsurelee

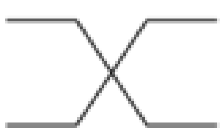
General approach: iSWAP gate

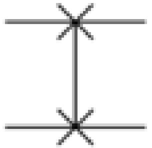
- A general task in (quantum) logic is to swap states. By adding a phase factor, one can create an iSWAP gate.

- Since this is a two-qubit gate, we realize it by coupling two qubits mutually on a chip.

- By controlling the interaction time, we can also implement sqrt(iSWAP) gates.

The swap gate between two qubits

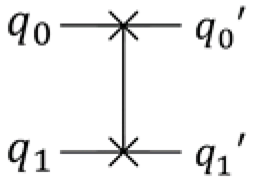




The swap gate exchange the state of two qubits. With respect to the basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle,$ it is represented by the matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

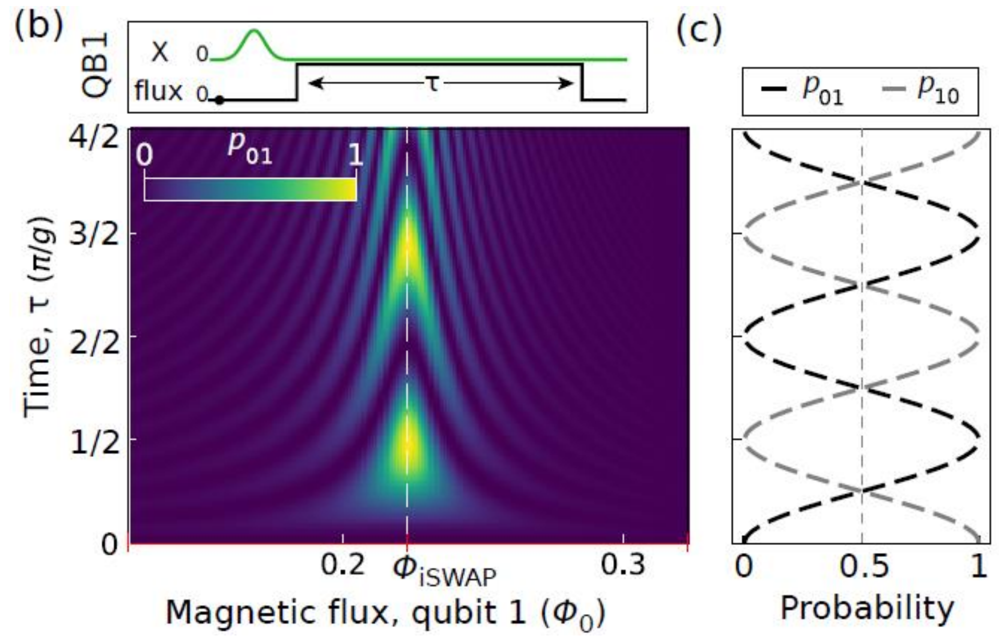
Applied to two qubits, the input-output relations are given as



Input		*Output*	
q_0	q_1	$q_0{}'$	$q_1{}'$
0	0	0	0
0	1	1	0
1	0	0	1
1	1	1	1

Since the swap gate is hard to realize, we first focus on the iSWAP gate.

The iSWAP gate between two qubits

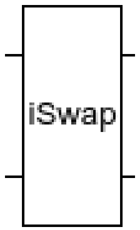


The iSWAP gate is a 2-qubit XX+YY interaction and a Clifford and symmetric gate. Its action is to swap two qubit states and puts phase of *-i* on the swapped elements.

The matrix representation of this system is given as

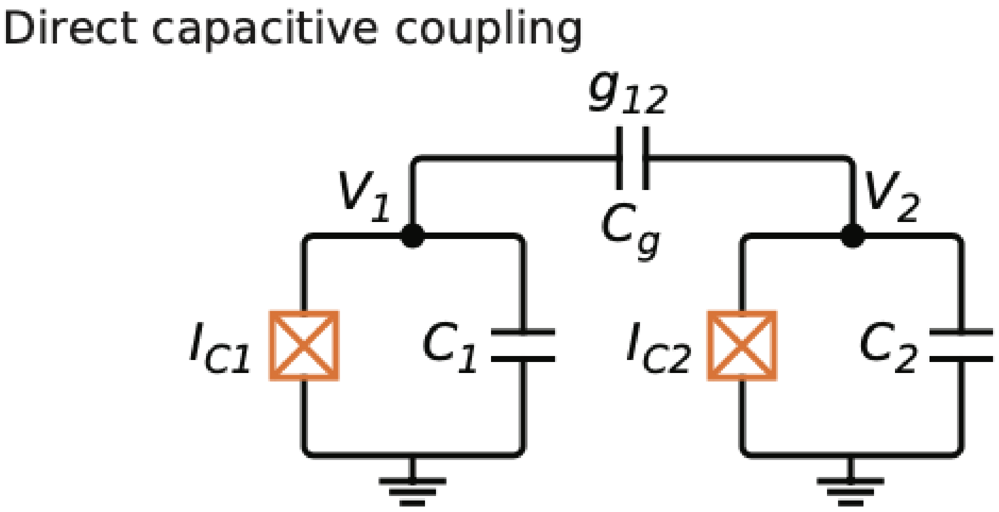
$$iSWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \equiv$$



arXiv:1904.06560v3

The iSWAP gate with tunable qubits



To realize the target unitary of the iSWAP gate, we consider two capacitively coupled transmon qubits in the limit $C_g \ll C_1, C_2$:

