Aalto University
School of Electrical
Engineering

Lecture 9:
Grid Converters: Power Angle Equation, LCL Filter

ELEC-E8402 Control of Electric Drives and Power Converters

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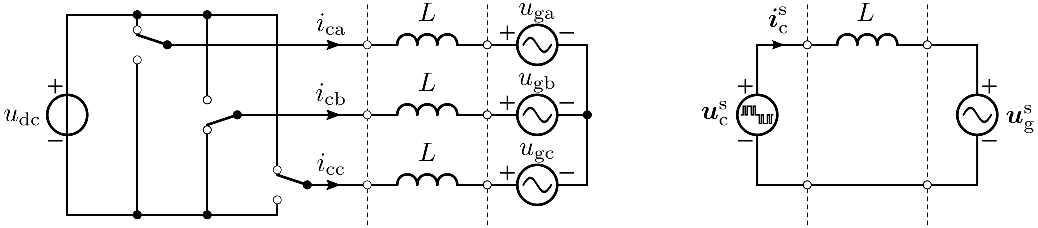
Learning Outcomes

After this lecture you will be able to:

- Derive the power angle equation and draw the corresponding vector diagrams
- Explain why a filter is needed between the converter and the grid
- Explain the most important characteristics of an LCL filter and compare it to an L filter

Some experimental examples are also included, but their details are not covered

Space-Vector Equivalent Circuit



In synchronous coordinates: $L\dfrac{\mathrm{d}\boldsymbol{i}_{\mathrm{c}}}{\mathrm{d}t} = \boldsymbol{u}_{\mathrm{c}} - \boldsymbol{u}_{\mathrm{g}} - \mathrm{j}\omega_{\mathrm{g}} L \boldsymbol{i}_{\mathrm{c}}$

The inductance L is the total inductance of the filter and the grid. It is also to be noted that the converter voltage $\boldsymbol{u}_{\mathrm{c}}$ contains PWM harmonics.

Power Angle Equation

- Power fed to the grid

$$p_{\mathrm{g}} = \frac{3}{2}\,\mathrm{Re}\left\{\boldsymbol{u}_{\mathrm{g}}\boldsymbol{i}_{\mathrm{c}}^{*}\right\} = \frac{3}{2}|\boldsymbol{u}_{\mathrm{g}}||\boldsymbol{i}_{\mathrm{c}}|\cos\varphi$$

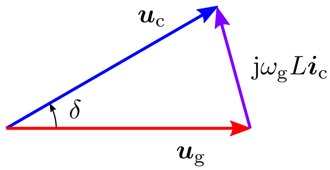
  where φ is the angle between $\boldsymbol{u}_{\mathrm{g}}$ and $\boldsymbol{i}_{\mathrm{c}}$

- Consider the steady state and the fundamental component

$$\boldsymbol{i}_{\mathrm{c}} = \frac{\boldsymbol{u}_{\mathrm{c}} - \boldsymbol{u}_{\mathrm{g}}}{\mathrm{j}\omega_{\mathrm{g}}L}$$

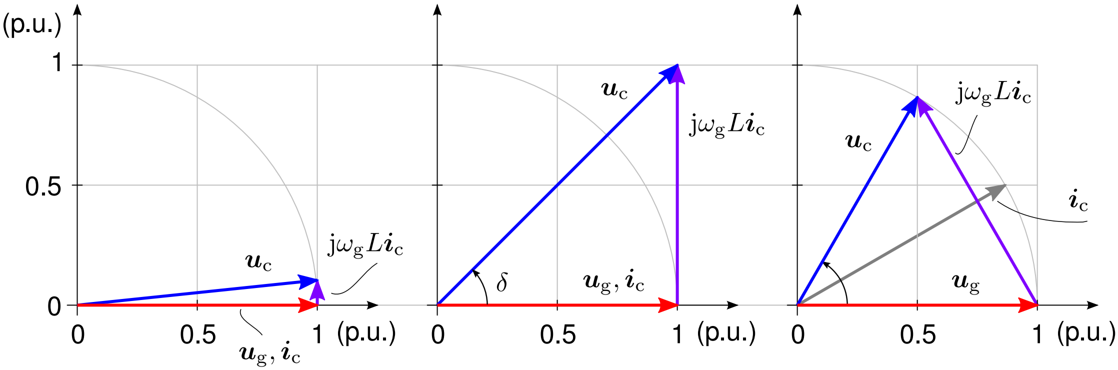
- Substituting $\boldsymbol{i}_{\mathrm{c}}$ to the power expression yields the power angle equation

