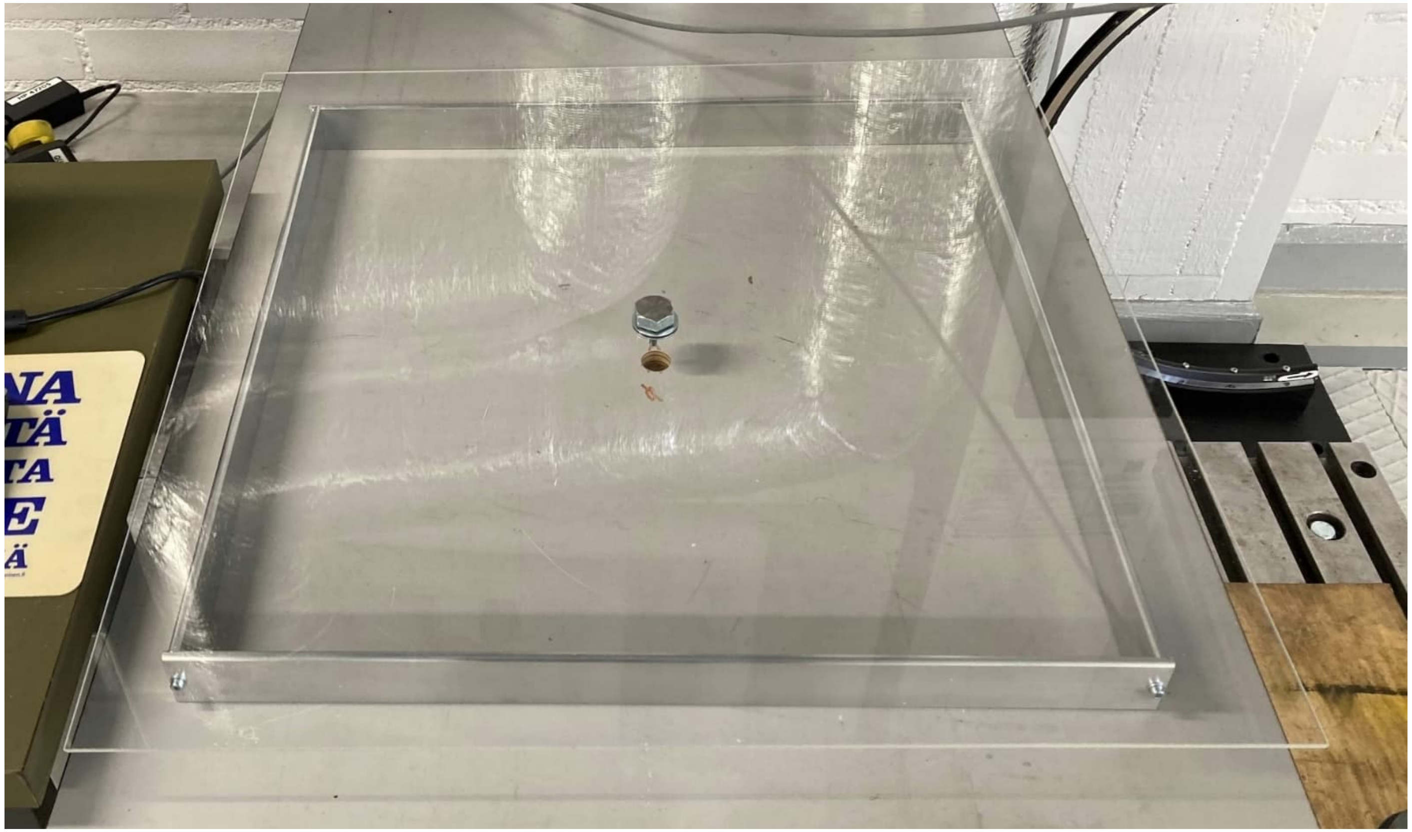
MEC-E1005

MODELLING IN APPLIED MECHANICS 2023

Weeks 21-22 RECTANGULAR PLATE RIGIDITY

ACRYLIC PLATE

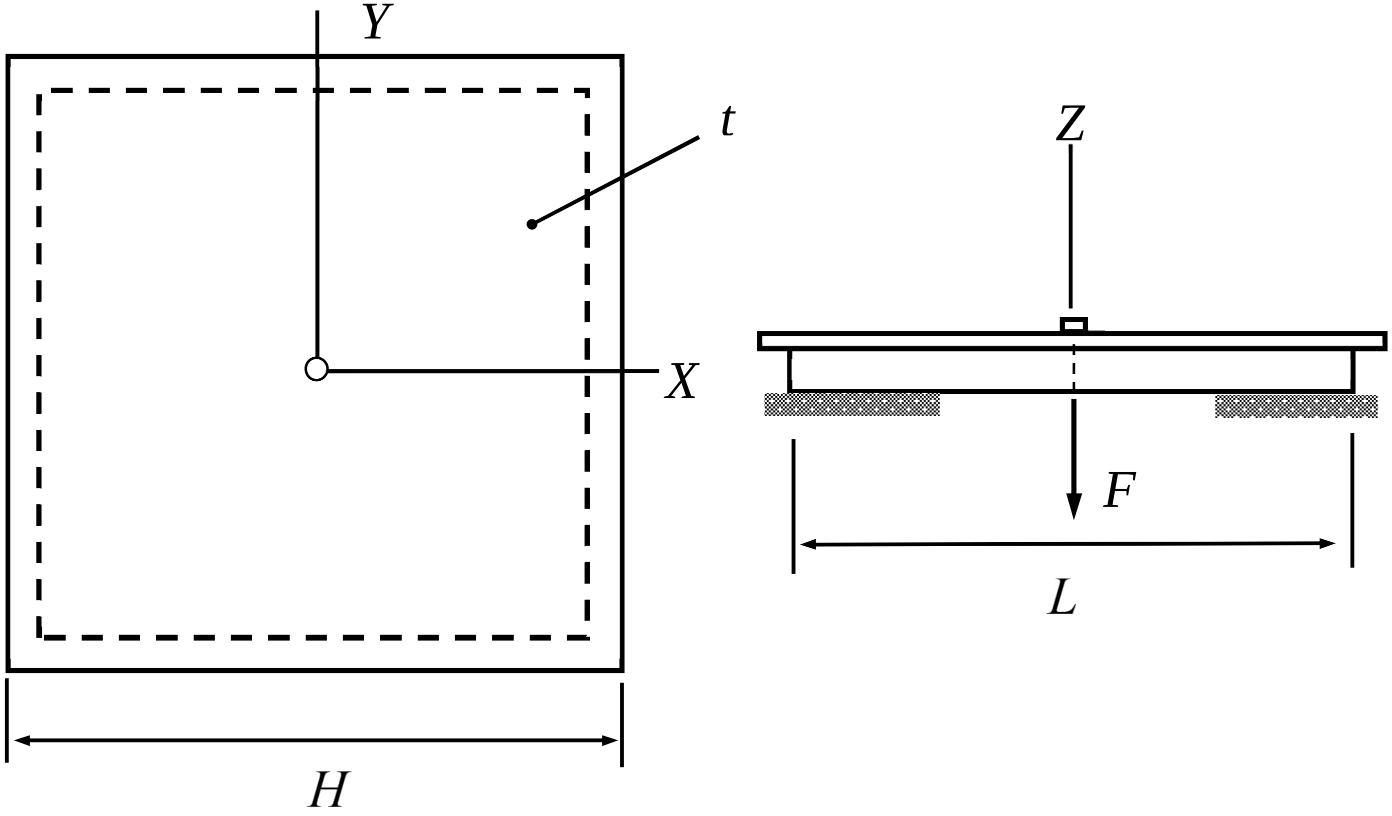


ASSIGNMENT

According to linear theory, rigidity of a simply supported plate is constant whose value depends on the plate thickness, size of the plate, and the plate material. Experiments indicate, however, that rigidity increases rapidly in the transverse displacement. The actual boundary conditions at the support may also affect the setting. For example, in rectangle geometry, the contact between the plate and support may be lost at the corner regions (if the displacement at the support is constrained only downwards like in the figure).

In the modelling assignment, you will study the effects of geometrical and material parameters, and displacement on rigidity of a rectangular plate on a rectangular support. The starting point is a generic expression predicted by dimension analysis. First, a simplified linear model is used for a more specific relationship. After that, analysis by FEM is used for a more precise picture. The final outcome is a design formula for rigidity. The modelled rigidities are compared with that given by an experiment.

IDEALIZATION AND PARAMETERIZATION



DIMENSION ANALYSIS

Assuming that the quantities related with the setting are E, ν, H, L, t, w, and F, dimension analysis implies the relationship

$$\frac{FL^2}{Et^4} = \alpha(\frac{w}{t}, \frac{H}{L}, \nu) \tag{1}$$

The dimensionless groups are based on plate theory. The expression on the right hand requires a more detailed analysis or/and additional assumptions. For example, truncated Taylor expansion with respect to the first argument gives

$$\alpha(\frac{w}{t}, \frac{H}{L}, \nu) = \alpha_1(\frac{H}{L}, \nu)\frac{w}{t} + \alpha_3(\frac{H}{L}, \nu)(\frac{w}{t})^3$$

if displacement vanishes without loading, and force-displacement response is similar for negative and positive force values.

