

# ARK-E2515 Parametric Design Surfaces

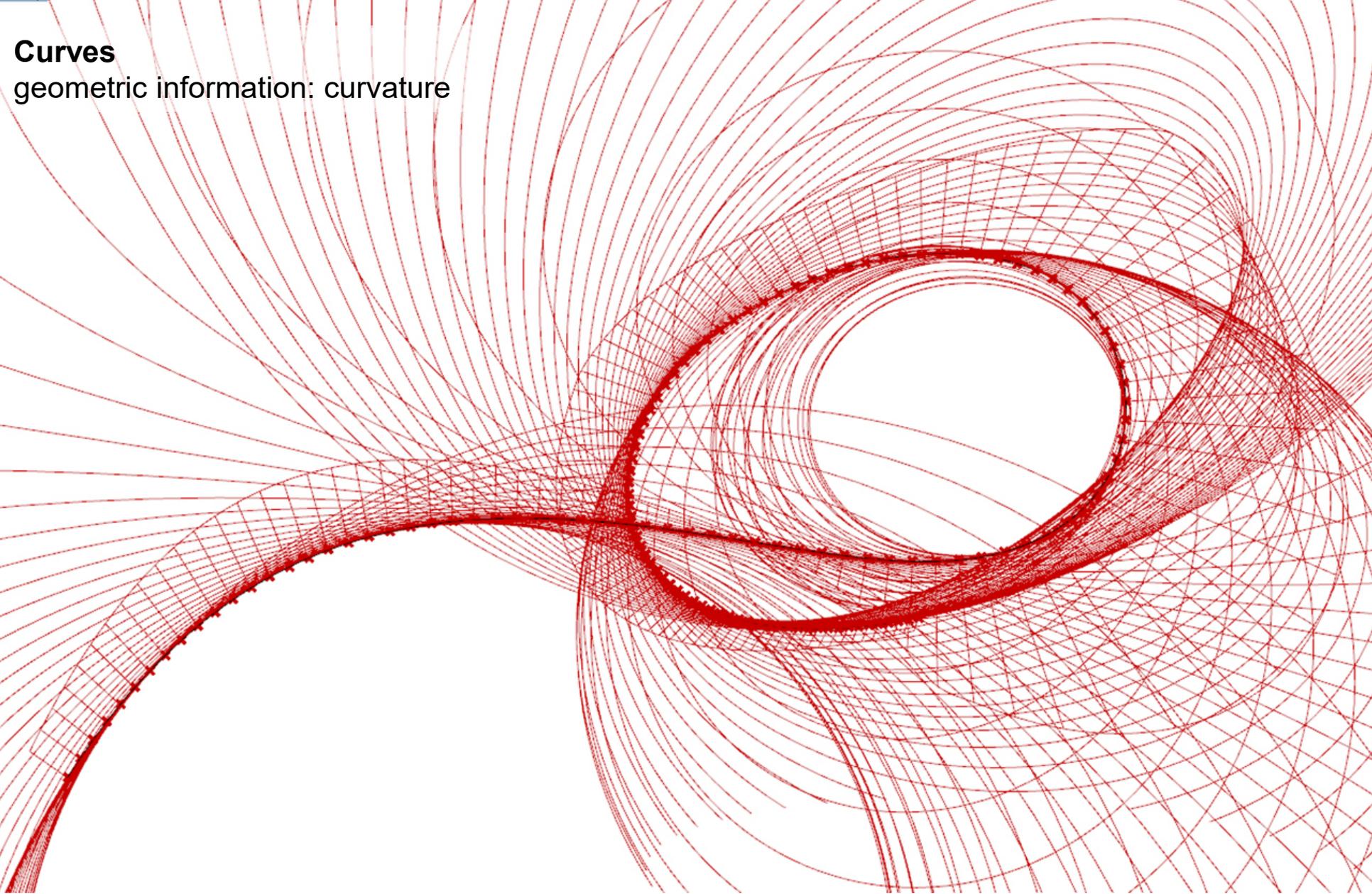
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Professor of Design of Structures

