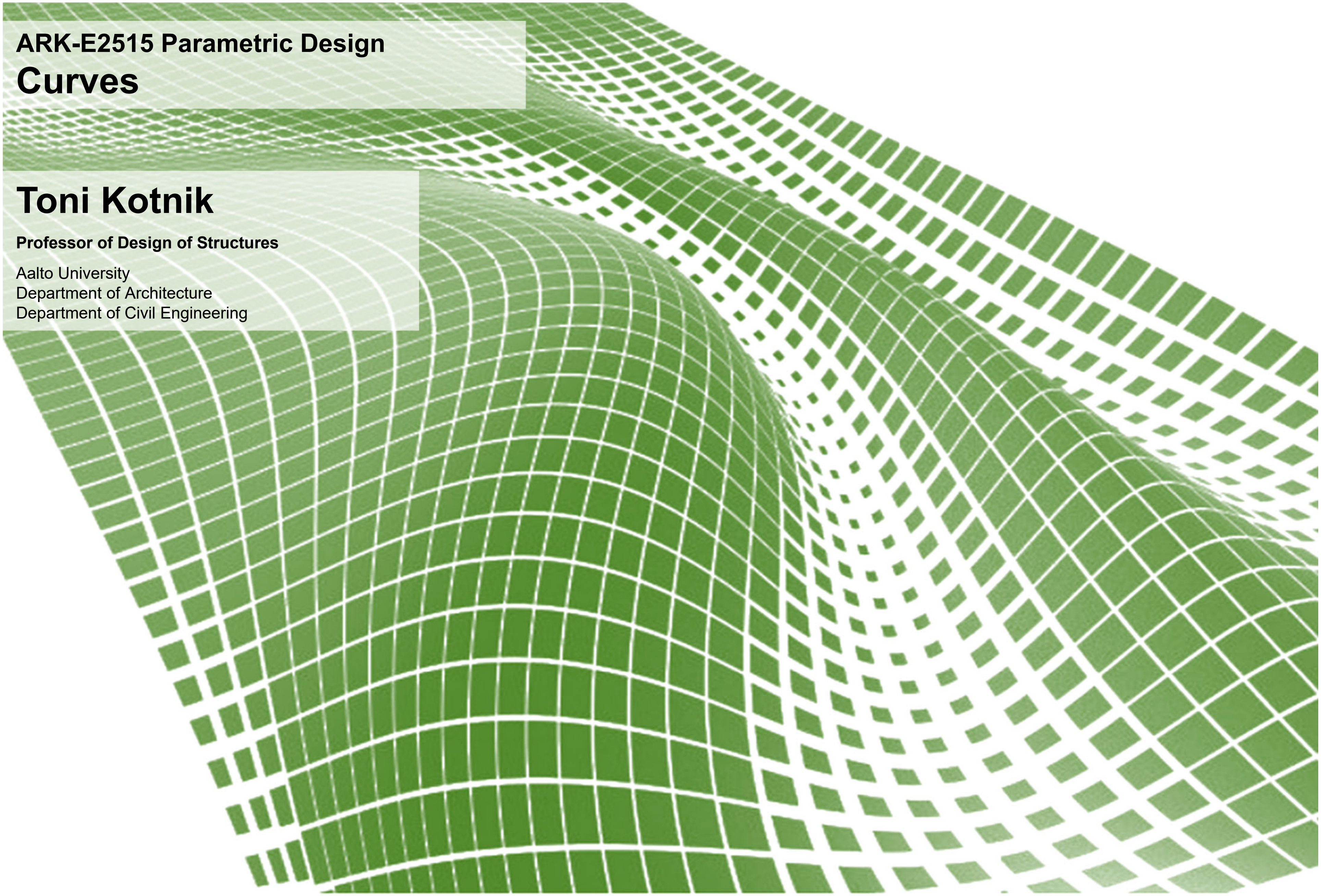


ARK-E2515 Parametric Design

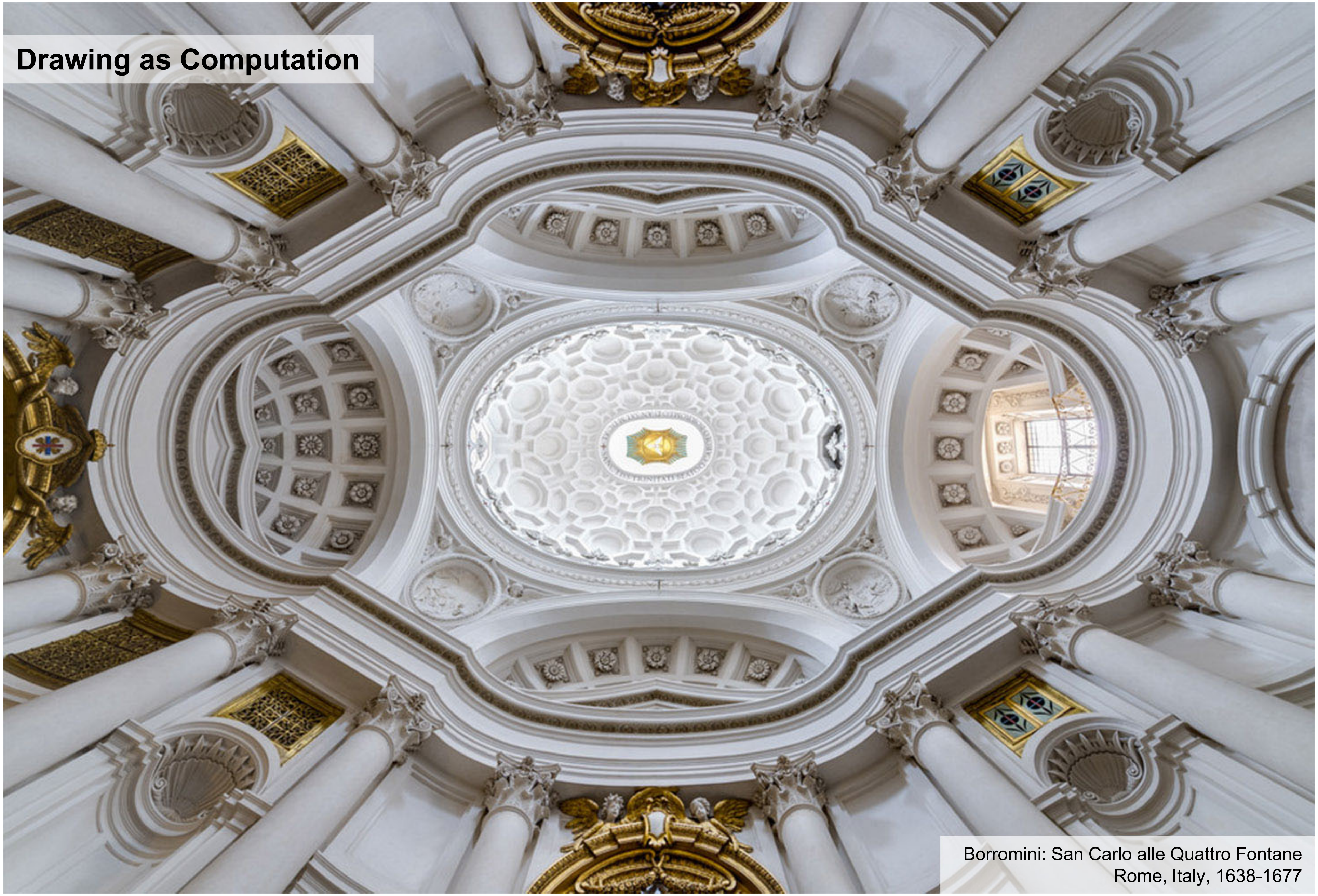
Curves

Toni Kotnik

Professor of Design of Structures

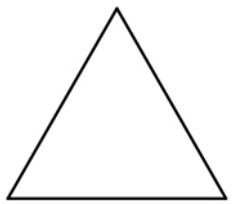
Aalto University
Department of Architecture
Department of Civil Engineering

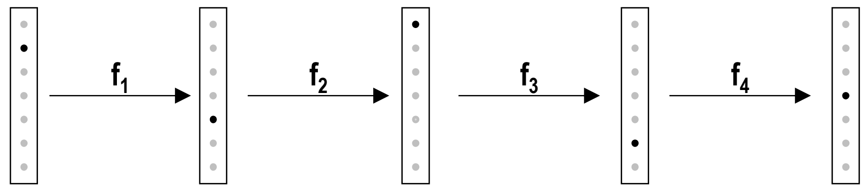
Drawing as Computation



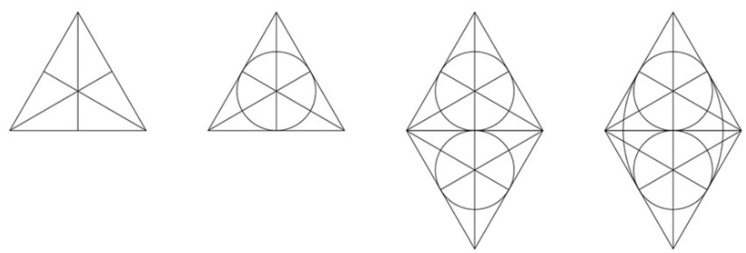
Borromini: San Carlo alle Quattro Fontane
Rome, Italy, 1638-1677