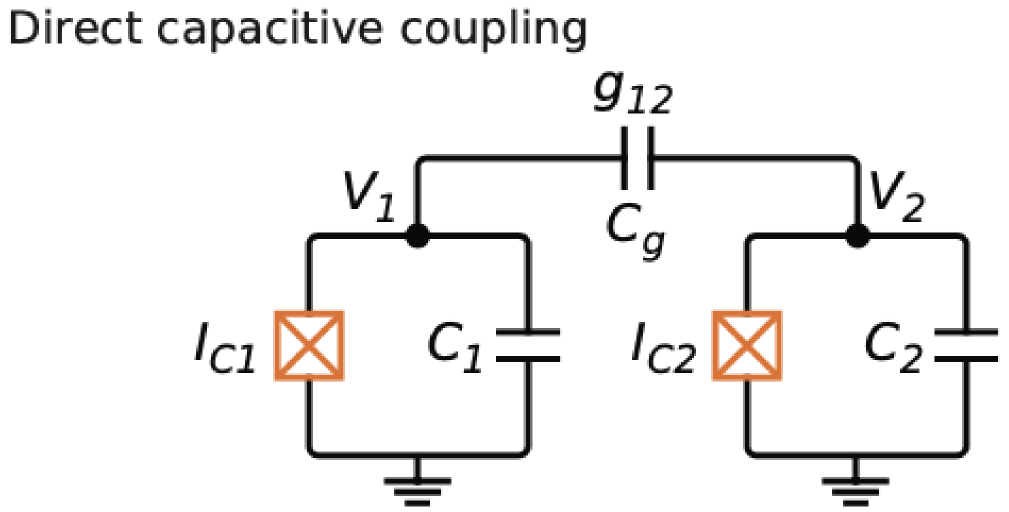
$$H = H_1 + H_2 + H_{\text{int}}$$

Similar to qubits coupled to resonators, the interaction is given by the respective voltage coupled through a mutual capacitance:

$$H_{\text{int}} = C_g V_1 V_2$$

IQM arXiv:1904.06560v3

The iSWAP gate with tunable qubits



The total system Hamiltonian therefore reads as

$$H = \sum_{i=1,2} \left[4E_{C,i} n_i^2 - E_{J,i}\cos(\phi_i)\right] + 4e^2 \frac{C_g}{C_1 C_2} n_1 n_2$$

In the language of second quantization we have $V \propto i(a - a^\dagger)$ and after Taylor expansion of the cosine:

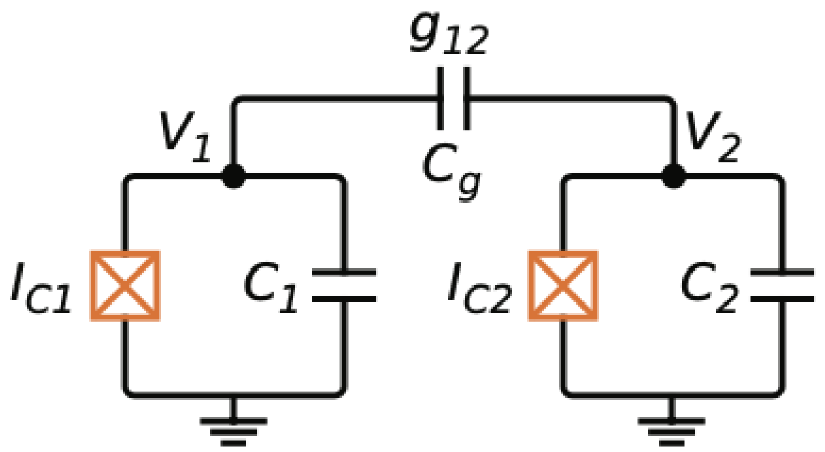
$$H = \sum_{i \in 1,2} \left[\omega_i a_i^\dagger a_i + \frac{\alpha_i}{2} a_i^\dagger a_i^\dagger a_i a_i\right] - g\left(a_1 - a_1^\dagger\right)\left(a_2 - a_2^\dagger\right)$$

This capacitive coupling is called transversal, it couples energy levels to each other, but not to itself. I.e. the operator is zero along the diagonal (like the X and Y operators).

Physically this is like a photon exchange between the two systems where one qubit loses a photon and the other one gains. When the coupling is to the environment such processes are responsible for energy loss of the system described by the T1 time.

