Structural Models in Behavioral Economics and Experimental Economics

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Introduction: Objective and Contents

Objective: Give an overview of applications of structural models in the context of experimental and behavioral economics.

Contents:

- Why use structural models?
- Basic estimation techniques commonly used on behavioral data
- Modeling of behavioral heterogeneity
    - Example: Social preferences
- Uncertainty in structural models
    - Example: Risk preferences

Basics

- Explicit modeling of preferences, beliefs, and constraints.
- The decision maker derives value: $V_c = V(c, \mathbf{x}, \boldsymbol{\theta})$.
  - c: choice alternatives in their choice set (whether discrete or continuous).
  - $\mathbf{x}$: variables defining the choice environment $\mathbf{x}$
  - $\boldsymbol{\theta}$ structural parameters (preferences, beliefs)
- Adding randomness generates a statistical model that provides a mapping between the distribution of choices (or moments of this distribution) and the structural parameters $\boldsymbol{\theta}$.

$\rightarrow$ impact of counterfactual changes in choice environment, $\mathbf{x}$, keeping preferences and beliefs, $\boldsymbol{\theta}$, constant.

Introduction: Advantages and Challenges

Advantages:

- Transparency in mechanisms driving predictions.

Challenges:

- Some often voiced concerns with structural models: i) takes time ii) stronger assumptions relative to reduced-form approaches
- *Behavioral economics* provides rich, explicit models and *experimental methods* allow researchers to exogenously vary components of the decision-making environment.
  - Experiments facilitates estimation under weaker assumptions
  - Experiments can be used to validate structural models
  - Structural models in Behavioral Economics do not need to be very complicated or hard to analyze

Example: A basic (principal-agent) model of worker motivation

A motivating example (Shearer, 2004)

Consider the following model of worker behavior in response to changes in compensation (see Shearer, 2004).

The economic model is based on a value function capturing utility of worker i at period t.

$$V_{it} = r_{it} y_{it} - C_i \left(e_{it} \right)$$

r_{it} is the piece-rate paid to the worker per unit of daily output y_{it}, and $C_i \left(e_{it} \right)$ is an increasing convex function capturing cost of effort e_{it}.

Assume that worker output follows a multiplicative production function:

$$y_{it} = e_{it} s_{it},$$

where s_{it} denotes random factors like weather conditions which influence worker output which are unrelated to the effort exerted.

A motivating example (Shearer, 2004)

To conveniently express the effort cost function $C_i\left(e_{it}\right)$, we can use:

$$C_i\left(e_{it}\right) = \kappa_i \frac{\gamma e_{it}^{(\gamma+1)/\gamma}}{(\gamma+1)}$$

where κ_i is a worker specific productivity parameter and γ reflects the elasticity of output to the piece-rate.

Solving for the optimal effort e_{it}^* leads to the following optimal output:

$$y_{it}^* = \frac{\left(r_{it} s_{it}\right)^{\gamma_i}}{\kappa_i^{\gamma_i}}.$$

Taking natural logs on both sides yields

$$\ln\left(y_{it}^*\right) = \gamma \ln\left(r_{it}\right) - \gamma \ln\left(\kappa_i\right) + \gamma \ln\left(s_{it}\right)$$

Can be written as a classical linear panel data regression model with unobserved individual heterogeneity. This transition from the economic model to a statistically estimable model is enabled by the stochastic element s_{it}.