Drawing as Computation

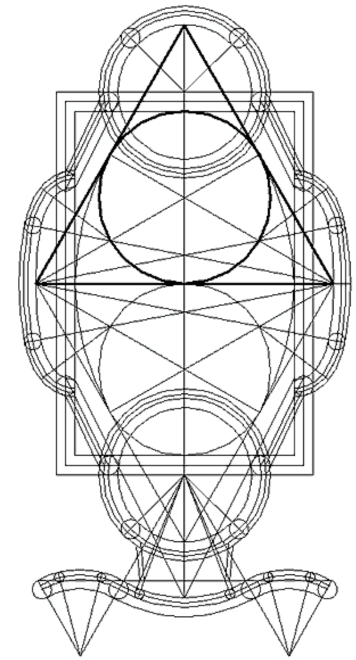




Associative Geometry
sequence of geometric operations
that built upon each other

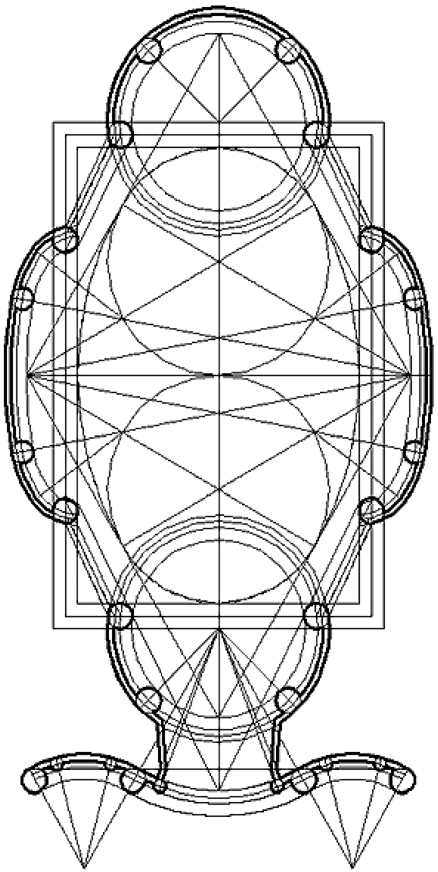


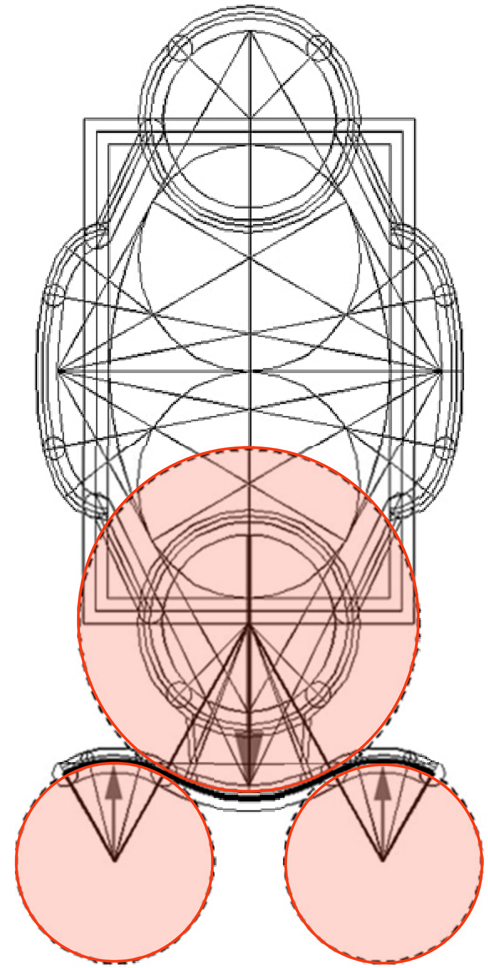
association (lat. *associare*: to unite, to ally)
uniting in a common purpose / work together for one goal



Borromini: San Carlo alle Quattro Fontane
Rome, Italy, 1638-1677

From Curve to Curvature





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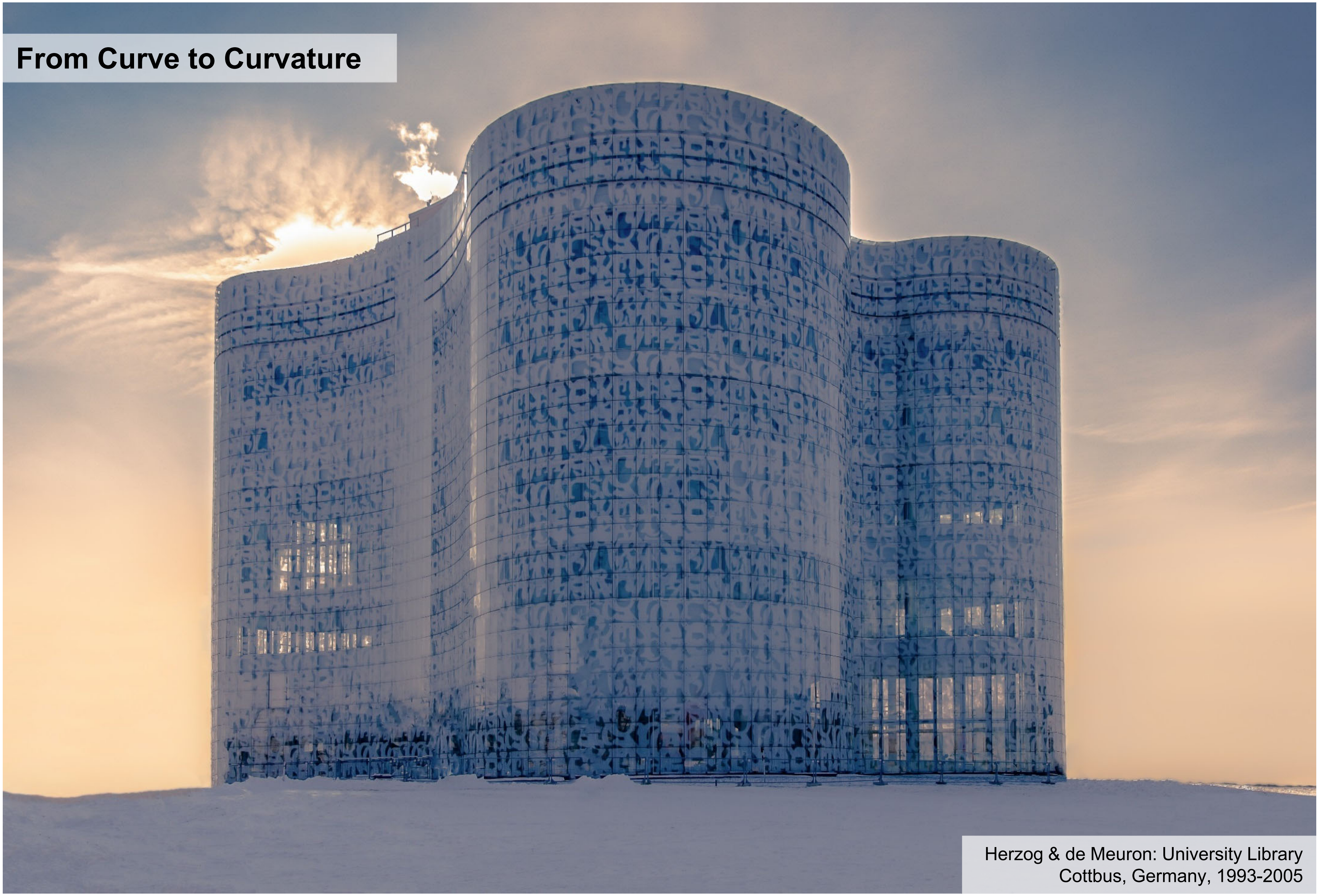
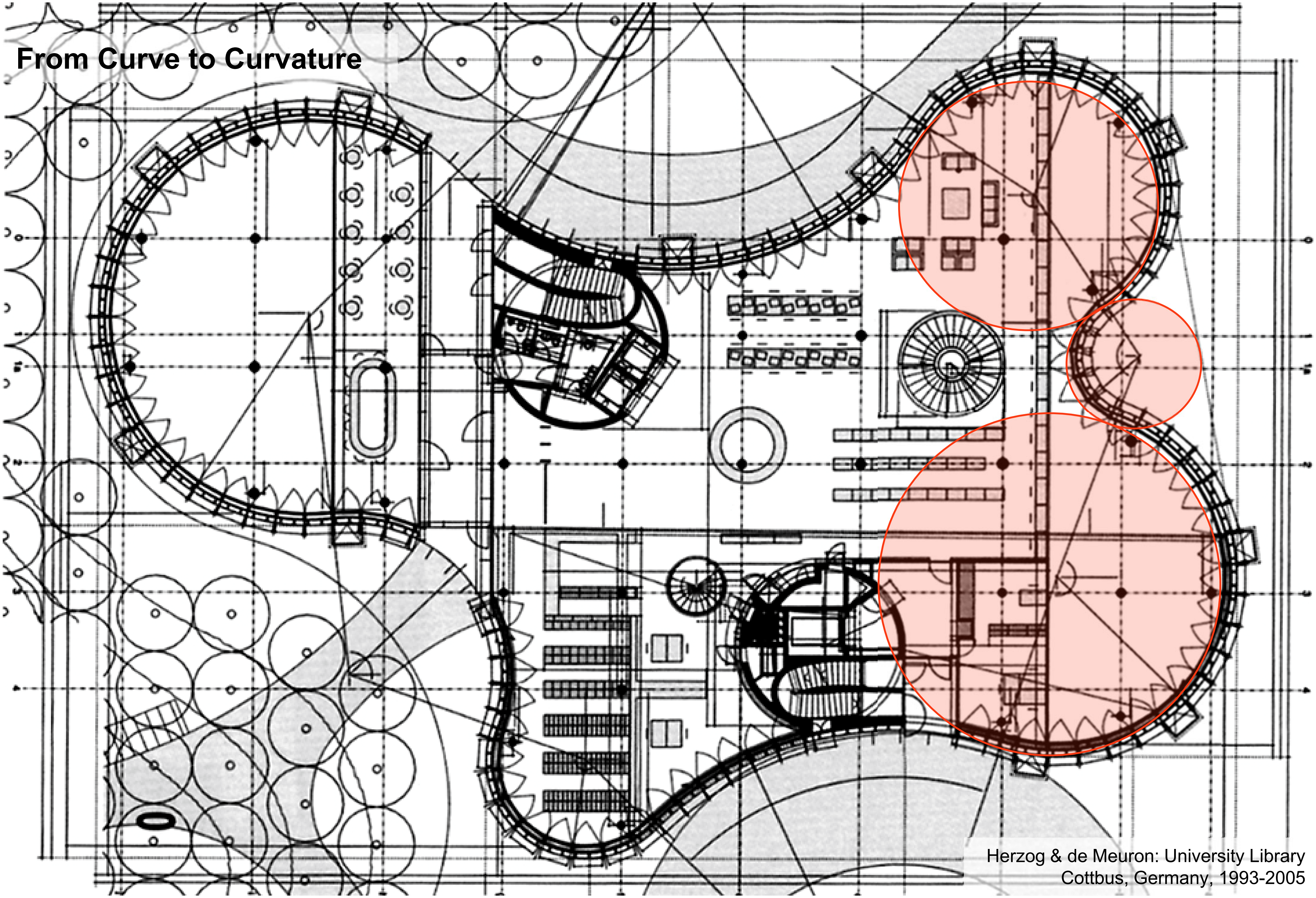