The iSWAP gate with tunable qubits

Direct capacitive coupling



Here, all the system parameters are absorbed in the coupling constant g:

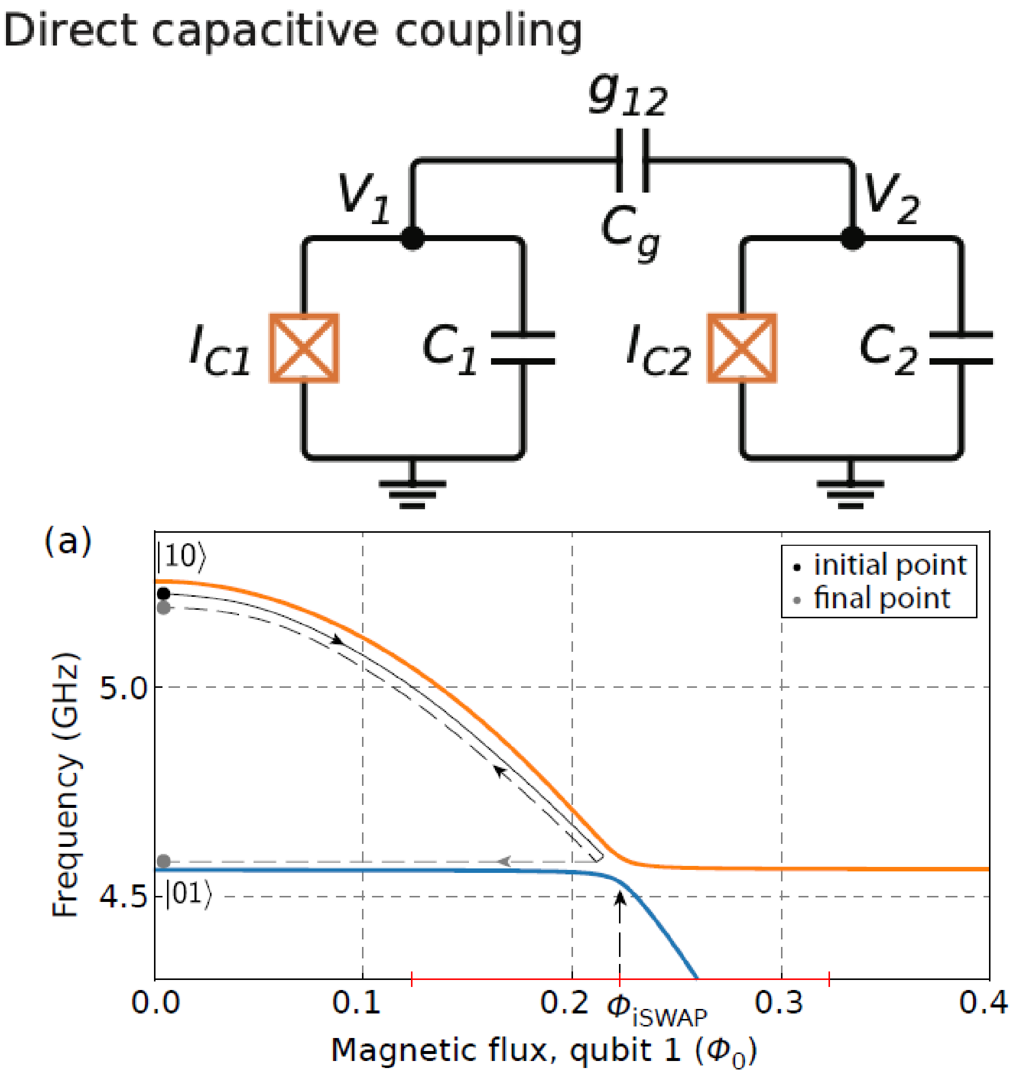
$$g \rightarrow g_{\text{q-q}} = \frac{1}{2}\sqrt{\omega_{\text{q}1}\omega_{\text{q}2}}\frac{C_{\text{q-q}}}{\sqrt{C_{\text{q-q}}+C_1}\sqrt{C_{\text{q-q}}+C_2}},$$

Truncating the system Hamiltonian to 2 levels (qubit approximation) yields:

$$H = \sum_{i\in 1,2}\frac{1}{2}\omega_i\sigma_{z,i} + g\sigma_{y,1}\sigma_{y,2}$$

Qubit truncation: We can choose eihter X or Y operators, because the phase goes into g. Most often X is used. When both inductive and capacitive coupling are used, you might need both X and Y operators.

The iSWAP gate with tunable qubits



Now, the qubit-qubit interaction Hamiltonian becomes

$$H_{\mathrm{qq}} = -g\left([\sigma^+ - \sigma^-] \otimes [\sigma^+ - \sigma^-]\right)$$

As earlier, we can apply a rotating-wave approximation (RWA) to ignore fast rotating terms like $\sigma^+\sigma^+$

$$H_{\mathrm{qq}} = g\left(e^{i\delta\omega_{12}t}\sigma^+\sigma^- + e^{-i\delta\omega_{12}t}\sigma^-\sigma^+\right)$$