Observation A: Reduced Behavioral Assumptions (Paarsch and Shearer)

- Paarsch and Shearer (1999) estimate γ using payroll data from a tree-planting firm in British Columbia.
- A key issue is *endogeneity*: the firm sets higher piece-rates r_{it} for more challenging planting blocks (lower s_{it}), introducing a negative correlation between s_{it} and r_{it}.
- To address this: added assumptions about how the firm determines r_{it}, specifically that it's set to make the least productive worker indifferent between working and minimum wage.
- Shearer (2004) instead uses experimental data which randomizes piece-rates across treatment blocks, ensuring variation in r_{it} for a given s_{it}. No need for additional assumptions.
- *This exemplifies the advantage of using experimental data to relax behavioral assumptions*

Observation B: Reduced Distributional Assumptions

- Above we observed that the experimental setup resulted in weaker behavioral assumptions. The experimental setup also gives weaker distributional assumptions.
- Paarsch and Shearer (1999) use Maximum Likelihood for their model, imposing extra distributional assumptions on s_{it}. Experimental data allows the use of simpler linear regression methods with minimal distributional assumptions (conditional moment restrictions).
- Both observations guide us to a simplified linear regression model, which can be implemented in standard statistical packages.
- This underscores the utility of experimental data in lessening behavioral and distributional assumptions during model estimation.

Observation C: Confounding Factors

- Consider an extension of a simple economic model of worker motivation above
- Role of gift-giving as an effort-inducing device (Akerlof, 1982;, Fehr et al., 1993; Gneezy & List, 2006)
- Surprise wage cuts triggers are stronger reaction than wage increases (Kube, Maréchal, and Puppe, 2013).
- Negative reciprocity dominates positive reciprocity?

Modeling Reciprocity

Consider a economic model that incorporates gifts and reciprocity:

$$V_{it} = r_{it} y_{it} - C_i\left(e_{it}\right) + \beta\left(y_{it} - y_{it}^{NG}\right) Gift_{it}$$

$Gift_{it}$: Unexpected wage increase or decrease; β reciprocity; y_{it}^{NG} production absent a gift.

Solving for optimal effort gives:

$$y_{it} = \left(\frac{\left[r_{it} + \beta Gift_{it}\right]}{\kappa_i}\right)^{\gamma} s_{it}^{\gamma+1}$$

Which can estimated using the following linear least squares equation:

$$\ln\left(y_{it}\right) = \alpha_0 + \gamma \log\left(r_{it} + \beta Gift_{it}\right) + \alpha_i + \epsilon_{it}$$

Structural modeling clarifies confounding factors

- Does finding a bigger productivity effect of wage decrease compared to wage increase provide evidence that negative reciprocity dominates?
- Recall optimal effort:

$$y_{it} = \left(\frac{[r_{it} + \beta Gift_{it}]}{\kappa_i} \right)^{\gamma} s_{it}^{\gamma+1}$$

- the structural model makes clear that the effect of a increase in wage/gift will depend on the cost of effort function γ.
- Using a piece rate of $r_{it} = 0.16$ and parameters values from Bellemare and Shearer (2011) $\beta = 0.001$ and $\gamma = 0.39$, a wage increase of \$100 leads to an increase average worker productivity by 20.8%, while a wage cut of \$100 is predicted to decrease average worker productivity by 31.7%.
- *Note*: Predictions based on given values of β, i.e. reciprocity is held constant.

Estimation Using First-Order Conditions

- First-order conditions (FOC) can be a basis for structural model estimation.
- FOCs often reveal the impact of model structure and variables on behavior more transparently than the utility function.
- Example: The FOCs for the cost-of-effort function can indicate how wage changes would affect effort, even before empirical estimation.

Estimation Using First-Order Conditions

- Suppose in an experiment, subject i has to choose y_i from a real interval and the value is given by:

$$V(y_i, \mathbf{x}_i, \boldsymbol{\theta})$$

- The optimal y_i^* solves:

$$V'(y_i^*, \mathbf{x}_i, \boldsymbol{\theta}) = 0$$

- The first-order condition approach requires that a closed-form expression for y_i^* can be obtained:

$$F(\mathbf{x}_i, \boldsymbol{\theta})$$

- Reduced-form models specify $F(\cdot)$ mainly for data-fitting and are thus not necessarily related to $V'(\cdot)$.
- Model is deterministic, assuming all subjects with same $\mathbf{x}_i$ choose the same y_i^*.
- Note: Incorporating heterogeneity in behavior is crucial for taking the model to data.
- Next: we discuss two approaches of introducing heterogeneity.

