Aalto University
Björn Ivarsson, 050-4067 832

Course Exam, Wednesday 19.04.2023, 09:00 - 12:00
Exam, Wednesday 19.04.2023, 09:00 - 12:00
Differential and Integral Calculus 3, MS-A0311

No calculators or tables of formulas allowed. **See backside for the allowed collection of formulas!**

Motivate your answers. Only giving answers gives no points. **The course exam** consists of the four exercises with best result out of **exercise 1,2,3,4, and 5**. **The exam** consists of **exercise 1,2,3,4, and 5**. If you prefer you can do all five exercises and I will evaluate using *"the result on the course exam + points given during the course"* or *"the result on the exam"*. The alternative giving the best grade will be used.

(1) (6p)

(2) (6p)

(3) (6p)

(4) (6p)

(5) (6p)

Good luck!

Useful theorems and formulas:

- Green's Theorem:

$$\iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \, dA = \oint_\gamma \mathbf{F} \cdot d\mathbf{r}$$

- Stokes's Theorem:

$$\iint_S (\operatorname{Curl} \mathbf{F}) \cdot \mathbf{n} \, dS = \oint_\gamma \mathbf{F} \cdot d\mathbf{r}$$

- Gauss's Theorem:

$$\iiint_D (\operatorname{div} \mathbf{F}) \, dV = \oiint_S \mathbf{F} \cdot \mathbf{n} \, dS$$

- Gradient in a orthogonal curvilinear coordinate system $[\hat{u}, \hat{v}, \hat{w}]$,

$$\nabla f = \frac{1}{h_u} \frac{\partial f}{\partial u} \hat{u} + \frac{1}{h_v} \frac{\partial f}{\partial v} \hat{v} + \frac{1}{h_w} \frac{\partial f}{\partial w} \hat{w}$$

- Divergence in a orthogonal curvilinear coordinate system $[\hat{u}, \hat{v}, \hat{w}]$,

$$\operatorname{div} \mathbf{F} = \frac{1}{h_u h_v h_w} \left(\frac{\partial}{\partial u}(F_u h_v h_w) + \frac{\partial}{\partial v}(F_v h_u h_w) + \frac{\partial}{\partial w}(F_w h_u h_v) \right)$$

- Curl in a positively oriented orthogonal curvilinear coordinate system $[\hat{u}, \hat{v}, \hat{w}]$,

$$\operatorname{Curl} \mathbf{F} = \frac{1}{h_u h_v h_w} \begin{vmatrix} h_u \hat{u} & h_v \hat{v} & h_w \hat{w} \\ \partial/\partial u & \partial/\partial v & \partial/\partial w \\ h_u F_u & h_v F_v & h_w F_w \end{vmatrix}$$