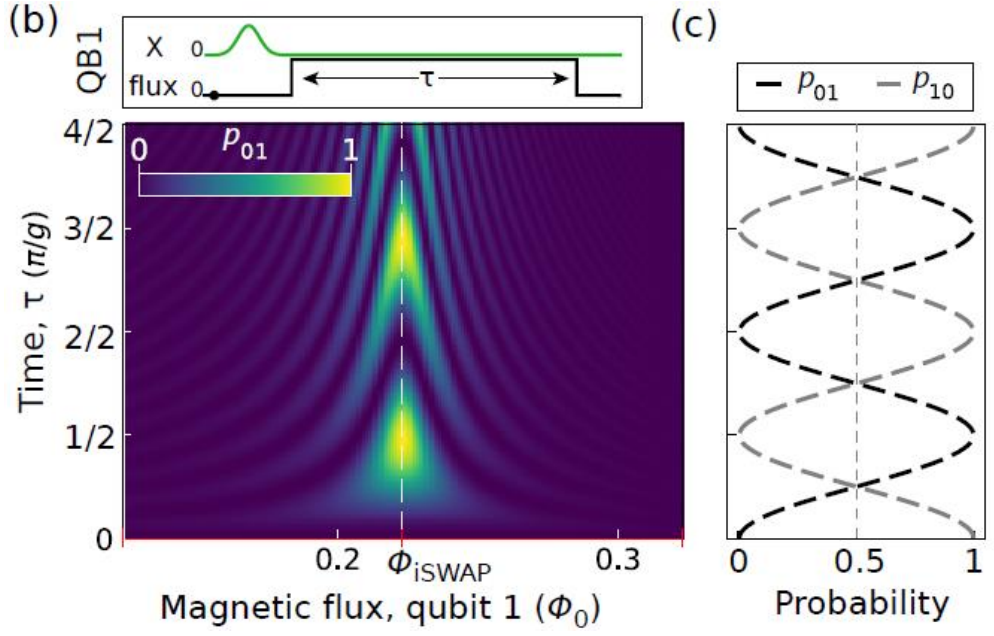
Here we have used the detuning of the two qubits

$$\delta\omega_{12} = \omega_{\mathrm{q1}} - \omega_{\mathrm{q2}}$$

This means that we get significant interaction only if the two qubits are on resonance. For large detuning, the interaction averages out due to the fast-rotating terms proportional to δ.

 arXiv:1904.06560v3

The iSWAP gate with tunable qubits



From this notion and assuming we are on resonance, we can derive the unitary expression

$$U_{qq}(t) = e^{-i\frac{g}{2}(\sigma_x\sigma_x+\sigma_y\sigma_y)t} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(gt) & -i\sin(gt) & 0 \\ 0 & -i\sin(gt) & \cos(gt) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To obtain the iSWAP gate, we need sin(gt) = 1 and cos(gt)=0. This is the case for $\boldsymbol{t} = \frac{\boldsymbol{\pi}}{\boldsymbol{2g}}$

$$U_{qq}\left(\frac{\pi}{2g}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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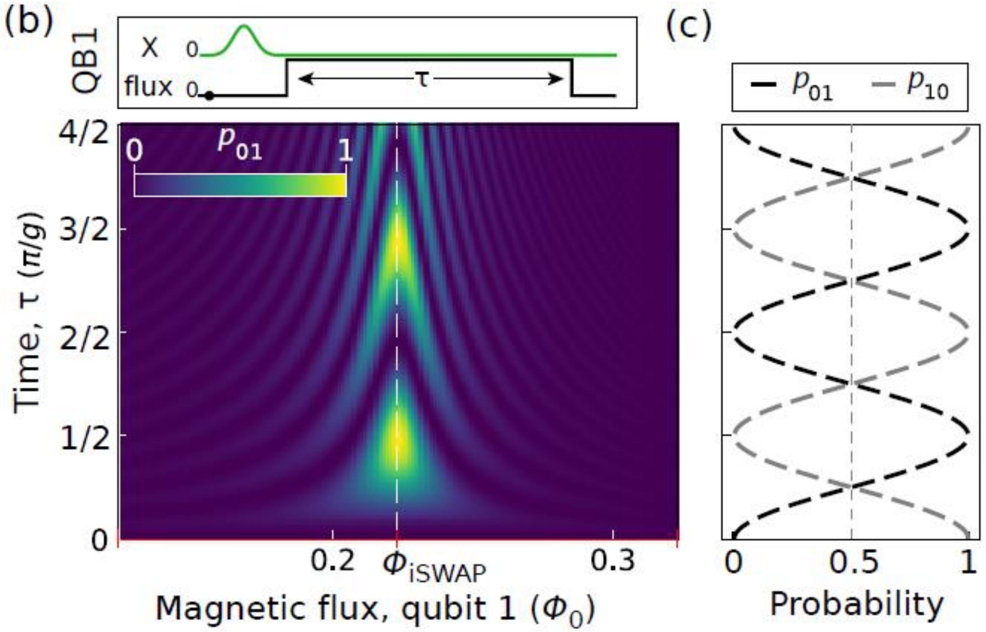
The sqrt(iSWAP) gate with tunable qubits



SQRT(SWAP) is an important entangling gate, yet so non-intuitive. Here we use the fact that U=exp(iHt) so H is the log of U. That makes a sqrt in the U into a simple division by 2 in the time.

To obtain the iSWAP gate, we need sin(gt) = 1 and cos(gt)=0. This is the case for $\boldsymbol{t} = \frac{\boldsymbol{\pi}}{\boldsymbol{4g}}$

$$U_{qq}\left(\frac{\pi}{4g}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -i/\sqrt{2} & 0 \\ 0 & -i/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \equiv \sqrt{i\text{SWAP}}$$

Review: iSWAP gate

- A general task in (quantum) logic is to swap states. By adding a phase factor, one can create an iSWAP gate.