Modeling Preference Heterogeneity

- Introducing heterogeneity through structural parameters is often of primary interest to researchers.
- Equation extended to capture heterogeneity across subjects:

$$y_i^* = F(\mathbf{x}_i, \boldsymbol{\theta}_i)$$

- $\boldsymbol{\theta}_i$ now indexed by i; assume for simplicity it is scalar θ_i.
- Simple model for preference heterogeneity:

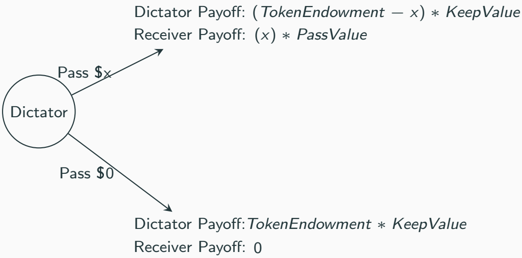
$$\theta_i = \mathbf{z}_i \boldsymbol{\theta} + \epsilon_i^{\theta}$$

- $\mathbf{z}_i$ is a vector of observable characteristics (e.g., age, gender).
- $\boldsymbol{\theta}$ captures the importance of these characteristics.
- ϵ_i^{θ} represents unobserved preference heterogeneity.
- Downside: The approach may not account for decision-making noise or other deviations.

Adding random disturbances

- Additive random disturbances capture deviations from model structure.
- In the worker motivation example above, random factors where incorporated in production function
- The error term do not need to come from the economic model
- Example: Andreoni and Miller (2002) test altruistic behavior in dictator games.

Dictator Game



Dictator Game Tree

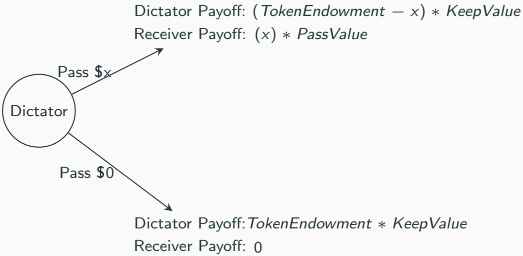


Figure 1: Dictator Game Tree

Allocation Choices in Andreoni and Miller (2002)

Budget	Token Endowment	Hold Value	Pass Value	Rel. Price of Giving	Avg Tokens Passed
1	40	3	1	3	8.0
2	40	1	3	0.33	12.8
3	60	1	1	1	12.7
4	60	1	2	0.5	19.4
5	75	2	1	2	15.5
6	75	1	2	0.5	22.7
7	60	1	1	1	14.6
8	100	1	1	1	23.0

- Dictators had multiple choices varying in budget and relative prices.
- This design allowed testing for rationality in giving and to investigate how the cost of giving affected decisions
- Moreover, subjects can be categorized according to their social preferences
- Classifies 43% of subjects as either selfish, Leontief, or perfect substitutes.
- Remaining 57% modeled with a structural approach.

Andreoni and Miller (2002)

Utility function defined over payoffs of dictators $\left(\pi_i^d\right)$ and the other subject (π_i^o):

$$V_i = \left[\alpha\left(\pi_i^d\right)^\rho + (1-\alpha)\left(\pi_i^o\right)^\rho\right]^{1/\rho}$$

where $\sigma = 1/(\rho - 1)$ is the elasticity of substitution between dictator and the other subject, which here captures the relative weight put inequality minimizing vs. efficiency.

α captures the weight put on own payoffs versus the other persons payoffs (i.e. the degree of selfishness).

Andreoni and Miller (2002)

Budget constraint is $m = \pi_d + p\pi_p$, with $m = M/p_d$ and $p = p_p/p_d$, where M denotes the budget, and (p_p, p_d) are the prices. First-order condition yields

$$\pi_{ig}^{d} = \frac{\gamma_0}{p^{\gamma_1} + \gamma_0} m_{ig}$$

where $\gamma_0 = \left(\frac{\alpha}{1-\alpha}\right)^{1/(1-\rho)}$ and $\gamma_1 = -\frac{\rho}{1-\rho}$ are estimated in a first step.