Herzog & de Meuron: University Library
Cottbus, Germany, 1993-2005

From Curve to Curvature



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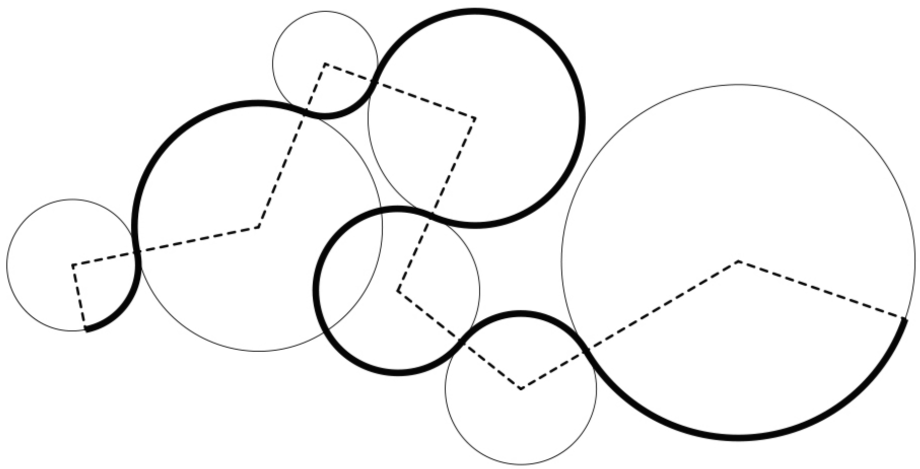
From Curve to Curvature

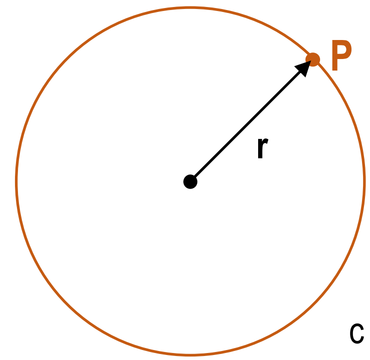


circles as well-known and easy-to-construct curvy curves with radius as measurement for curvature!

Curvature

for a circle the **curvature** ***κ*** is defined as the invers of the radius **r**

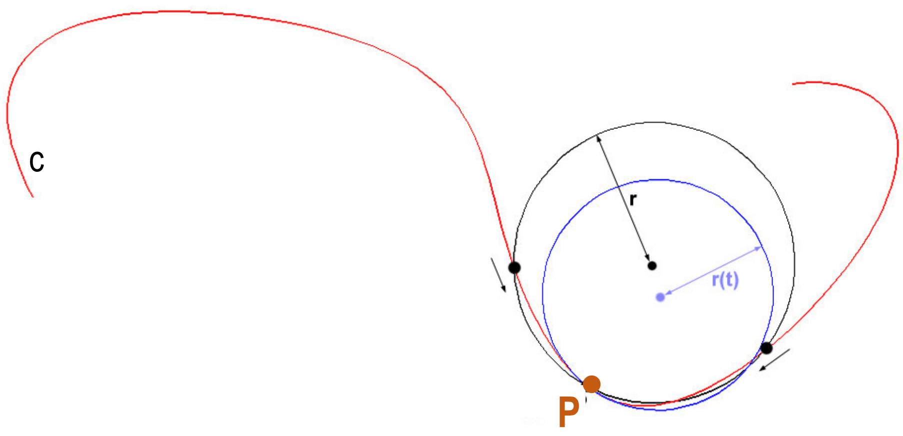
$$\kappa = 1/r$$



the curvature ***κ*** is a measure for the roundness of the circle

by means of the **limit circle r(t)** the local behaviour of a curve at point c(t) can be approximated

$$\kappa(t) = 1/r(t)$$



Curvature

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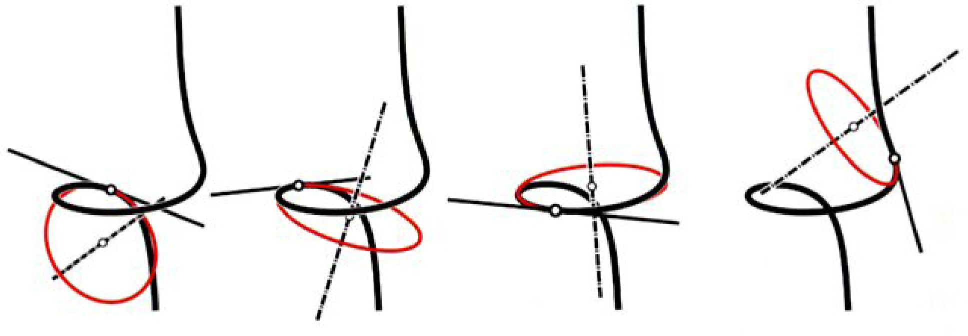

the concept of limit circle is also valid in 3d and enables the definition of a curvature for a curve