$$p_{\mathrm{g}} = \frac{3}{2}\frac{1}{\omega_{\mathrm{g}}L}\,\mathrm{Im}\left\{\boldsymbol{u}_{\mathrm{c}}\boldsymbol{u}_{\mathrm{g}}^{*}\right\} = \frac{3}{2}\frac{|\boldsymbol{u}_{\mathrm{c}}||\boldsymbol{u}_{\mathrm{g}}|}{\omega_{\mathrm{g}}L}\sin\delta$$



If per-unit values are used, the factor $3/2$ in the power equation and the factor $3n_{\mathrm{p}}/2$ in the torque equation disappear.

Examples: Strong Grid and Very Weak Grid



Strong grid: $L = 0.1$ p.u.
$\cos\varphi = 1$, $p_\mathrm{g} = 1$ p.u.

Very weak grid: $L = 1$ p.u.
$\cos\varphi = 1$, $p_\mathrm{g} = 1$ p.u.

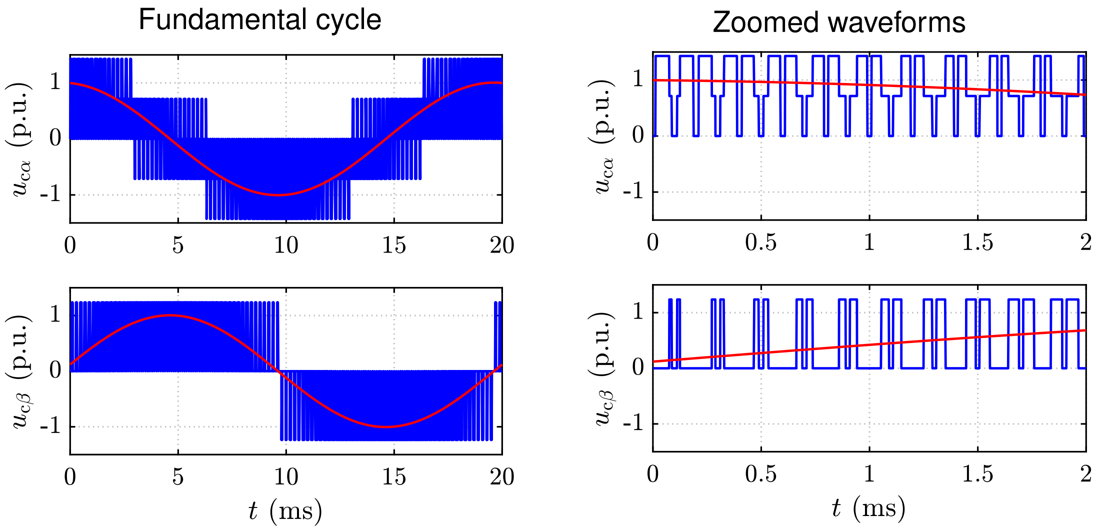
Very weak grid: $L = 1$ p.u.
$|\boldsymbol{u}_\mathrm{c}| = 1$ p.u.

The converter current $|\boldsymbol{i}_\mathrm{c}| = 1$ p.u., the grid voltage $|\boldsymbol{u}_\mathrm{g}| = 1$ p.u., and the grid frequency $\omega_\mathrm{g} = 1$ p.u. are assumed in all three example cases.