An error term is added to the first order conditions yielding a statistical model which is estimated by Maximum Likelihood

Based on the first-stage estimates γ_0, γ_1, structural parameters can be recovered in a second step using

$$\rho = \frac{\gamma_1}{\gamma_1 - 1}, \quad \sigma = (\gamma_1 - 1), \quad \alpha = \frac{1}{1 + \gamma_0^{1/(\gamma_1 - 1)}}$$

Standard errors of estimated structural parameters can be computed using the delta method.

Andreoni and Miller (2002)

TABLE IV

ESTIMATES OF PARAMETERS (STANDARD ERRORS) FOR CES UTILITY FUNCTIONS FOR THE THREE WEAK TYPES

	Weak Selfish	Weak Leontief	Weak Perf. Subst.
$A = [a/(1-a)]^{1/(1-\rho)}$	20.183	1.6023	2.536
	(5.586)	(0.081)	(0.311)
$r = -\rho/(1-\rho)$	−1.636	0.259	−2.022
	(0.265)	(0.067)	(0.188)
a	0.758	0.654	0.576
ρ	0.621	−0.350	0.669
σ	−2.636	−0.741	−3.022
s.e.-self	0.2216	0.179	0.244
	(0.011)	(0.009)	(0.014)
ln likelihood	−107.620	52.117	−69.583
Number of cases	380	230	242

Combining Preference Heterogeneity and Random Disturbances

- Exclusively focusing on preference heterogeneity or random disturbances can be limiting.
- Preference heterogeneity observed in laboratory and field experiments.
- Importance of accommodating departures that capture unobserved preferences and model deviations.
- Proposal: Combine preference heterogeneity and random disturbances.

Example: Gift giving revisited

Recall: We could estimate the model using:

$$\ln\left(y_{it}\right) = \alpha_0 + \gamma \log\left(r_{it} + \beta Gift_{it}\right) + \alpha_i + \epsilon_{it}$$

Reciprocity parameter can vary with respect to covariates x (e.g., gender, age, tenure).

That is, we can allow for heterogeneity of β with respect to covariates x_{it}, by assuming $\beta_i = \beta_0 + \beta_1 x_{it}$. This leads to:

$$\log\left(y_{it}\right) = \alpha_0 + \gamma \log\left(r_{it} + \beta_0 Gift_{it} + \beta_1 Gift_{it} x_{it}\right) + \alpha_i + \varepsilon_{it}$$

Introduction to Estimation Using Discrete Choice Models

- Discrete choice approach is predominant in behavioral economics.
- Structural models sometimes lack easily interpretable or usable first-order conditions.
- Discrete choice models naturally fit many experimental designs where subjects make discrete decisions.
- We build upon the random utility framework.
- Introduction to the basic random utility framework followed by its extensions for more heterogeneity.

Random Utility Framework

- Subjects derive value V_{ij} for each alternative j in set J.
- Value function: $V_{ij} = \mathbf{x}_{ij}\boldsymbol{\theta}$, linear in observable characteristics x_{ij}.
- Basic random utility model (RUM) introduces random disturbances (Fechner errors):

$$\tilde{V}_{ij} = V_{ij} + \lambda\varepsilon_{ij} = \mathbf{x}_{ij}\boldsymbol{\theta} + \lambda\varepsilon_{ij}$$

- ε_{ij}: random disturbance and λ: noise parameter
- Subject choose the alternative with highest valuation (including noise)
- Noise becomes more important when subject are close to indifferent
- Can be interpreted as either decision errors or unobserved preferences

Application to Ultimatum Game

Description

The Ultimatum Game involves two players. The proposer offer the responder a discrete share of an amount Π. The responder can accept ($j = a$) or reject ($j = r$) the proposal. If the responder accepts, the money is split as proposed. If rejected, both get nothing.