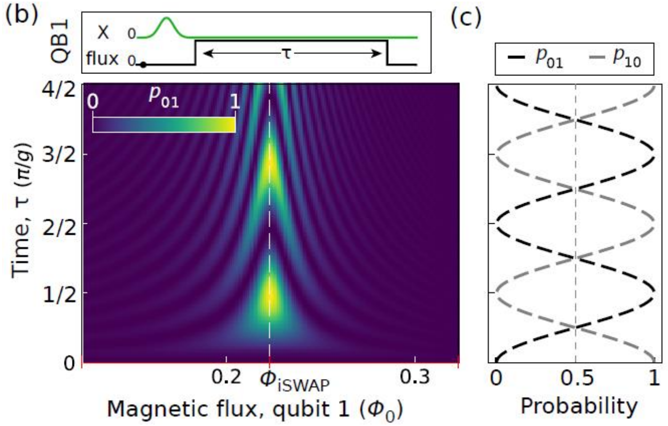
- Since this is a two-qubit gate, we realize it by coupling two qubits mutually on a chip.

- By controlling the interaction time, we can also implement sqrt(iSWAP) gates.

Agenda for today

10. Two-qubit operations: Architectures for 2-qubit gates 4th DiVincenzo criteria
    a. iSWAP
    b. cPhase
    c. cNot

a.



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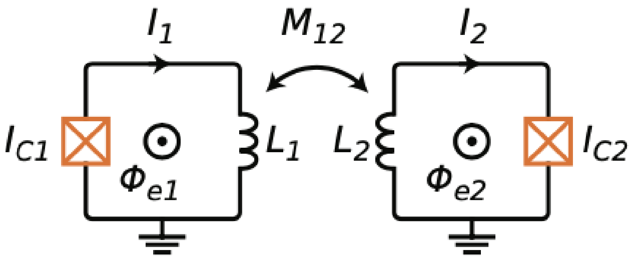

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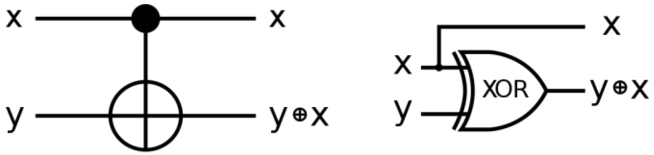


input x	input y	output x	output y+x
|0⟩	|0⟩	|0⟩	|0⟩
|0⟩	|1⟩	|0⟩	|1⟩
|1⟩	|0⟩	|1⟩	|1⟩
|1⟩	|1⟩	|1⟩	|0⟩

input x	input y	output x	output y+x
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

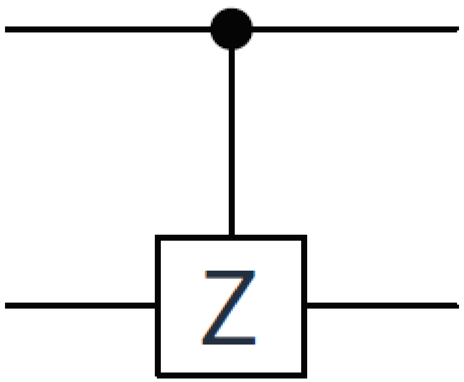
General approach: cPhase gate

- The cPhase gate is a diagonal and symmetric gate that induces a phase on the state of the target qubit, depending on the control state.

- Since this is a two-qubit gate, we realize it by coupling two qubits mutually on a chip.

- By controlling the interaction time and coupling strength, we can control the exact phase induced to the target qubit.

The cPhase gate between two qubits



The controlled phase (cPhase) gate is a diagonal and symmetric gate that induces a phase on the state of the target qubit, depending on the control state:

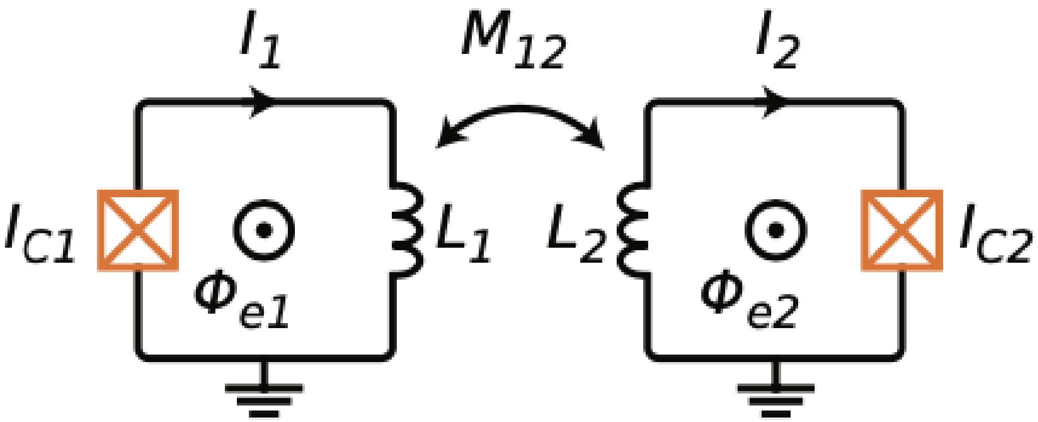
$$\mathrm{CPHASE} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Applied to two qubits, the input-output relations are given as

Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|10\rangle$
$\|11\rangle$	$-\|11\rangle$

There are no swap operations involved in this gate.

The cPhase gate with tunable qubits



To realize the cPhase gate, we need longitudinal interaction.