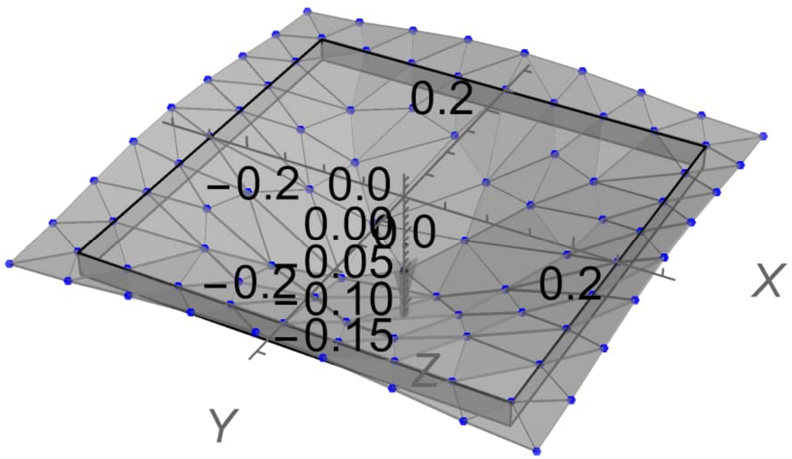
SIMPLIFIED ANALYSIS

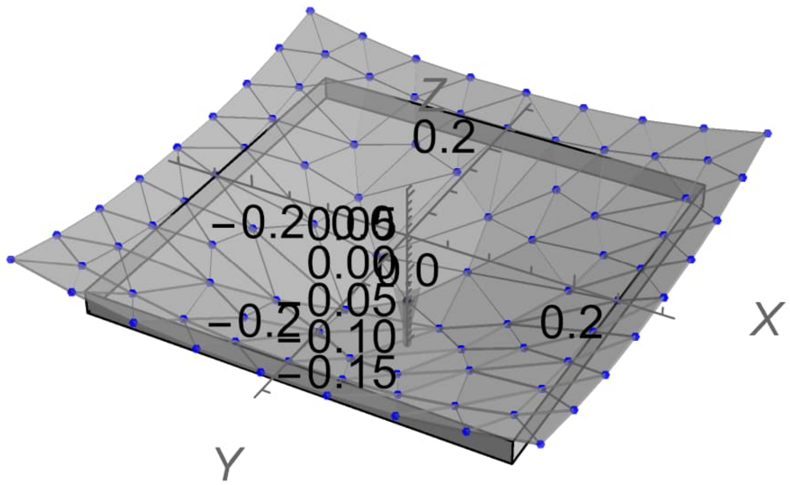
Simplified small displacement analysis can be based on the double-sine series solution to a rectangular simply supported plate. One may also use the virtual work density of Kirchhoff plate (either small or large displacement) and a few-parameter displacement approximation (MEC-E1050, MEC-E8001, MEC-E8003)

$$\delta W = -\int_{\Omega} \begin{Bmatrix} \dfrac{\partial^2 \delta w}{\partial x^2} \\ \dfrac{\partial^2 \delta w}{\partial y^2} \\ 2\dfrac{\partial^2 \delta w}{\partial x \partial y} \end{Bmatrix}^{\mathrm{T}} D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} dA - \delta w(x_F, y_F) F$$

FINITE ELEMENT ANALYSIS

Analysis by the finite element method and solid or plate/shell elements gives the displacement without (too many) simplifying assumptions. Numerical method requires numerical values for all the problem parameters, but one may consider the effects of non-linearity due to large displacement and one-sided boundary condition at the support.





EXPERIMENT

The set-up is located in Puumiehenkuja 5L (Konemiehentie side of the building). The hall is open for the experiment during the office hours (09:00-16:00) from Thu of week 21 to Thu of week 22.

Place a mass on the loading tray and record the displacement shown on the laptop display. Disk material is not purely elastic so wait for the displacement reading to settle (almost). Gather enough mass-displacement data to find the displacement-force relationship reliably. For example, you may repeat a measurement with certain loading several times to reduce the effect of random error by averaging etc. You may also consider different loading sequences (like increasing and decreasing the mass) to minimize the effect of the viscous part of material response.