Assume the utility of each alternative is given by:

$$\tilde{V}_{ia} = V_{ia} + \lambda\varepsilon_{ia} = \mathbf{x}_{ia}\boldsymbol{\theta} + \lambda\varepsilon_{ia}$$
$$\tilde{V}_{ir} = V_{ir} + \lambda\varepsilon_{ir} = \mathbf{x}_{ir}\boldsymbol{\theta} + \lambda\varepsilon_{ir}$$

Where $\mathbf{x}_{ij}$ are alternative-specific explanatory variables, λ is the noise parameter

Responders choose option $j \in (a, r)$ that maximizes their value functions with error.

Ultimatum Game Example: Estimation

With i.i.d. extreme value Type 1 errors with constant main across alternatives we obtain the following Conditional Logit formula:

$$\Pr\left(y_i = a \mid \mathbf{x}_i\right) = \frac{\exp\left(\frac{\mathbf{x}_i \boldsymbol{\theta}}{\lambda}\right)}{\exp\left(\frac{\mathbf{x}_{ia} \boldsymbol{\theta}}{\lambda}\right) + \exp\left(\frac{\mathbf{x}_{ir} \boldsymbol{\theta}}{\lambda}\right)}$$

- $\boldsymbol{\theta}$ and λ can not all be estimated
- Common to normalize one coefficient to 1.

Ultimatum Game Example: Fehr and Schmidt (1999) preferences

Assume the responder has Fehr and Schmidt (1999) inequality averse preferences. The utility of rejecting is $V_{ir} = 0$ and utility of accepting is:

$$\begin{aligned} V_{ia} &= \pi_{ia} - \alpha \max\left(0, \Pi - 2\pi_{ia}\right) - \beta \max\left(0, 2\pi_{ia} - \Pi\right) \\ &= \mathbf{x}_{ia}\boldsymbol{\theta} \end{aligned}$$

i.e. we have

$$\mathbf{x}_{ia} = \left[\pi_{ia}, \max\left(0, \Pi - 2\pi_{ia}\right), \max\left(0, 2\pi_{ia} - \Pi\right)\right]$$

$$\boldsymbol{\theta} = (1, -\alpha, -\beta)'$$

Here coefficient of own payoffs π_{ia} is normalized to 1 which allows λ to be estimated freely. Also allows interpreting α and β as the the price the subject is willing to pay for reducing inequality by one unit.

Random Utility Model with Preference Heterogeneity

Part of heterogeneity in behavior in a given treatment condition possibly reflects preference heterogeneity, implying that $\boldsymbol{\theta}_i$ varies across i.

Often, we wish to decompose preference heterogeneity:

- Observable individual characteristics.
- Unobserved heterogeneity.

The unobservable part is usually larger, and capturing it is more complex (see for example von Gaudecker, van Soest and Wengström, 2011).

Approaches for Unobserved Heterogeneity

Three main methods in behavioral economics:

1. Parametric approach: E.g. assume that parameters are normally distributed and estimate the distribution parameters.
2. Finite mixture (non-parametric): Each subject belong to a class k defined by a common parameter vector θ_k
3. Individual-level estimation (non-parametric): Estimate the model on individual separately

Uncertainty in Structural Models

Decision-making under uncertainty is central in experimental and behavioral economics.

- So far: Models without uncertainty.
- Now: Estimation of structural models with uncertainty.
- Challenge: Value functions aren't simple linear combinations.

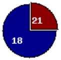
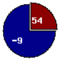
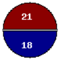
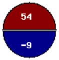
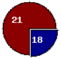
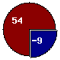
von Gaudecker, van Soest and Wengström, 2011

- Measuring risk preferences via experimental designs.
- Builds on earlier work by Hey and Orme (1994), Holt and Laury (2002) and others
- Online experiment with $N = 1422$ (CentERpanel)
- Unbalanced panel of binary decisions between lotteries π_j^A, π_j^B, $j = 1 \ldots J_i$ with $J_i \in \{28, 32, \ldots, 56\}$
- Large set of controls (CentERpanel): sex, age, education, household income, wealth, financial experience/knowledge, short / long completion time.