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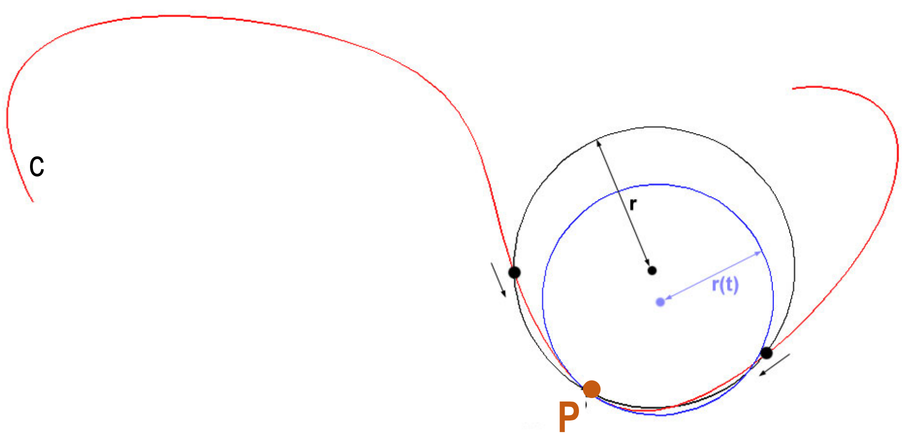
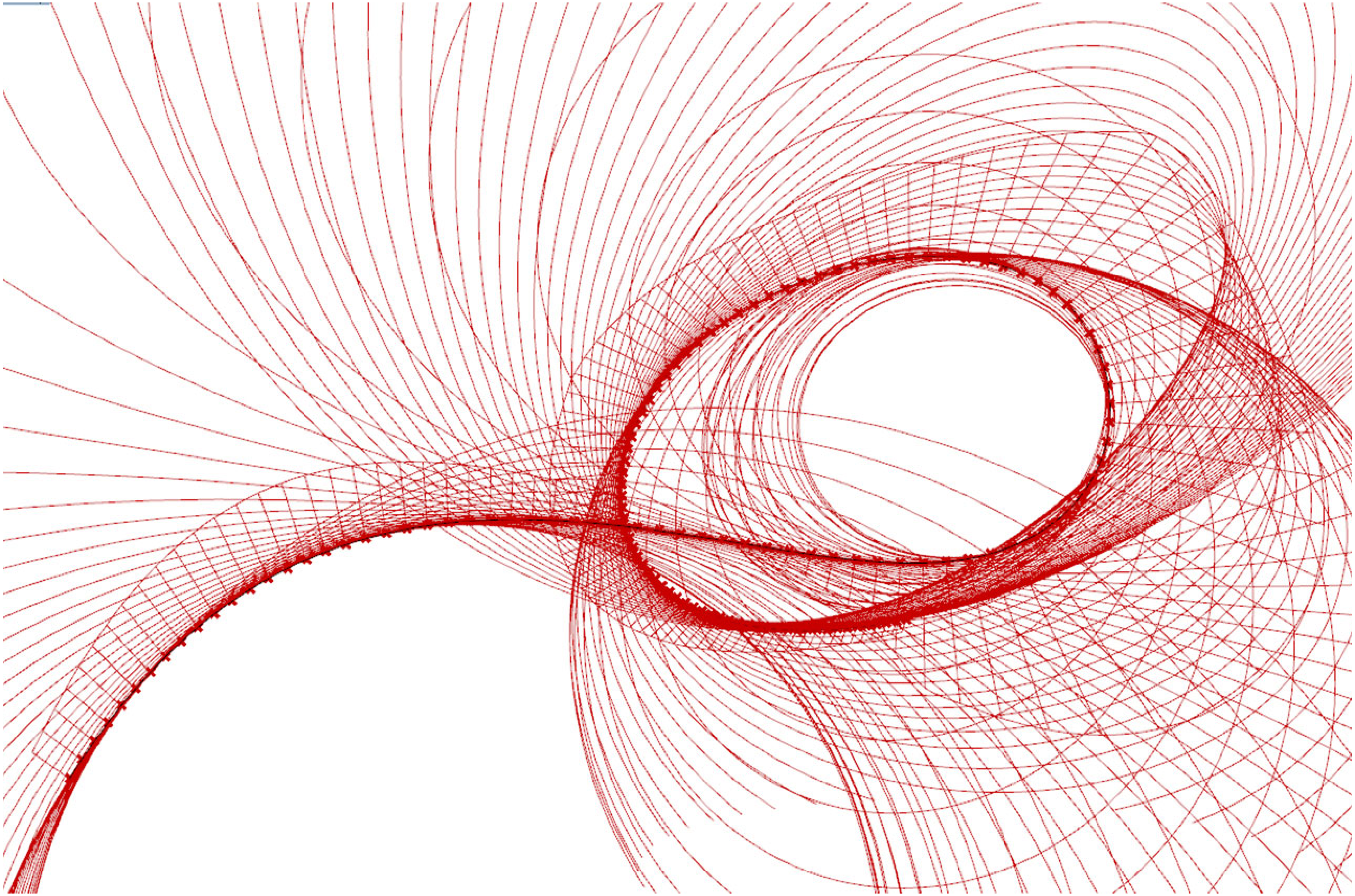
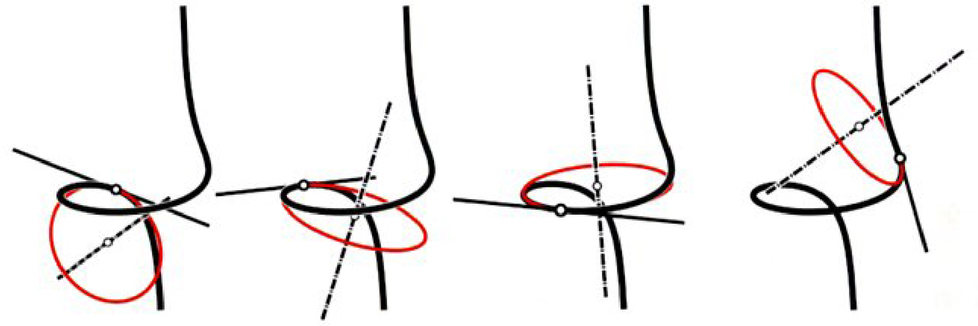
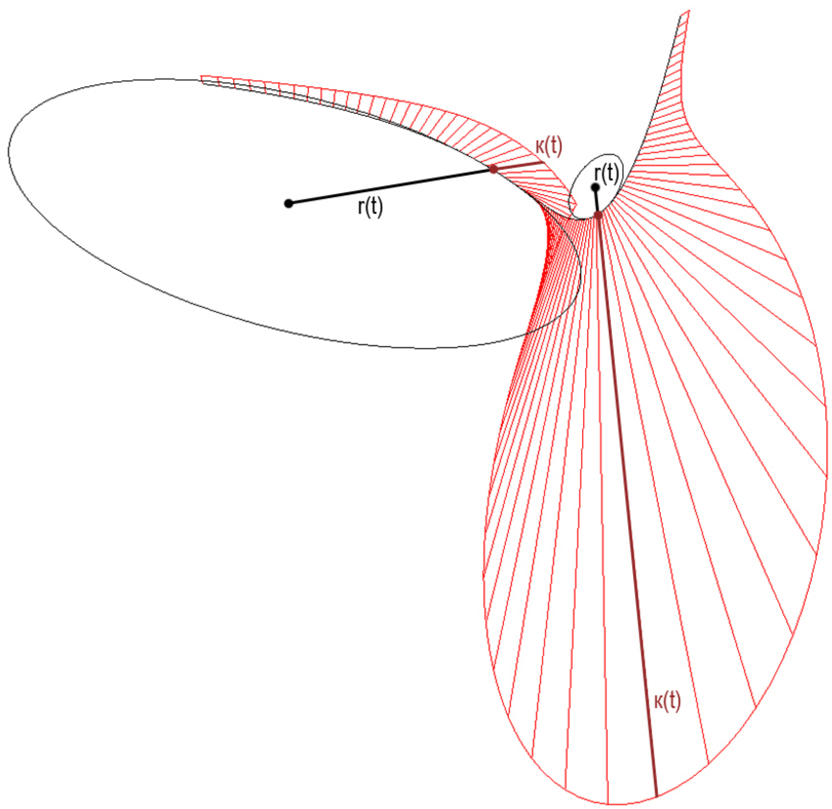




Curvature Graph

Check 4: construct a curvature graph for a curve c





the concept of limit circle is also valid in 3d and enables the definition of a curvature for a curve