Observations

- Capability to feed active power to the grid reduces with the increasing grid inductance for the given voltage and current capacity
  - Case 1: $|\boldsymbol{u}_\mathrm{c}| \approx$ 1.005 p.u.
  - Case 2: $|\boldsymbol{u}_\mathrm{c}| = \sqrt{2}$ p.u. $\approx$ 1.41 p.u., need for higher u_dc and voltage rating
  - Case 3: $|\boldsymbol{u}_\mathrm{c}| =$ 1 p.u. but the power $p_\mathrm{g} = \sqrt{3}/2$ p.u. $\approx$ 0.87 p.u. is limited
- Maximum available converter voltage is $u_\mathrm{c,max} = u_\mathrm{dc}/\sqrt{3}$
- Short-circuit ratio (SCR) is the inverse of L in per units
  - Grid is considered weak if SCR $<$ 3 (corresponding to $L >$ 0.33 p.u.)
- Stability of the control system may also limit the maximum power in weak grids
- These considerations are approximately valid also for an LCL filter, which behaves as an L filter at lower frequencies

PWM Voltage: Symmetrical Suboscillation Method (SVPWM)



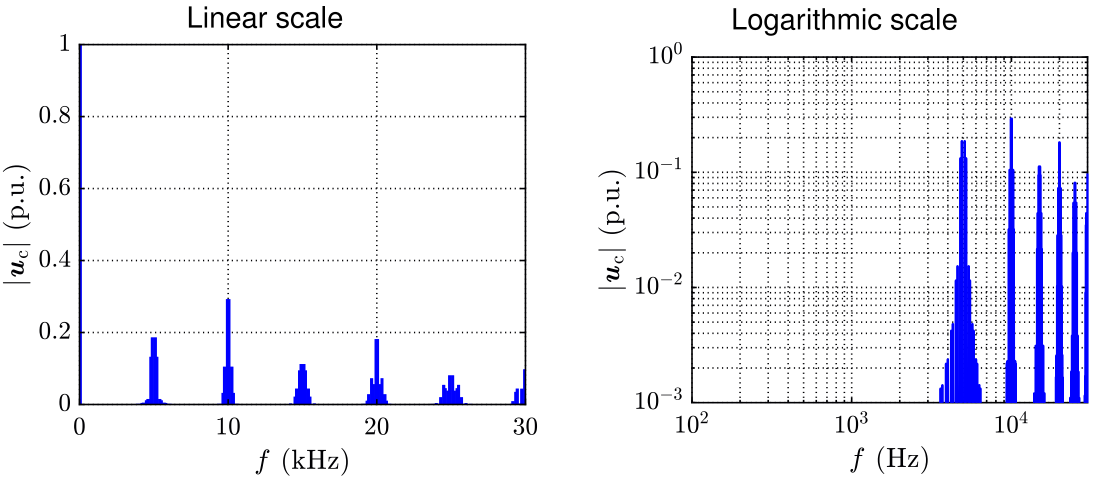
PWM Voltage Spectrum

Spectrum depends on

- Modulation method (SVPWM, DPWM, ...)
- Operating point (DC-bus voltage, AC voltage)
- Ratio between the switching frequency and the fundamental frequency
- Parasitic issues (inverter dead time, clock resolution, ...)

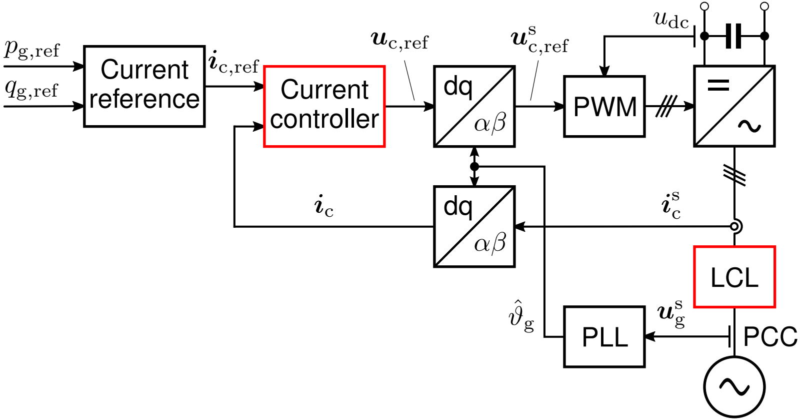
Spectrum can be determined, e.g., by means of the discrete Fourier transform

- Fundamental frequency 50 Hz, switching frequency 5 kHz



LCL Filters are Commonly Used in Grid Converters

- Current controller has to damp the LCL resonance (unless damping resistors in the filter are used)
- Converter current or grid current is measured