Screenshot of Lottery 5, First Screen

Progress: 70% **Instructions** **Help**

Please, make a choice between A and B for each of the decision problems below.

Option A -outcome IMMEDIATELY revealed		Option B -outcome revealed in THREE MONTHS		Choice A B
	€ 21 with probability 25% € 18 with probability 75%		€ 54 with probability 25% € -9 with probability 75%	○ ○
	€ 21 with probability 50% € 18 with probability 50%		€ 54 with probability 50% € -9 with probability 50%	○ ○
	€ 21 with probability 75% € 18 with probability 25%		€ 54 with probability 75% € -9 with probability 25%	○ ○
	€ 21 with probability 100% € 18 with probability 0%		€ 54 with probability 100% € -9 with probability 0%	○ ○

Continue

Payoffs from the Seven Lotteries

	Option A			Option B		
Payoff Configuration	Uncertainty Resolution	Low Payoff	High Payoff	Uncertainty Resolution	Low Payoff	High Payoff
1	early	27	33	early	0	69
2	early	39	48	early	9	87
3	early	12	15	early	-15	48
4	early	33	36	late	6	69
5	early	18	21	late	-9	54
6	early	24	27	early	-3	60
7	late	15	18	late	-12	51

Note: The order was randomised.

Expected Utility of Income

- Start from a simple exponential utility model with loss aversion:

$$u(z, \gamma, \lambda) = \begin{cases} -\frac{1}{\gamma} e^{-\gamma z} & \text{for } z \geq 0 \\ \frac{\lambda - 1}{\gamma} - \frac{\lambda}{\gamma} e^{-\gamma z} & \text{for } z < 0 \end{cases}$$

  where $z \in \mathbb{R}$ denote lottery outcomes, $\gamma \in \mathbb{R}$ is the coefficient of absolute risk aversion, $\lambda \in \mathbb{R}_+$ is the loss aversion parameter

Expected Utility of Income

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  where $z \in \mathbb{R}$ denote lottery outcomes, $\gamma \in \mathbb{R}$ is the coefficient of absolute risk aversion, $\lambda \in \mathbb{R}_+$ is the loss aversion parameter

- Why not power utility? Problems around the origin, difficult to incorporate uncertainty resolution preferences with positive and negative payoffs. But some robustness checks in the paper (worse fit).

Uncertainty Resolution Preferences

- We also estimate a preference of uncertainty resolution ρ using the framework of Kreps & Porteus (1978):
- This gives a slightly modified utility function.

$$v(z,\gamma,\lambda,\rho) = \begin{cases} \max\{-\frac{\lambda}{\gamma},0\} \qquad\qquad - \frac{1}{\gamma}e^{-\gamma\rho^S z} & \text{for } z \geq 0 \\ \max\{-\frac{\lambda}{\gamma},0\} + \frac{\lambda-1}{\gamma} - \frac{\lambda}{\gamma}e^{-\gamma\rho^S z} & \text{for } z < 0 \end{cases}$$

Overall Likelihood: Certainty Equivalents and Errors

- Binary choice between lotteries π^A and π^B. Take the difference in certainty equivalents between the lotteries for choice j by individual i:

$$\Delta\mathsf{CE}_{ij} = \mathsf{CE}(\pi_j^B, \gamma_i, \lambda_i, \rho_i) - \mathsf{CE}(\pi_j^A, \gamma_i, \lambda_i, \rho_i)$$

- The actual choice is then: $Y_{ij} = \mathbb{I}\left\{\Delta\mathsf{CE}_{ij} + \tau\varepsilon_{ij} > 0\right\}$; $\varepsilon_{ij} \sim \Lambda$

