This is a suggested list of projects. They are of mainly two types: one involves studying some material beyond, but related to, what was discussed in class (typically one research article that we can provide to you) and elaborate the content in the form of a short report to be then discussed. The second types involves the practical implementation, through some programming language, of methods discussed in class: the resulting codes should be used to perform some proposed experiments, whose results should again be summarised in a short report to be discussed.

Students interested in this path to complete the course can either pick one of the proposed themes, or propose one themselves.

Theoretical projects

1. Study the details of the Benzi-Klymko Theorem on the universal behaviour of f-total communicabilites and f-subgraph centralities. Bibliography: M. Benzi and C. Klymko. On the limiting behavior of parameter-dependent network centrality measures. SIAM J. Matrix Anal. Appl. 36(2), 2015, pp. 686–706.

2. Investigate further the spectral theory of the deformed graph Laplacian. Bibliography: P. Grindrod, D. J. Higham and V. Noferini. The deformed graph Laplacian and its applications to network centrality analysis. SIAM J. Matrix Analysis Appl. 39(1), 2018, pp. 310–341.

3. Study the extension of NBT centrality to directed graph. Bibliography: F. Arrigo, P. Grindrod, D. J. Higham and V. Noferini. Nonbacktracking walk centrality for directed networks. J. Complex Networks 6(1), 2018, pp. 54–78.

4. Study the NBT analougue of f-subgraph centrality. Bibliography: F. Arrigo, P. Grindrod, D. J. Higham and V. Noferini. On the exponential generating function for non-backtracking walks. Linear Algebra Appl. 556, 2018, pp. 381–399.

5. Study the use of nonnegative matrix factorization for clustering, and explain how it is related to the spectral clustering performed using the Laplacian matrix. Bibliography:
`-https://www.cs.cmu.edu/~aarti/Class/10701/readings/Luxburg06_`

TR.pdf
- Ding, C., He, X., Simon, H.D.: On the equivalence of nonnegative matrix factorization and spectral clustering. In: SDM 2005: Proc. of SIAM Int. Conf. on Data Mining, pp. 606–610

Implementative projects

1. Write a program that, given in input the adjacency matrix of a graph, computes: degree centrality, Katz centrality, NBT centrality, and eigenvector centrality. Then, do a comparison on the rankings produced to these four centralities (possibly using a few distinct fraction of the maximum allowed value of the parameter for Katz and NBT) on a collection of real-life networks. We suggest the folllowing method: download a sample of graphs coming from applications from `https://sparse.tamu.edu/` (use the keyword "undirected graph" to search them, and if the graph is weighted construct via another code the corresponding unweighted version); compare the rankings obtained one each graph by e.g. Kendall's τ correlation parameter.

2. Write a program that, given in input the adjacency matrix of a graph, computes Katz centrality and NBT centrality. Test it on a relatively small sample graph (you can download examples of real-life networks from from `https://sparse.tamu.edu/` as described in the above project: if necessary, consider a subgraph). Study the behaviour of the centrality measure of each node as a function of the parameters (α and t respectively). Plot the results in a graph, using different colours for each node (limit yourself to the top vertices if appropriate). Discuss whether there are many takeovers or not, possibly depending on the graph.

3. Study and implement the algorithm described in "Brandes, Ulrik (2001). "A faster algorithm for betweenness centrality". Journal of Mathematical Sociology. 25 (2): 163–177" to efficiently compute the betweenness centrality of a graph. Test it on a relatively small sample graph (you can download examples of real-life networks from from `https://sparse.tamu.edu/` as described in the above project: if necessary, consider a subgraph). Discuss the computational complexity of the algorithm.

4. Study and implement the NMF (Nonnegative Matrix Factorization) methods presented in Chapter 6 of the article "Kuang, D.; Yun, S.; Park, H. SymNMF: Nonnegative low-rank approximation of a similarity matrix for graph clustering. J. Glob. Optim. 2015, 62, 545–574." Apply the algorithm to the dataset of images `http://www.cs.columbia.edu/CAVE/databases/SLAM_coil-20_coil-100/coil-20/coil-20-proc.zip` Use the right function for your programming language in order to convert the images into pixel-wise data. Plot the clustering accuracy, explained in Section 7.4, for different number of clusters k and comment the results.