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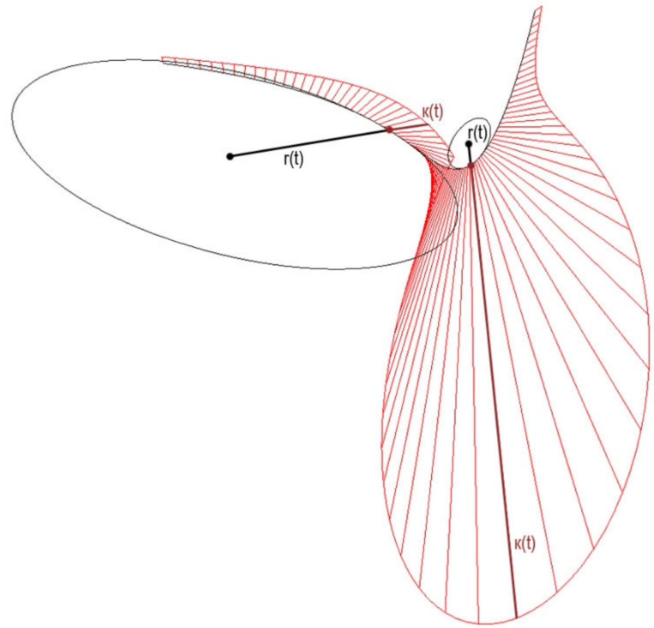
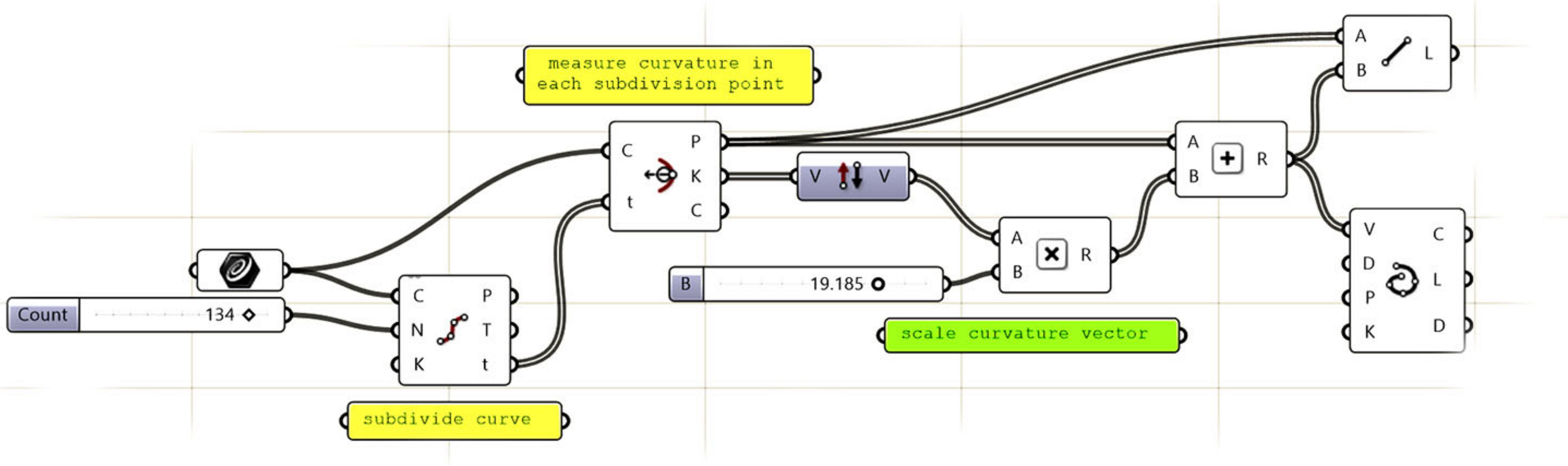
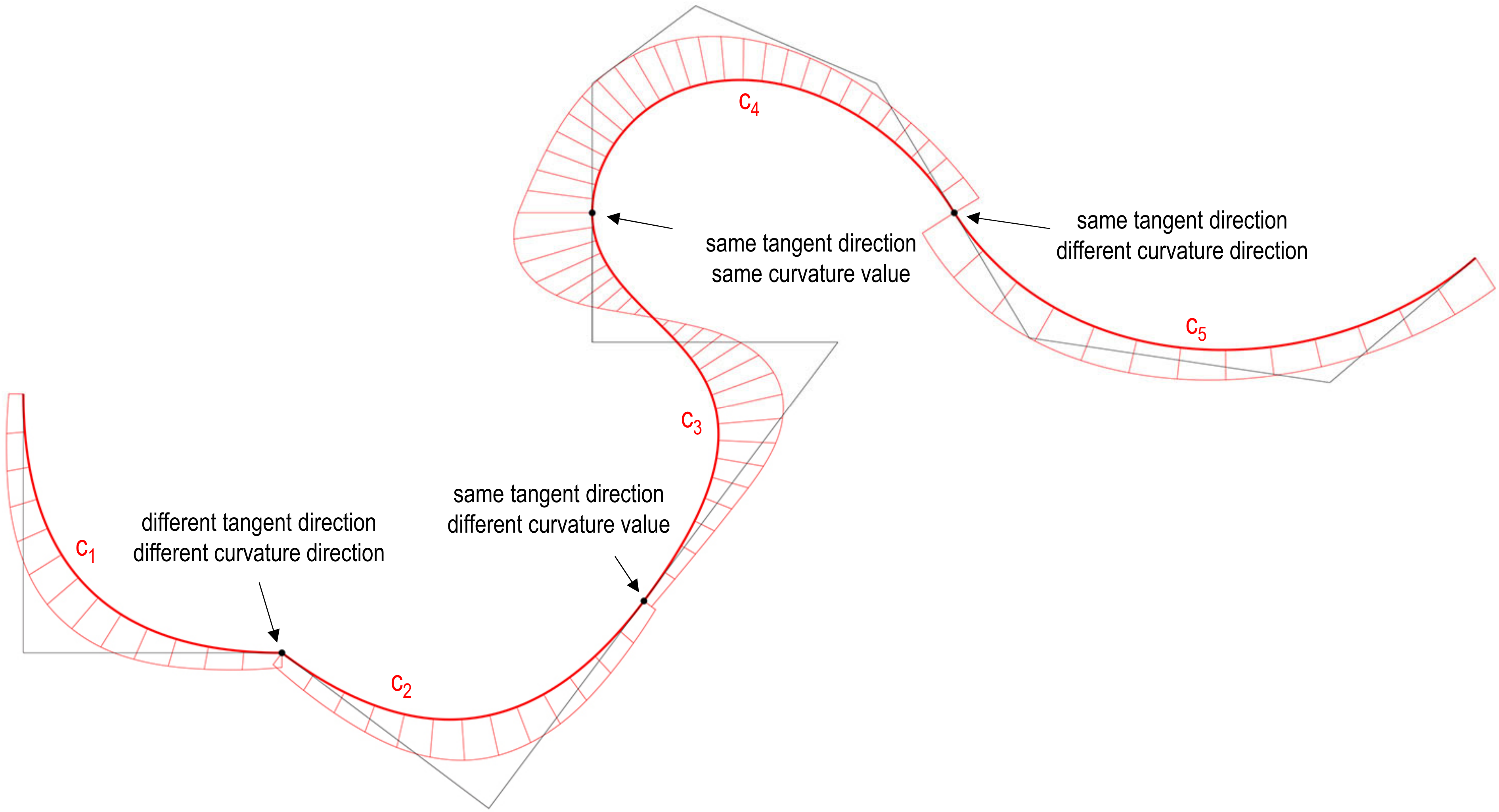
Curvature Graph