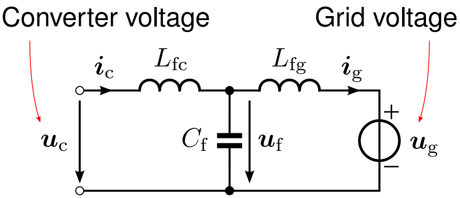
LCL Filter: Space-Vector Model

- Model is presented in stator coordinates
- Superscript s is dropped to simplify notation
- Model can be easily transformed to synchronous coordinates

$$\frac{\mathrm{d}}{\mathrm{d}t} \leftarrow \frac{\mathrm{d}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{g} \qquad \text{in the time domain}$$

$$s \leftarrow s + \mathrm{j}\omega_\mathrm{g} \qquad \text{in the Laplace domain}$$

- Parasitic losses are omitted



$$L_\mathrm{fc}\frac{\mathrm{d}\boldsymbol{i}_\mathrm{c}}{\mathrm{d}t} = \boldsymbol{u}_\mathrm{c} - \boldsymbol{u}_\mathrm{f}$$

$$C_\mathrm{f}\frac{\mathrm{d}\boldsymbol{u}_\mathrm{f}}{\mathrm{d}t} = \boldsymbol{i}_\mathrm{c} - \boldsymbol{i}_\mathrm{g}$$

$$L_\mathrm{fg}\frac{\mathrm{d}\boldsymbol{i}_\mathrm{g}}{\mathrm{d}t} = \boldsymbol{u}_\mathrm{f} - \boldsymbol{u}_\mathrm{g}$$

- State-space form

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{bmatrix}\boldsymbol{i}_\mathrm{c}\\ \boldsymbol{u}_\mathrm{f}\\ \boldsymbol{i}_\mathrm{g}\end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix}0 & -1/L_\mathrm{fc} & 0\\ 1/C_\mathrm{f} & 0 & -1/C_\mathrm{f}\\ 0 & 1/L_\mathrm{fg} & 0\end{bmatrix}}_{\boldsymbol{A}}\begin{bmatrix}\boldsymbol{i}_\mathrm{c}\\ \boldsymbol{u}_\mathrm{f}\\ \boldsymbol{i}_\mathrm{g}\end{bmatrix} + \underbrace{\begin{bmatrix}1/L_\mathrm{fc}\\ 0\\ 0\end{bmatrix}}_{\boldsymbol{B}_\mathrm{c}}\boldsymbol{u}_\mathrm{c} + \underbrace{\begin{bmatrix}0\\ 0\\ -1/L_\mathrm{fg}\end{bmatrix}}_{\boldsymbol{B}_\mathrm{g}}\boldsymbol{u}_\mathrm{g}$$

$$\boldsymbol{i}_\mathrm{c} = \underbrace{\begin{bmatrix}1 & 0 & 0\end{bmatrix}}_{\boldsymbol{C}_\mathrm{c}}\boldsymbol{x} \qquad \boldsymbol{i}_\mathrm{g} = \underbrace{\begin{bmatrix}0 & 0 & 1\end{bmatrix}}_{\boldsymbol{C}_\mathrm{g}}\boldsymbol{x}$$

- Transfer functions can be expressed using system matrices, for example

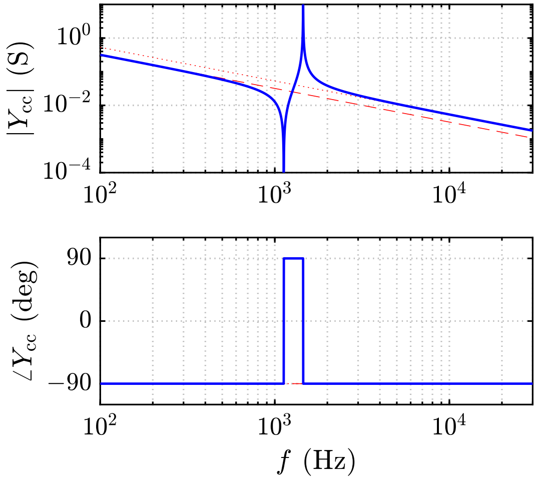
$$\frac{\boldsymbol{i}_\mathrm{c}(s)}{\boldsymbol{u}_\mathrm{c}(s)} = Y_\mathrm{cc}(s) = \boldsymbol{C}_\mathrm{c}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}_\mathrm{c} \qquad \frac{\boldsymbol{i}_\mathrm{g}(s)}{\boldsymbol{u}_\mathrm{c}(s)} = Y_\mathrm{gc}(s) = \boldsymbol{C}_\mathrm{g}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}_\mathrm{c}$$