A longitudinal interaction is purely diagonal (like the Z operator, i.e. magnetic dipole coupling, i.e. inductive coupling) and affects the energy level splitting, but does not couple states to one another. This kind of coupling is responsible for the T2 time of the system when there is a magnetic coupling to an environment.

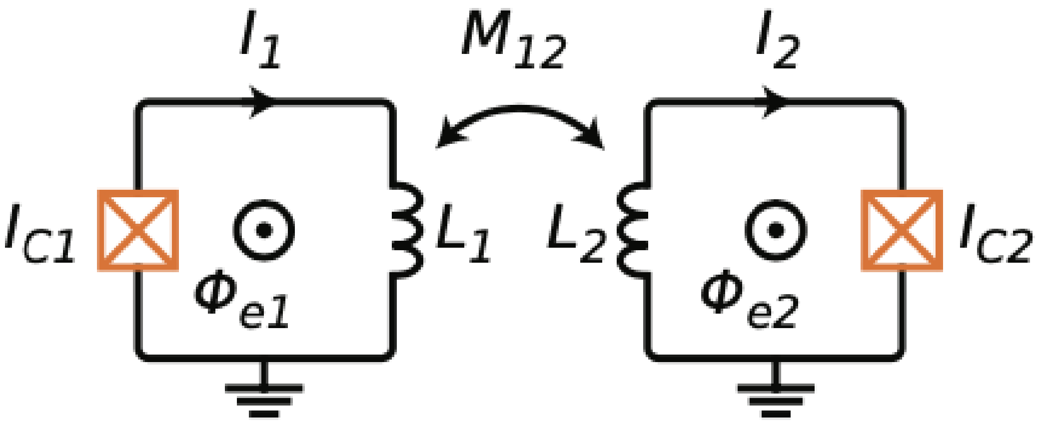
To implement this interaction, we consider two qubits inductively coupled to each other

$$H_{\mathrm{int}} = M_{12} I_1 I_2$$

It can be shown that they follow a ZZ interaction:

$$H = \sum_{i=1,2} \frac{1}{2} \omega_i \sigma_{zi} + g \sigma_{z1} \sigma_{z2}$$

The cPhase gate with tunable qubits



$$H = \sum_{i=1,2} \frac{1}{2}\omega_i \sigma_{zi} + g\sigma_{z1}\sigma_{z2}$$

In matrix form, the bare qubit terms read:

$$\frac{1}{2}\omega_1\sigma_{z1} + \frac{1}{2}\omega_2\sigma_{z2} = \frac{1}{2}\omega_1 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} + \frac{1}{2}\omega_2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

In a similar fashion, the interaction term is proportional to

$$\sigma_{z1}\sigma_{z2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence, the total system Hamiltonian is given as

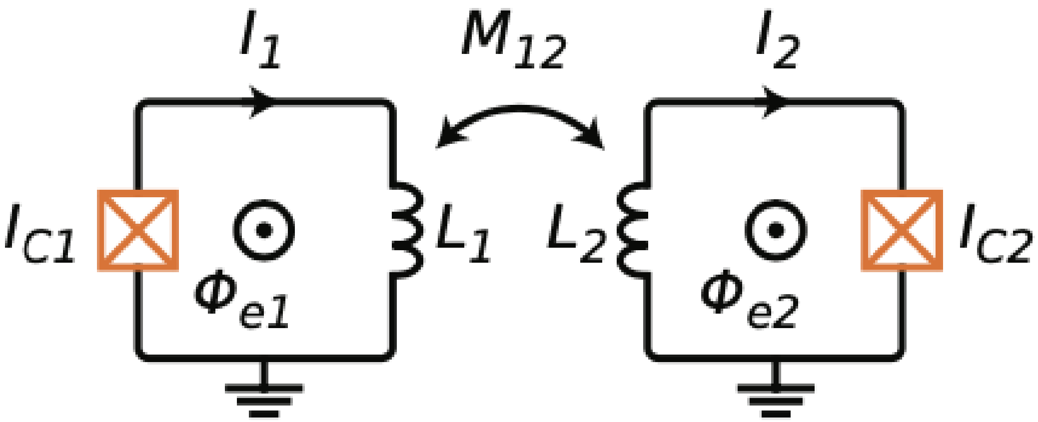
$$H = \frac{1}{2} \begin{bmatrix} \omega_1 + \omega_2 & 0 & 0 & 0 \\ 0 & \omega_1 - \omega_2 & 0 & 0 \\ 0 & 0 & -\omega_1 + \omega_2 & 0 \\ 0 & 0 & 0 & -(\omega_1 + \omega_2) \end{bmatrix} + g \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

https://inst.eecs.berkeley.edu/~cs191/fa14/lectures/lecture5.pdf

The cPhase gate with tunable qubits

Then, by controlling the qubit frequencies we set $\frac{1}{2}\omega_{1/2} = -g$
Which leads to

$$H = g\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = g\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$



$$H = \sum_{i=1,2} \frac{1}{2}\omega_i \sigma_{zi} + g\sigma_{z1}\sigma_{z2}$$

Hence, the cPhase gate is achieved by choosing

$$U(t) = e^{-iHt} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-4igt} \end{bmatrix}$$

$$U(\frac{\pi}{4g}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = CPHASE$$

Review: cPhase gate

- The cPhase gate is a diagonal and symmetric gate that induces a phase on the state of the target qubit, depending on the control state.