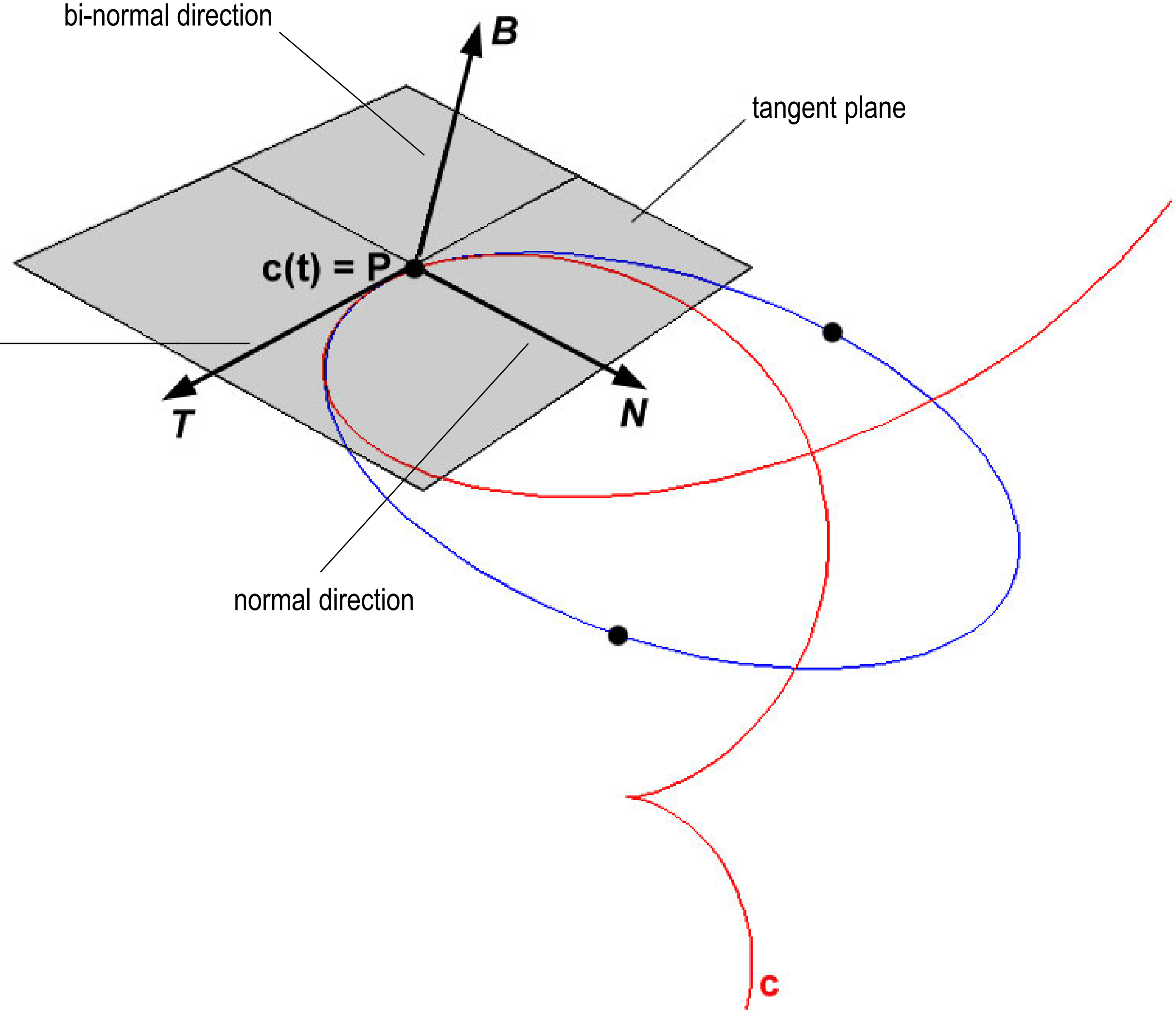
Curvature Graph
joining curves

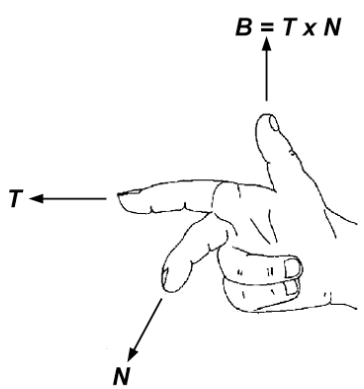
different degrees of smoothness of joining two curves are possible
dependent on the continuity of the curvature graph



Frenet-Frame
curve frame

based on the limit circle a point P = c(t) a local coordinate-system at P can be defined

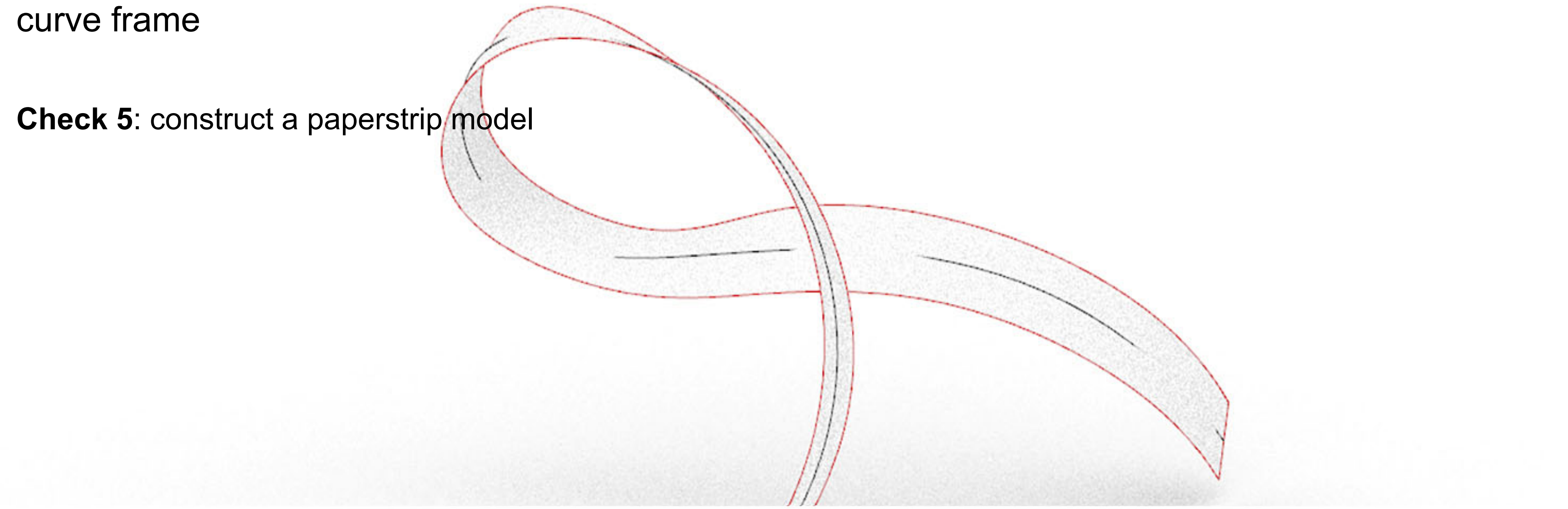


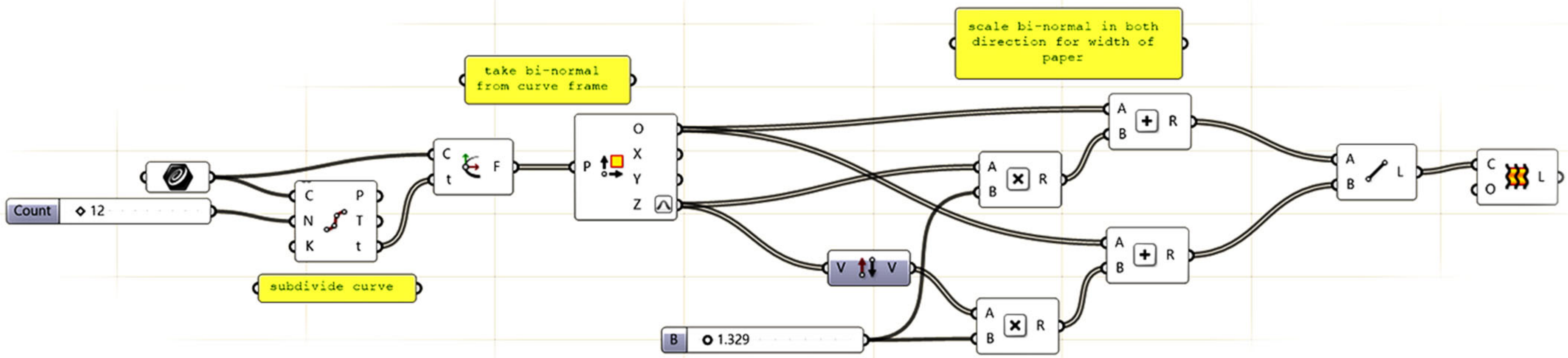


orientation by right-hand rule

Frenet-Frame
curve frame

Check 5: construct a paperstrip model

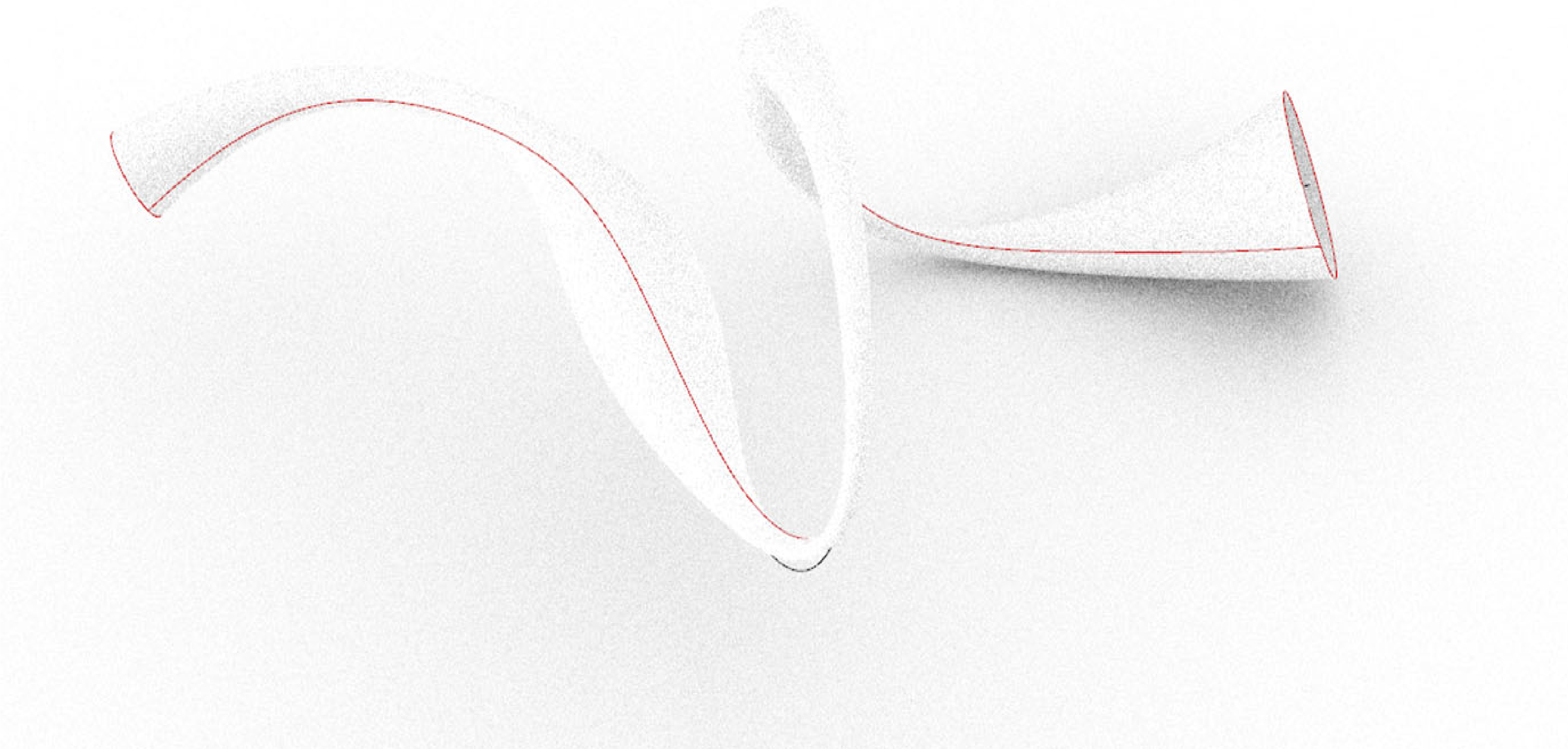




Aalto University
Design of Structures

Toni Kotnik, Professor of Design of Structures

Exercise 2: construct a pipe with varying diameter defined by the inverse curvature of the guiding curve

