Advanced Microeconomics 2
Helsinki GSE
Juuso Välimäki

Problem Set 3, Due December 5, 2023

1. Consider the following exchange economy with two agents and three goods (real Edgeworth Box). Agent 1 has linear preferences represented by the utility function

$$u_1(x_{11}, x_{12}, x_{13}) = x_{11} + 2x_{12} + 5x_{13},$$

   and agent 2 has utility function

$$u_2(x_{21}, x_{22}, x_{23}) = 3x_{21} + 3x_{22} + 7x_{23}.$$

   (a) Let the total resources of the three goods be given by: $\overline{x}_1 = \overline{x}_2 = \overline{x}_3 = 3$. What are the Pareto-efficient allocations?

   (b) Suppose that the initial endowments of the two agents are: $\omega_{11} = 2, \omega_{12} = 2, \omega_{13} = 1$ and $\omega_{21} = 1, \omega_{22} = 1, \omega_{23} = 2$. Compute the equilibrium prices and the equilibrium allocation for this economy.

2. Two consumers have identical Cobb-Douglas preferences for L goods $(x_{i1}, ..., x_{iL})$ given by:

$$u_i(\boldsymbol{x}_i) = \prod_{l=1}^{L} x_{il}^{\alpha_l},\ 0 < \alpha_l < 1 \text{ for all } l.$$

   (a) Let $\boldsymbol{\omega}$ denote the vector of total resources for the economy and find the Pareto-efficient allocations.

   (b) For Pareto-efficient allocations, compute the shares ($s_{21} = \frac{x_{2l}}{\omega_l}$ across the different goods l)

3. Consider the following exchange economy.

   (a) In an economy, two consumers have utility functions:

$$u_1(x_{11}, x_{12}) = \ln(x_{11}) + x_{12},$$

$$u_2(x_{21}, x_{22}) = \ln(x_{21}) + x_{22}.$$

Find the Pareto-efficient allocations for total resources of 2 units of good 1 and 4 units of good 2.

(b) Find a competitive equilibrium allocation and price for the economy where the agents have utility functions as above and the initial endowments are: $\omega_1 = (0, 3)$, $\omega_2 = (2, 1)$.

(c) Add a third consumer with utility function $u_3(x_{31}, x_{32}) = \ln(2x_{31} + x_{32})$ and $\omega_3 = (2, 2)$). Find the Pareto-efficient allocations and the competitive equilibrium.

4. Consider an economy where all three consumers $i \in \{1, 2, 3\}$ have the same utility functions $u_i(x_{i1}, x_{i2}) = x_{i1}x_{i2}$, and the initial endowments of the three consumers are $\omega_1 = (1, 14)$, $\omega_2 = (1, 14)$, $\omega_3 = (27, 1)$).

   (a) Show that the allocation $\boldsymbol{x} = ((6, 6), (7, 7), (16, 16))$ is Pareto-efficient.

   (b) Show that this allocation is in the core of the economy.

   (c) Consider a replica economy where you have identical copies to the original three consumers added to the economy. Denote an allocation for this economy by $\boldsymbol{x}^{(2)} = (\boldsymbol{x}, \boldsymbol{x}')$, where $\boldsymbol{x}'$ is the allocation for the copied consumers. Is the allocation

$$(((6, 6), (7, 7), (16, 16)), ((6, 6), (7, 7), (16, 16)))$$

   in the core for this replica economy?

5. M intermediate goods $j \in \{1, ..., M\}$ are produced using input vectors $\boldsymbol{z}_j$ with $\boldsymbol{z}_j \in \mathbb{R}^L_+$ and the production function is given by $q_j(\boldsymbol{z}_j) = f_j(\boldsymbol{z}_j)$ for some strictly increasing and concave function f_j. The final product q is produced from the intermediate goods according to the production function $q = \min\{q_1, ..., q_M\}$. Find the cost function for q in terms of the individual cost functions c_j.

6. Consider a production economy with a fixed size of available land L. All agents in the economy either work or enjoy leisure. Total amount of time available is $T \leq 2L$. Working hours are divided between cultivating barley b, denoted by t_b or cultivating rye r denoted by t_r so that leisure amounts to $T - t_b - t_r$. Land is also divided amongst barley and rye into l_b and l_r.

(a) Suppose that all agents have the same preferences given by

$$u^i(b^i, r^i, t_b^i, t_r^i) = b^i r^i (T - t_b^i + t_r^i)$$

and the production functions for firms producing b and r are Leontieff:

$$b = \min\{\frac{1}{2}t_b, l_b\}, \quad r = \min\{t_r, l_r\},$$

where t_j denotes the aggregate time spent cultivating j. Show that if a competitive solution to the firms' problems exists, the firms make zero profit.

(b) Define a competitive equilibrium for this economy.

(c) Show that in any competitive equilibrium, all agents work the same total hours.

(d) Solve for the competitive equilibrium prices and allocation.