Combinatorial Network Analysis - Introduction

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- Example 1. The world wide web is a network: the elements are the web pages and the connections are the hyperlinks that point from one web page to another.
- Example 2. The network made by the behaviour of the reindeer in a herd in a given day. Each reindeer is an actor, and we place a weighted interaction between any two reindeer: the weight is n if they come closer than a given threshold (say, 1 meter) n times in a day.

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- Example of question addressed by network science: which elements of the network are the most important?
- Example of question addressed by network science: are there any naturally identified subnetworks that emerge from the main network?

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The questions on the previous slides were admittedly vaguely formulated. One goal of network analysis is to mathematize them so that they can be precisely formulated (and hopefully answered). In doing so one takes into account various aspects:

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- The underlying application – the mathematical model should be as realistic as possible;
- Mathematical issues – it should be complex enough to capture as much as possible of the reality, but also simple enough that mathematicians can do something with it;
- Computational issues – real-life networks are often huge; they may evolve with time; practitioners may want very quick answers on them.

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- Edges = abstract connections. Information spreading among a group of people; airports linked by airline routes.

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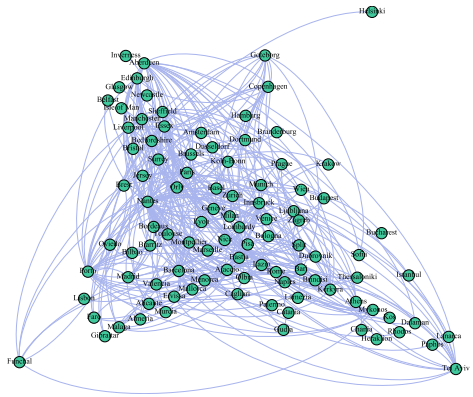
- Edges = other kinds of relations. Exchange of goods/services in an economic network; correlation between stocks in the market exchange.

Some real-world examples



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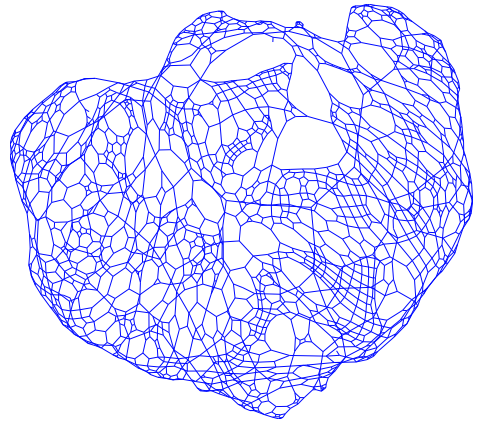

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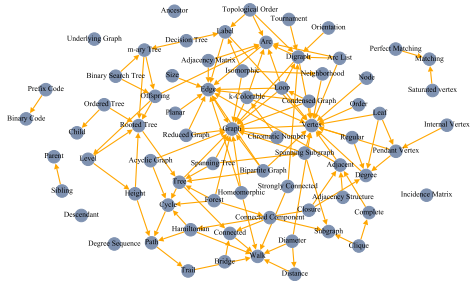

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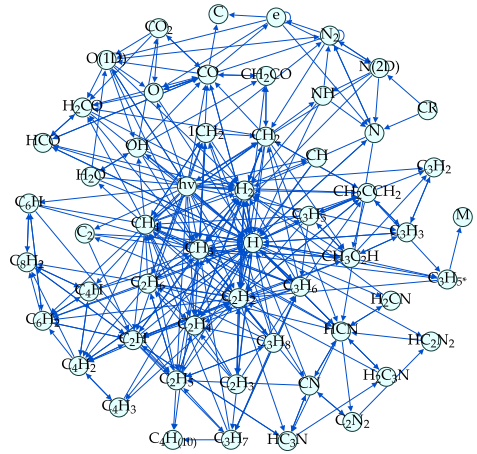

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