

Aalto University

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Exam, Monday, December 4, 2023, 09:00 - 12:00

Complex Analysis, MS-C1300

Motivate your answers. Only giving answers gives no points. No calculators or books are allowed. **Good luck!**

- | | |
|---------|------|
| (1) (a) | (2p) |
| (b) | (2p) |
| (c) | (2p) |
| (2) (a) | (2p) |
| (b) | (2p) |
| (c) | (2p) |
| (3) (a) | (3p) |
| (b) | (3p) |
| (4) | (6p) |

Useful formulas

- Cauchy–Riemann equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

- Cauchy's Integral Formula

$$n(\gamma, z_0)f^{(k)}(z_0) = \frac{k!}{2\pi i} \int_{\gamma} \frac{f(z)}{(z - z_0)^{k+1}} dz$$

- Residue for a pole of order m

$$\text{Res}(z_0, f) = \frac{1}{(m-1)!} \lim_{z \rightarrow z_0} \frac{d^{m-1}}{dz^{m-1}} [(z - z_0)^m f(z)]$$

- Some Taylor series

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n, \text{ when } |z| < 1$$

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}, \text{ when } z \in \mathbb{C}$$

$$\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}, \text{ when } z \in \mathbb{C}$$

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}, \text{ when } z \in \mathbb{C}$$