Transfer Function from $\boldsymbol{u}_{\mathrm{c}}$ to $\boldsymbol{i}_{\mathrm{c}}$

$$\frac{\boldsymbol{i}_{\mathrm{c}}(s)}{\boldsymbol{u}_{\mathrm{c}}(s)} = Y_{\mathrm{cc}}(s) = \frac{1}{sL_{\mathrm{fc}}} \frac{s^2 + \omega_{\mathrm{z}}^2}{s^2 + \omega_{\mathrm{p}}^2}$$

- Antiresonance frequency and resonance frequency

$$\omega_{\mathrm{z}} = \sqrt{\frac{1}{L_{\mathrm{fg}} C_{\mathrm{f}}}} \qquad \omega_{\mathrm{p}} = \sqrt{\frac{L_{\mathrm{fc}} + L_{\mathrm{fg}}}{L_{\mathrm{fc}} L_{\mathrm{fg}} C_{\mathrm{f}}}}$$



Transfer Function from $\boldsymbol{u}_\mathrm{c}$ to $\boldsymbol{i}_\mathrm{g}$

$$\frac{\boldsymbol{i}_\mathrm{g}(s)}{\boldsymbol{u}_\mathrm{c}(s)} = Y_\mathrm{gc}(s) = \frac{1}{sL_\mathrm{fc}} \frac{\omega_\mathrm{z}^2}{s^2 + \omega_\mathrm{p}^2}$$

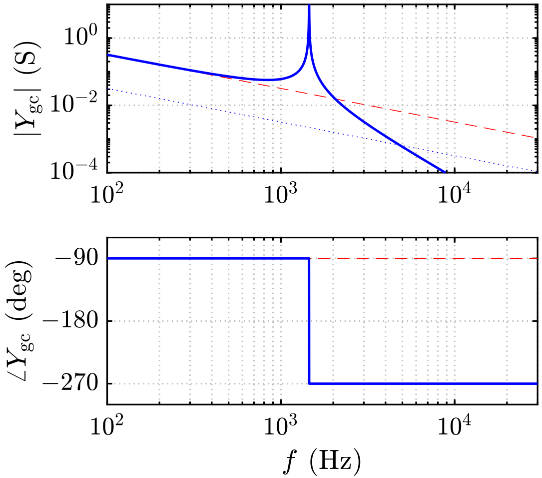
- Parameters in these examples

$$L_\mathrm{fc} = 3 \text{ mH} \quad C_\mathrm{f} = 10\ \mu\text{F} \quad L_\mathrm{fg} = 2 \text{ mH}$$

- Resonance frequencies

$$f_\mathrm{z} = \omega_\mathrm{z}/(2\pi) = 1.13 \text{ kHz}$$
$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = 1.45 \text{ kHz}$$

- Red dashed line: inductor $L_\mathrm{fc} + L_\mathrm{fg}$
- Blue dotted line: $10(L_\mathrm{fc} + L_\mathrm{fg})$



LCL Filter vs. L Filter

- Harmonics of the grid current at the PCC should meet the grid code or standard requirements (e.g. THD $< 5\%$)
- Compare $|Y_{\mathrm{gc}}|$ of an LCL filter at 5 kHz to the inductor $10(L_{\mathrm{fc}} + L_{\mathrm{fg}})$
- For the same attenuation, the total inductance of the LCL filter can be made much smaller than the pure inductance of the L filter
- How the inductance value is related to the physical size of the inductor?[1,2]

[1] Kazimierczuk, *High-frequency magnetic components*. Wiley, 2013.

[2] Koppinen, Rahman, and Hinkkanen, "Effects of the switching frequency of a grid converter on the LCL filter design," in *Proc. IET PEMD*, 2016.