- Since this is a two-qubit gate, we realize it by coupling two qubits mutually on a chip.

- By controlling the interaction time and coupling strength, we can control the exact phase induced to the target qubit.

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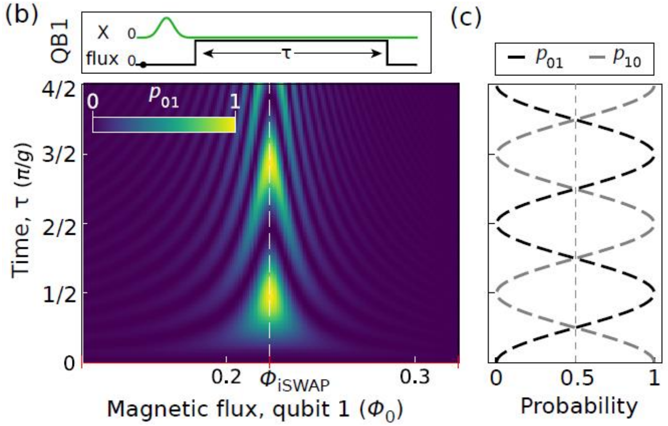
    a. iSWAP
    b. cPhase
    c. cNot

a.



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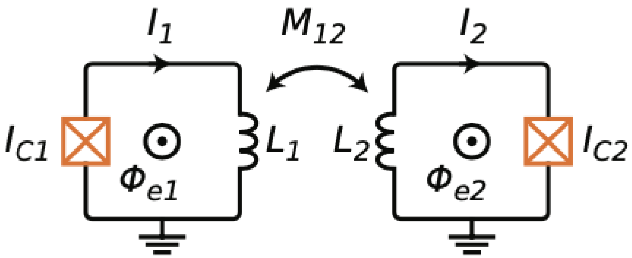
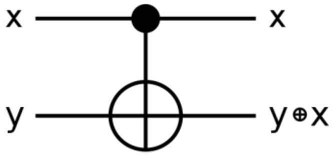
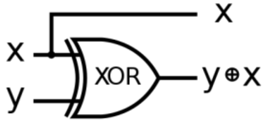


c.



input		output	
x	y	x	y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$



input		output	
x	y	x	y+x
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

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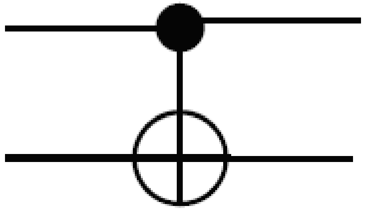
General approach: cNot gate

- In computer science, the controlled NOT gate (also C-NOT or CNOT) is a quantum logic gate that can be used to entangle and disentangle qubits.

- The CNOT gate operates on a quantum register consisting of 2 qubits. The CNOT gate flips the second qubit (the target qubit) if and only if the first qubit (the control qubit) is $|1\rangle$.

- Any quantum circuit can be simulated to an arbitrary degree of accuracy using a combination of CNOT gates and single qubit rotations.

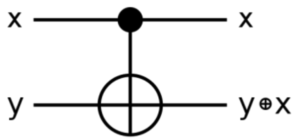
The cNOT gate between two qubits

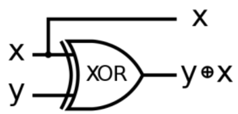


Controlled-NOT gate applies a swap to the target qubit if the control qubit is in state $|1\rangle$

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Applied to two qubits, the input-output relations are given as

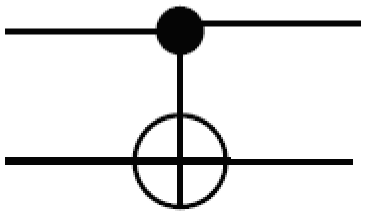




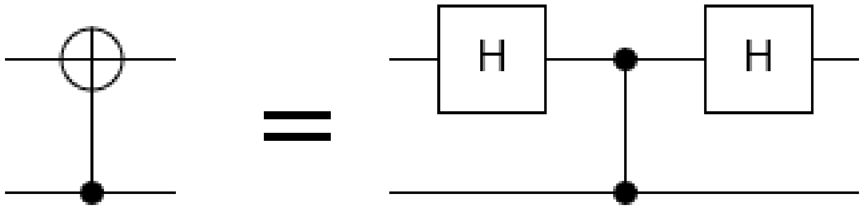
input		output	
x	y	x	y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

input		output	
x	y	x	y+x
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

The cNOT gate between two qubits



It can be shown that a cNOT gate can be implemented as a series of Hadamard and cPhase gates



Altogether, we can now create a universal quantum gate set

$$\mathcal{G}_1 = \{\mathsf{H}, \mathsf{S}, \mathsf{T}, \mathsf{CNOT}\}$$

Informally, a set of **universal quantum gates** is any set of gates to which any operation possible on a quantum computer can be reduced, that is, any other unitary operation can be expressed as a finite sequence of gates from the set.

