# Hints for exercise set #1

2. In part b), the following facts will be useful:

$$\mathbb{E}(y_i) = \int_0^\infty \frac{y^3}{2\theta} e^{-\frac{y}{\theta}} dy = 3\theta.$$

$$\mathbb{E}(y_i^2) = \int_0^\infty \frac{y^4}{2\theta} e^{-\frac{y}{\theta}} dy = 12\theta^2$$

   The integrals can be solved via multiple iterations of integration by parts, if you wish to convince yourself.

   Recall also the identity $\mathbb{E}(X^2) = \mathrm{var}(X) + [\mathbb{E}(X)]^2$ for any random variable $X$.

4. In part b), you may need the expectation $\mathbb{E}(x_i)$ of an Erlang distributed random variable $x_i$. By definition, expected value is given by the integral

$$\mathbb{E}(x_i) = \frac{1}{(m-1)!} \int_0^\infty \left(\frac{x}{\theta}\right)^m e^{-\frac{x}{\theta}} dx.$$

   Introducing the substitution $u = x/\theta$, we have $dx = \theta du$ and

$$\mathbb{E}(x_i) = \frac{\theta}{(m-1)!} \underbrace{\int_0^\infty u^m e^{-u} du}_{I_m}.$$

   Denoting the resulting integral by $I_m$, we can integrate by parts and obtain

$$I_m = -u^m e^{-u} \Big|_0^\infty + \int_0^\infty m u^{m-1} e^{-u} du = 0 + m I_{m-1}.$$

   Continuing the recursion, and observing that $I_0 = 1$, we obtain $I_m = m!$, and hence

$$E(x_i) = m\theta.$$