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- The actual choice is then: $Y_{ij} = \mathbb{I}\left\{\Delta \mathrm{CE}_{ij} + \tau \varepsilon_{ij} > 0\right\}$; $\varepsilon_{ij} \sim \Lambda$
- Likelihood of each observation:

$$l_{ij} = (1-\omega_i)\,\Lambda\left((2Y_{ij}-1)\frac{1}{\tau}\Delta \mathrm{CE}_{ij}\left(\pi_j^A, \pi_j^B, \gamma_i, \lambda_i, \rho_i\right)\right) + \frac{\omega_i}{2},$$

- Two sources of error: Monetary cost of "wrong" choice τ, probability for random behaviour ω_i.

Distributional Assumptions

- Write: $\eta_i \quad = \quad g_\eta(X_i^\eta \beta^\eta + \xi_i^\eta), \quad \eta_i = \{\gamma_i, \lambda_i, \rho_i, \omega_i\}$

  where the $g_\eta(\cdot)$ serve to impose the theoretical parameter restrictions.
- Assume joint normality of:

$$\begin{pmatrix} g_\gamma^{-1}(\gamma_i) \\ g_\lambda^{-1}(\lambda_i) \\ g_\rho^{-1}(\rho_i) \\ g_\omega^{-1}(\omega_i) \end{pmatrix} \sim N\left(\begin{pmatrix} X_i^\gamma \beta^\gamma \\ X_i^\lambda \beta^\lambda \\ X_i^\rho \beta^\rho \\ X_i^\omega \beta^\omega \end{pmatrix}, \Sigma'\Sigma \right)$$

Individual Likelihoods and Estimation Details

- Group the 4 unobserved components in ξ_i, define $\xi^* = (\Sigma')^{-1}\xi$ and get the individual likelihood:

$$l_i = \int_{\mathbb{R}^4} \left[\prod_{j \in J_i} l_{ij} \left(\pi_j^A, \pi_j^B, Y_{ij}, \tau, g(X_i\beta + \xi_i^*) \right) \right] \phi(\xi^*) d\xi^*$$

- Approximate the integral by numerical integration with Halton sequences.

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- Approximate the integral by numerical integration with Halton sequences.
- Maximise overall likelihood using BFGS algorithm with numerical derivatives, variance-covariance matrix via OPG, delta method for transformed parameters.

Aggregate Facts: Parameter Estimates

- Median utility function concave ($\gamma = .032$), has a kink at zero ($\lambda = 2.4$), uncertainty resolution does not matter for the median subject ($\rho = 1$)
- Implied risk premia comparable to other studies
- Median random choice propensity of about 8.3%. Together with $\tau = 4.1$, this implies:
  - $P(Y_j) = .88$ for $\Delta\text{CE}_j = 10$ Euros and
  - $P(Y_j) = .55$ for $\Delta\text{CE}_j = 1$ Euro.

Aside: Observable Correlates

- Women are more risk averse and loss averse, more inconsistencies.
- Positive age gradient of risk aversion and error frequency. Loss aversion peaks at ages 35-44 and decreases thereafter.
- Higher educated persons: less risk averse, substantially fewer mistakes.
- Little effects of income and wealth – but errors decrease in wealth.
- No significant associations for uncertainty resolution preferences.

Identification issues

Apesteguia and Ballester (2018) discussed identification problems associated with estimation of risk preferences based on random utility models.

Consider the following lotteries:

- Lottery A: Pays \$1 with probability 0.9, and \$60 with probability 0.1.
- Lottery B: Pays \$5 with certainty.
- Add type 1 extreme value random disturbances scaled by λ to the expected utility.

Probability of Choosing Lottery A

We can express the probability of choosing the risk lottery as

$$\Pr(\text{Choose } = A) = \frac{\exp\left(EU_{iA}(\theta)/\lambda\right)}{\exp\left(EU_{iA}(\theta)/\lambda\right) + \exp\left(EU_{iB}(\theta)/\lambda\right)}$$

Expectation: Probability of choosing A decreases with risk aversion θ.