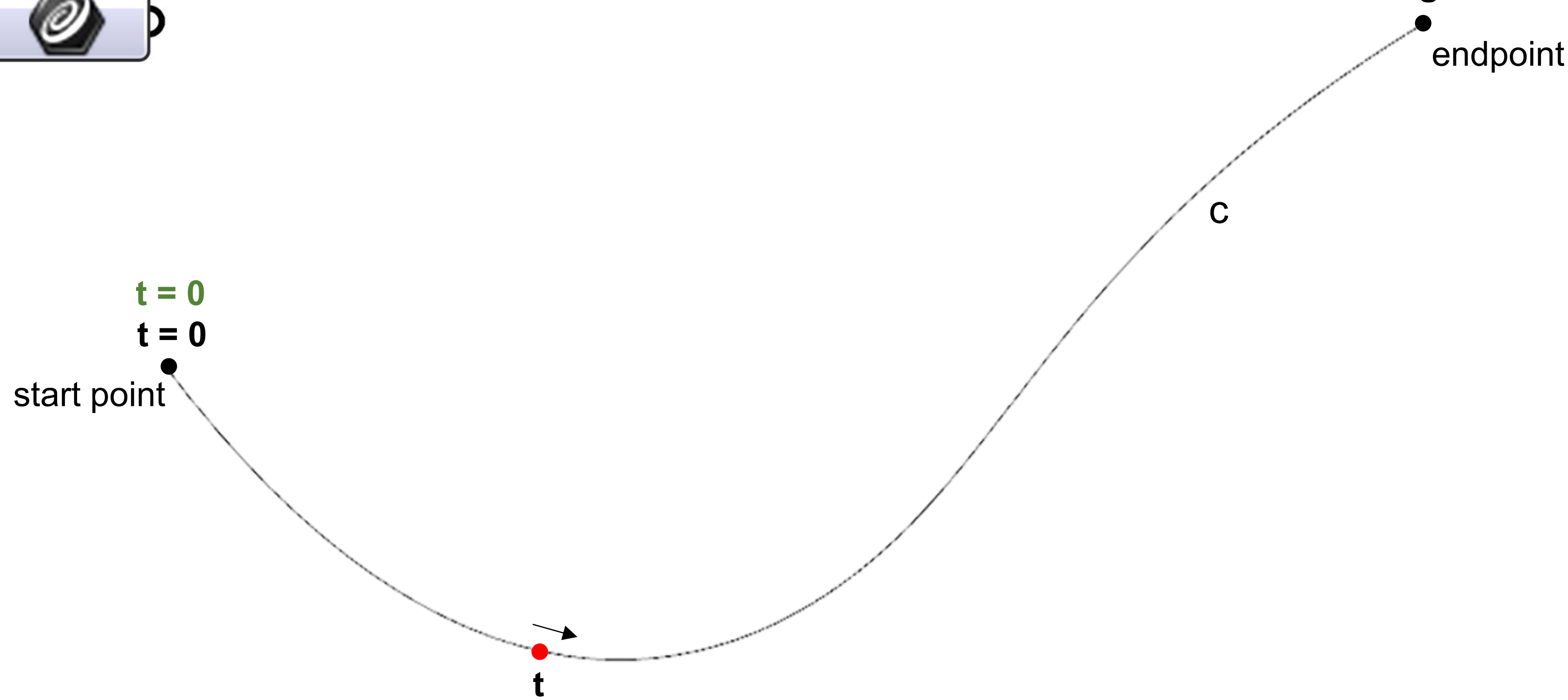


Parametrization

Re-Parametrization

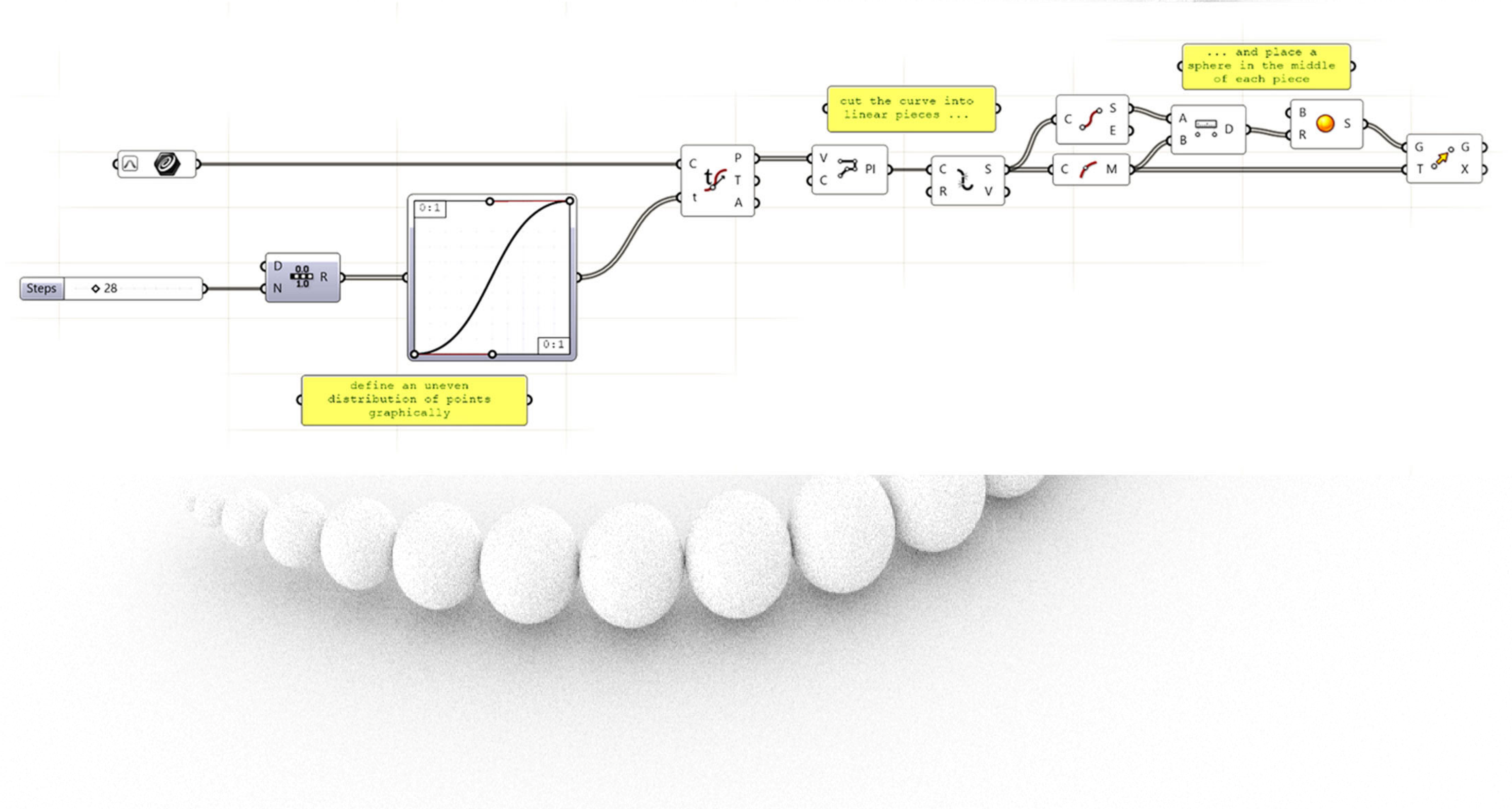
absolute length

relative length

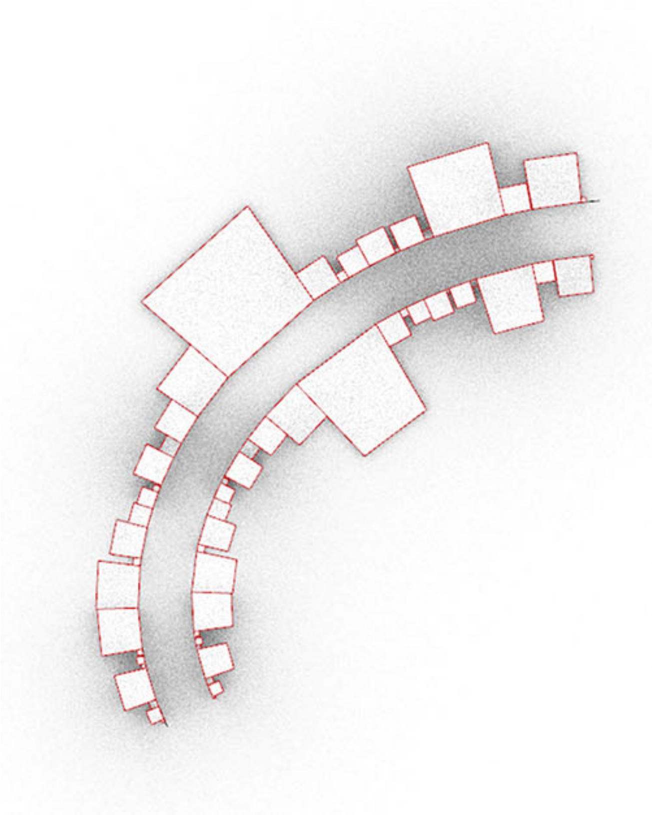


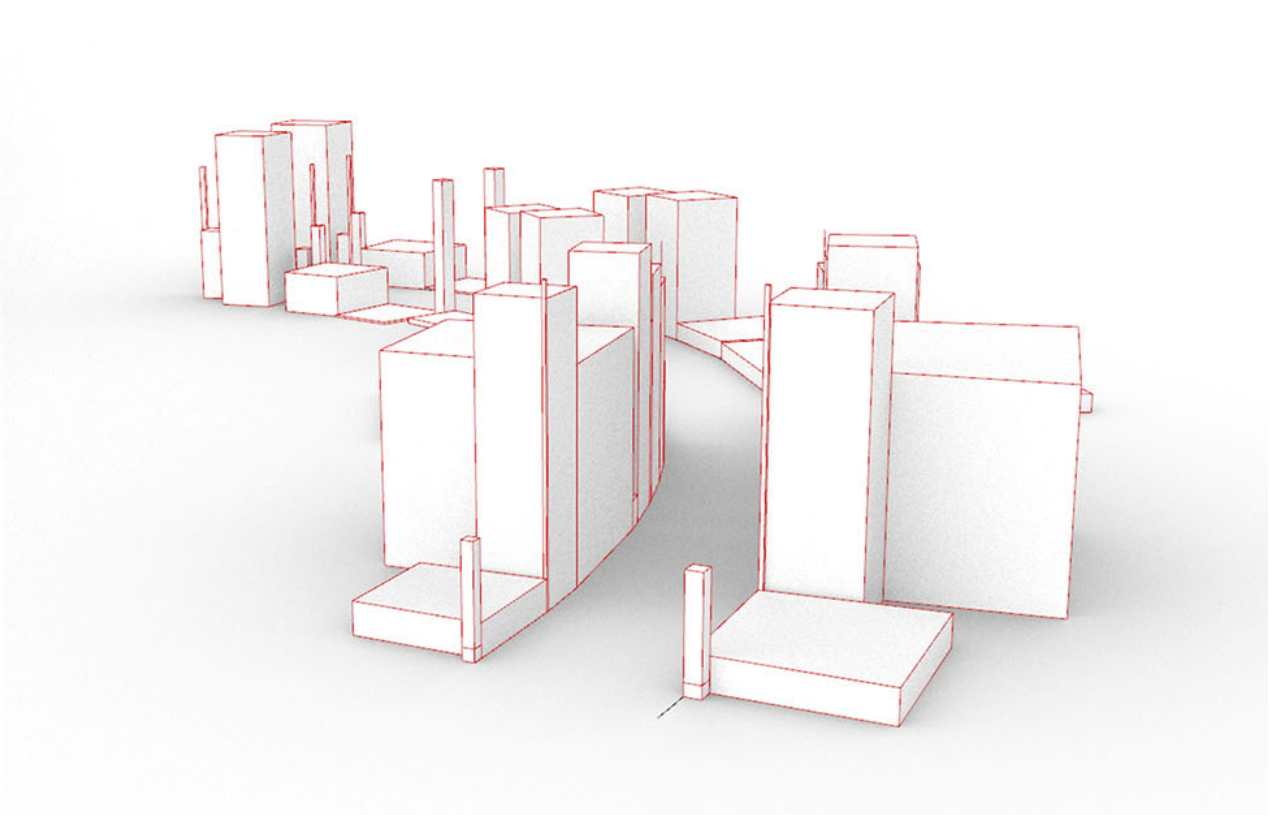
curve as trajectory of a point moving
from start point to endpoint

