ELEC-E8422 An Introduction to Electric Energy
Exercises - Lecture 2 3-phases system

## EX 1 wye and delta connections

The phase voltage of a wye connected 3-phases source is $\bar{V}_{a n}=230 \angle-40^{\circ}$. The source is feeding a delta connected 3-phases load, the impedance of which is $\bar{Z}=10 \angle-30^{\circ}$.
a. Draw the circuit sketch of this situation
b. calculate the line-to-line voltages of the source ( $\bar{V}_{a b}, \bar{V}_{b c}, \bar{V}_{c a}$ ) and the phase currents of the load $\left(\bar{I}_{a b}, \bar{I}_{b c}, \bar{I}_{c a}\right)$

## EX 2 impedances

The phase voltage of a wye connected 3-phases source is $\bar{V}_{a n}=230 \angle 0^{\circ}$. The source is feeding a delta connected 3 -phases load, the impedance of which is unknown. The measurement of the cphase current gives $\bar{I}_{c}=10 \angle 75^{\circ}$. Calculate the load impedance, i.e., the impedance of one branch of the delta connected load.

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Exercises - Lecture 1 AC circuits

## EX 1 wye and delta connections

The phase voltage of a wye connected 3-phases source is $\bar{V}_{a n}=230 \angle-40^{\circ}$. The source is feeding a delta connected 3-phases load, the impedance of which is $\bar{Z}=10 \angle-30^{\circ}$.
c. Draw the circuit sketch of this situation.
d. calculate the line-to-line voltages of the source $\left(\bar{V}_{a b}, \bar{V}_{b c}, \bar{V}_{c a}\right)$ and the phase currents of the load $\left(\bar{I}_{a b}, \bar{I}_{b c}, \bar{I}_{c a}\right)$

## Solution



From th electure slides, one can find an equation for the line-to-line voltage:

$$
\begin{aligned}
\bar{V}_{a b} & =\bar{V}_{a n}-\bar{V}_{b n} \\
& =230 Ð-40^{\circ}-230 Đ(-40-120)^{\circ} \\
& =398.4 Ð-10^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
\bar{V}_{b c} & =\bar{V}_{b n}-\bar{V}_{c n} \\
& =230 Đ(-40-120)^{\circ}-230 Đ(-40-240)^{\circ} \\
& =398.4 \mathrm{Ð}-130^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
\bar{V}_{c a} & =\bar{V}_{c n}-\bar{V}_{a n} \\
& =230 Đ(-40-240)^{\circ}-230 Đ-40^{\circ} \\
& =398.4 Đ 110^{\circ}
\end{aligned}
$$

the angle of the line-to-line voltage ca could also be $-250^{\circ}(-250+360=110)$
The currents are calculated from the voltage over the impedance, i.e.,
$\bar{I}_{a b}=\frac{\bar{V}_{a b}}{\bar{Z}}$
$\bar{I}_{b c}=\frac{\bar{V}_{b c}}{\bar{Z}}$
$\bar{I}_{c a}=\frac{\bar{V}_{c a}}{\bar{Z}}$
$=\frac{398.4 \mathrm{Ð}-10^{\circ}}{10 Ð-30^{\circ}}$
$=\frac{398.4 \mathrm{Đ}-130^{\circ}}{10 \mathrm{Ð}-30^{\circ}}$
$=\frac{398.4 \mathrm{Đ} 110^{\circ}}{10 \mathrm{Ð}-30^{\circ}}$
$=39.84 \mathrm{Ð} 20^{\circ}$
$=39.84 \mathrm{Ð}-100^{\circ}$
$=39.84 \mathrm{Đ} 140^{\circ}$

Note that the currents are also 120 degrees phase-shifted from each other.

## EX 2: impedances

The phase voltage of a wye connected 3-phases source is $\bar{V}_{a n}=230 \angle 0^{\circ}$. The source is feeding a delta connected 3 -phases load, the impedance of which is unknown. The measurement of the cphase current gives $\bar{I}_{c}=10 \angle 75^{\circ}$. Calculate the load impedance, i.e., the impedance of one branch of the delta connected load.

## Solution

Let us first calculate the voltage over on of the branches of the delta-connected load, i.e., the line-to-line voltage, e.g.

$$
\begin{aligned}
\bar{V}_{a b} & =\sqrt{3} \bar{V}_{a n} \mathrm{Đ} 30^{\circ} \\
& =\sqrt{3}\left(V_{a n} \mathrm{Ð} 0^{\circ}\right) \mathrm{Ð} 30^{\circ} \\
& =398.4 \mathrm{Ð} 30^{\circ}
\end{aligned}
$$

The load current can be calculated from the source current as, e.g.,
$\bar{I}_{a b}=\frac{\bar{I}_{a} Đ 30^{\circ}}{\sqrt{3}}$, where

$$
\begin{aligned}
\bar{I}_{a} & =\bar{I}_{c} Đ-120^{\circ} \\
& =\left(10 Ð 75^{\circ}\right) Ð-120^{\circ} \\
& =10 Ð-45^{\circ}
\end{aligned}
$$

so that

$$
\begin{aligned}
\bar{I}_{a b} & =\frac{\bar{I}_{a} Đ 30^{\circ}}{\sqrt{3}} \\
& =\frac{\left(10 Ð-45^{\circ}\right) Ð 30^{\circ}}{\sqrt{3}} \\
& =5.77 Ð-15^{\circ}
\end{aligned}
$$

The impedance can then be calculated as:
$\bar{Z}=\frac{\bar{V}_{a b}}{\bar{I}_{a b}}$
$=\frac{398.4 \angle 30^{\circ}}{5.77 \angle-15^{\circ}}$
$=69.05 \angle 45^{\circ} \Omega$