Apesteguia and Ballester (2018) showed that this may not always be the case.

Monotonicity Issues

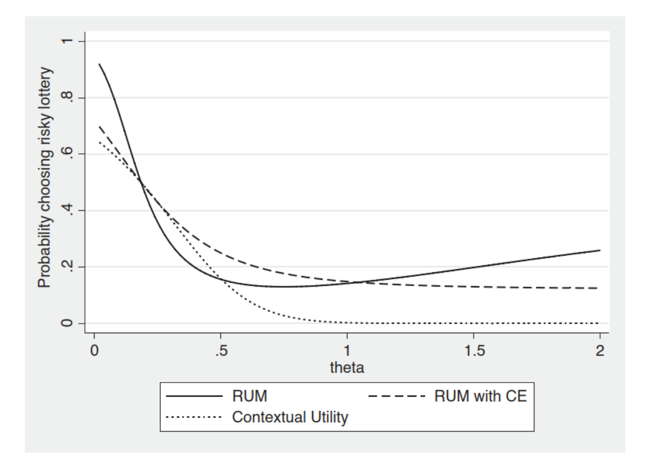

Figure 2: Predicted probability of choosing risky lottery A

Monotonicity Issues

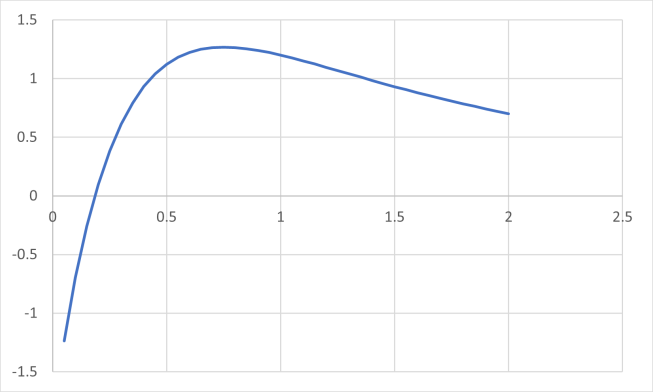

Figure 3: Expected Utility Difference between Safe and Risky lottery

Understanding Non-Monotonicity

- Probability of choosing the risky lottery initially decreases with risk aversion before increasing for high levels of risk aversion.
- Non-monotonicity arises due to varying scale of utility with risk aversion.
- Cardinal value of difference in expected utility diminishes and flattens out when θ increases.
- Noise level may dominate differences in value functions for high risk aversion.

Proposed Solutions

- Wilcox (2011): RUM approach called "contextual utility" that re-scales valuations.
- Certainty equivalents provide a common metric regardless of θ.
- Both methods can restore monotonicity, but not always.
- Apesteguia and Ballester (2018) introduced RPM as an alternative to RUM.

Random Parameter Model (RPM)

Preserves monotonicity. Randomness is introduced through unobserved heterogeneity of the risk preferences of subjects.

$$U\left(x_{ij};\theta_i\right) = U\left(x_{ij};\theta + \epsilon_i\right)$$

where ϵ_i follows a monotone cumulative distribution function over the space of risk preferences.

The model is estimated by Maximum Likelihood

Which approach to use?

- Clear objectives essential before implementing a design and conducting experiments.
- For some designs, all approaches work
- RPM preferable in more general choice settings, but can be harder to implement when having more than one parameter to estimate

Conclusions

- Experiments enable exogenous variation of subjects' choice environment
- Behavioral Economics provide a rich set of models of human behavior
- Choice between reduced-form or structural approaches should align with research questions
- Important to decide empirical approach prior to experiment design

Conclusions

- Careful experimental design (including simulations) critical for separate identification of model features.
- Experimental methods can reduce assumptions and validate models.
- Synergies between experiments and structural modeling