Check 6: Create a necklace with one big pearl in the middle, and gradually smaller size pearls towards the ends.



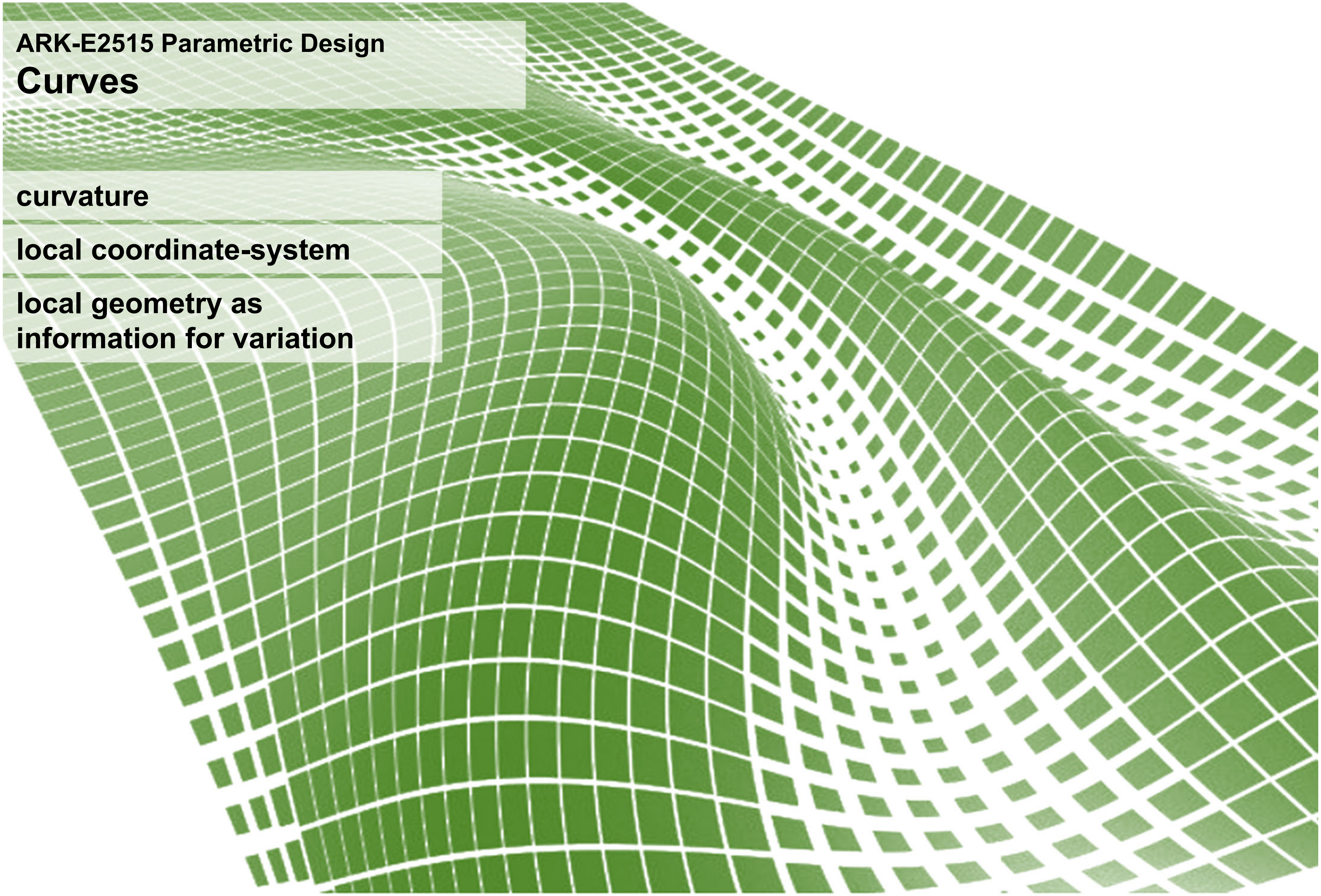
Exercise 3: for a planar curve construct a streetscape with a randamozed almost-squared footprint and randomized height.





ARK-E2515 Parametric Design

Curves



curvature

local coordinate-system

local geometry as information for variation