arXiv:1904.06560v3

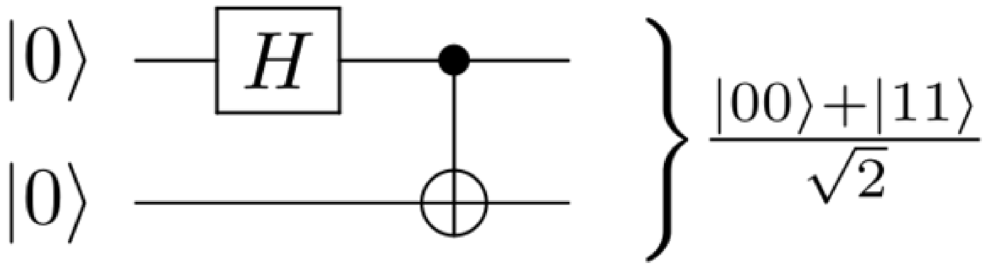
Short review: cNot gate

- In computer science, the controlled NOT gate (also C-NOT or CNOT) is a quantum logic gate that can be used to entangle and disentangle qubits.

- The CNOT gate operates on a quantum register consisting of 2 qubits. The CNOT gate flips the second qubit (the target qubit) if and only if the first qubit (the control qubit) is $|1\rangle$.

- Any quantum circuit can be simulated to an arbitrary degree of accuracy using a combination of CNOT gates and single qubit rotations.

Example: How to generate a Bell state



The Bell states are four specific maximally entangled quantum states of two qubits. They are in a superposition of 0 and 1, their entanglement means the following:

The first qubit can be 0 as well as 1 and measuring the qubit in the standard basis, the outcome would be perfectly random, either possibility 0 or 1 having probability 1/2.

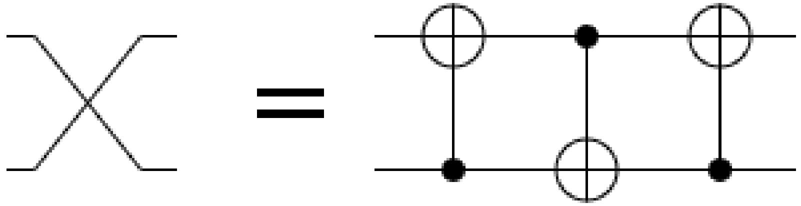
If the second qubit is measured, the outcome would be random on first sight. But if the results are communicated, they would find out that, although their outcomes seemed random, they are perfectly correlated.

This perfect correlation at a distance is special: maybe the two particles "agreed" in advance, when the pair was created (before the qubits were separated), which outcome they would show in case of a measurement.

Hence, following Einstein, Podolsky, and Rosen in 1935 in their famous "EPR paper", there is something missing in the description of the qubit pair given above—namely this "agreement", called more formally a hidden variable.

Example: How to apply a SWAP gate

Swap implemented with Hadamards and CZs



Review: Two-qubit gates

- The 4th DiVicenzo criterion is being able to generate entanglement via two-qubit gates (operations)

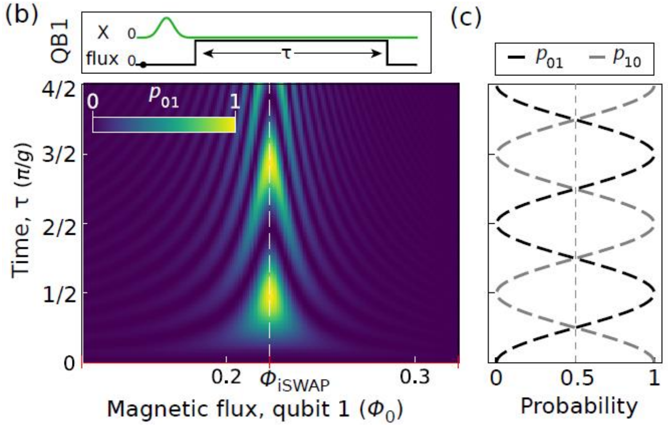
- Three favorite gates with transmon qubits are:
  - iSWAP & entangling sqrt(iSWAP)
  - CPhase
  - CNOT

- They are realized by creating a mutual interaction between the qubits, either directly or through a coupling element.

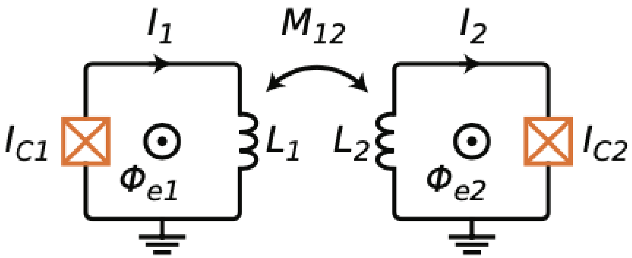
Agenda for today

10. Two-qubit operations: Architectures for 2-qubit gates 4th DiVincenzo criteria
    a. iSWAP
    b. cPhase
    c. cNot

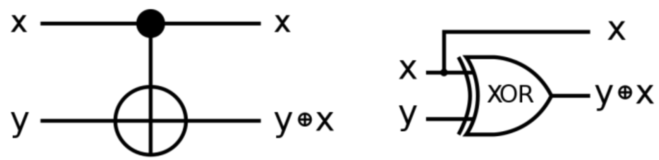
a.



b.



c.



input x	input y	output x	output y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

input x	input y	output x	output y+x
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

IQM