Measure for an Inductor Size: Area Product

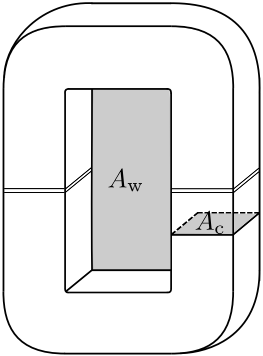
- Stored energy of the inductor

$$W_L = \frac{1}{2}Li^2 = \frac{1}{2}\psi i = \frac{1}{2}(NBA_\mathrm{c})(K_\mathrm{u}JA_\mathrm{w}/N)$$

- B is the peak value of the core flux density [T] (depends on the core material)
- J is the peak value of the current density [A/m^2] (depends on the cooling)
- K_u is the window utilization factor
- For given maximum current i, the inductance is proportional to the area product $A_\mathrm{w}A_\mathrm{c}$ [m^4]

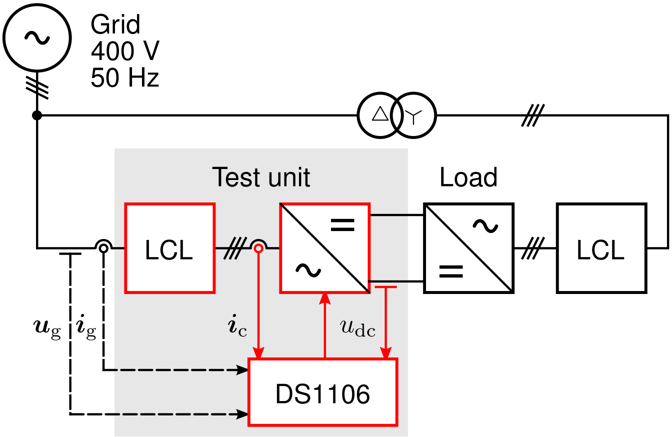
$$L = \frac{K_\mathrm{u}JB}{i^2}A_\mathrm{w}A_\mathrm{c}$$

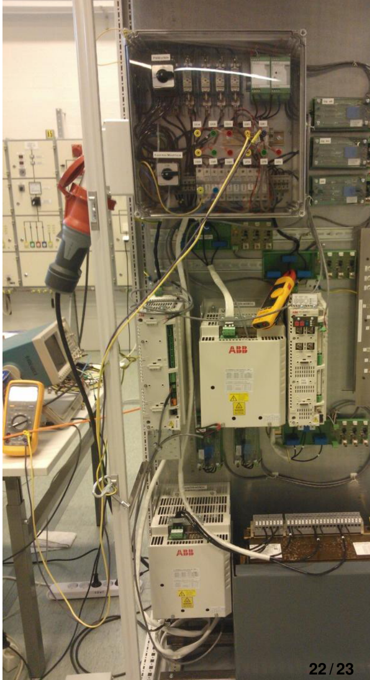
- Required L depends on the switching frequency



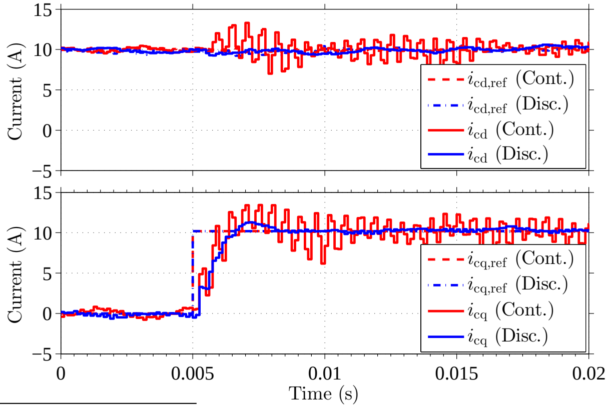
Example of Experimental Results

- 12.5-kVA test converter
- Switching frequency $f_{\mathrm{sw}} = 4$ kHz
- Filter resonance frequency: $f_{\mathrm{p}} = 1.5$ kHz





- Low ratio $f_{\mathrm{sw}}/f_{\mathrm{p}}$ makes control more difficult
- Digital delays can be effectively taken into account in the discrete-time design
- Two state-feedback controllers (same feedback signals and design specs): continuous-time design with the Tustin method vs. direct discrete-time design[3]



[3] Kukkola, Hinkkanen, and Zenger, "Observer-based state-space current controller for a grid converter equipped with an LCL filter: Analytical method for direct discrete-time design," *IEEE Trans. Ind. Appl.*, 2015.