Aalto University  
Department of Architecture  
Department of Civil Engineering

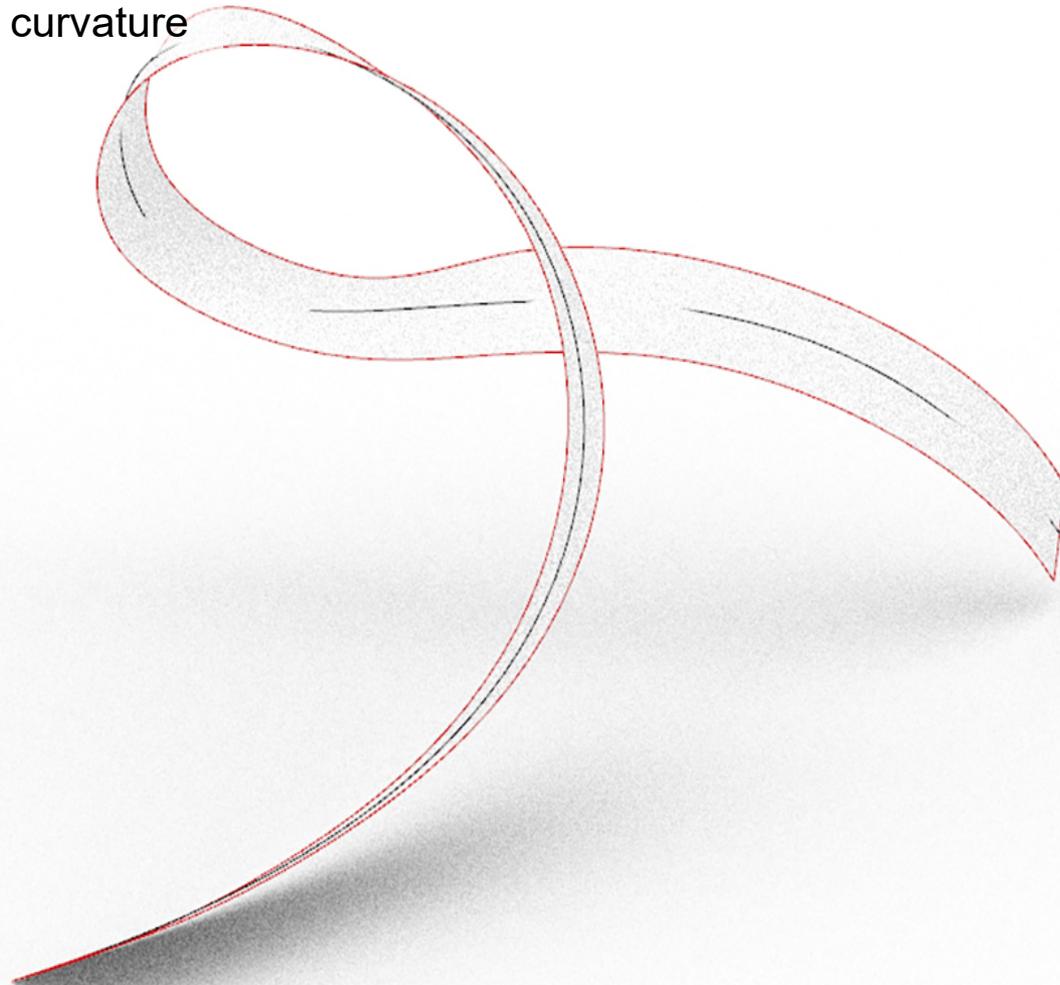
# Curves

geometric information: curvature



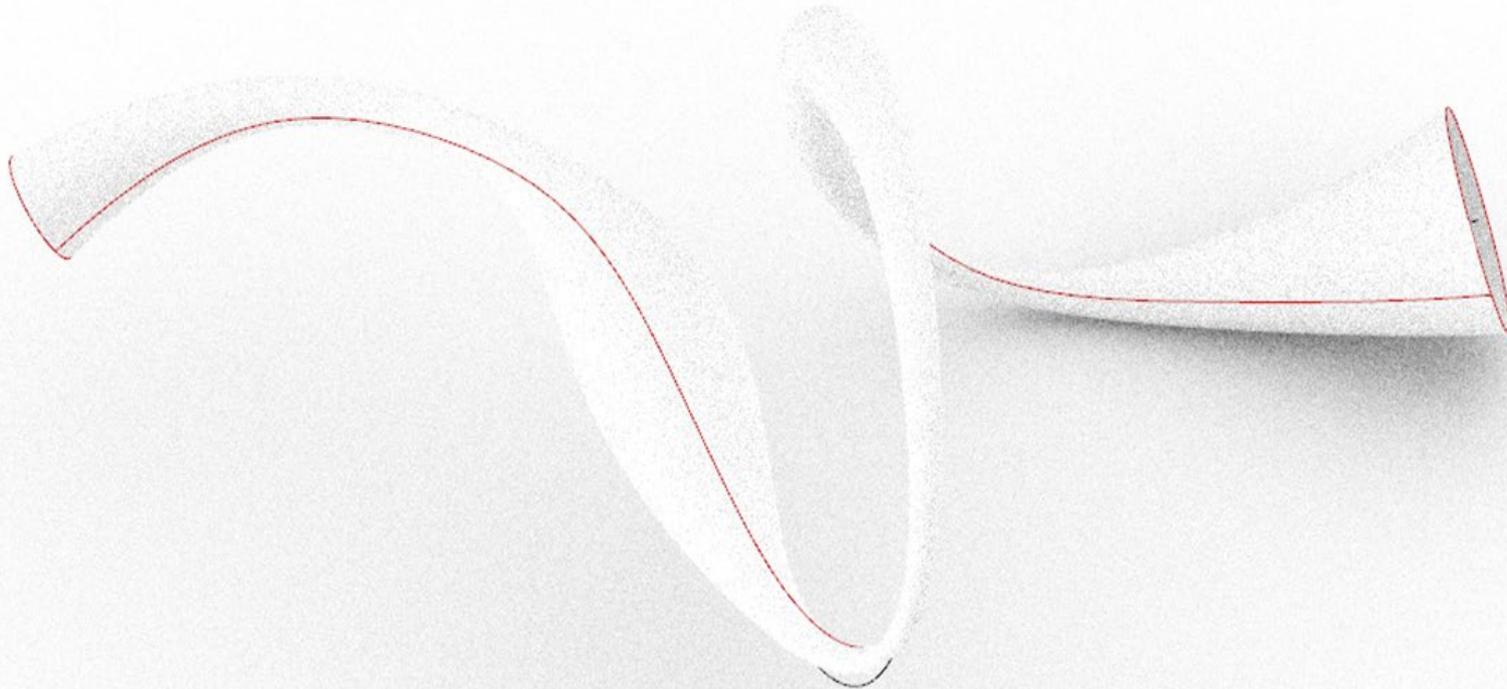
# From Curves to Surfaces

geometric information: curvature



# From Curves to Surfaces

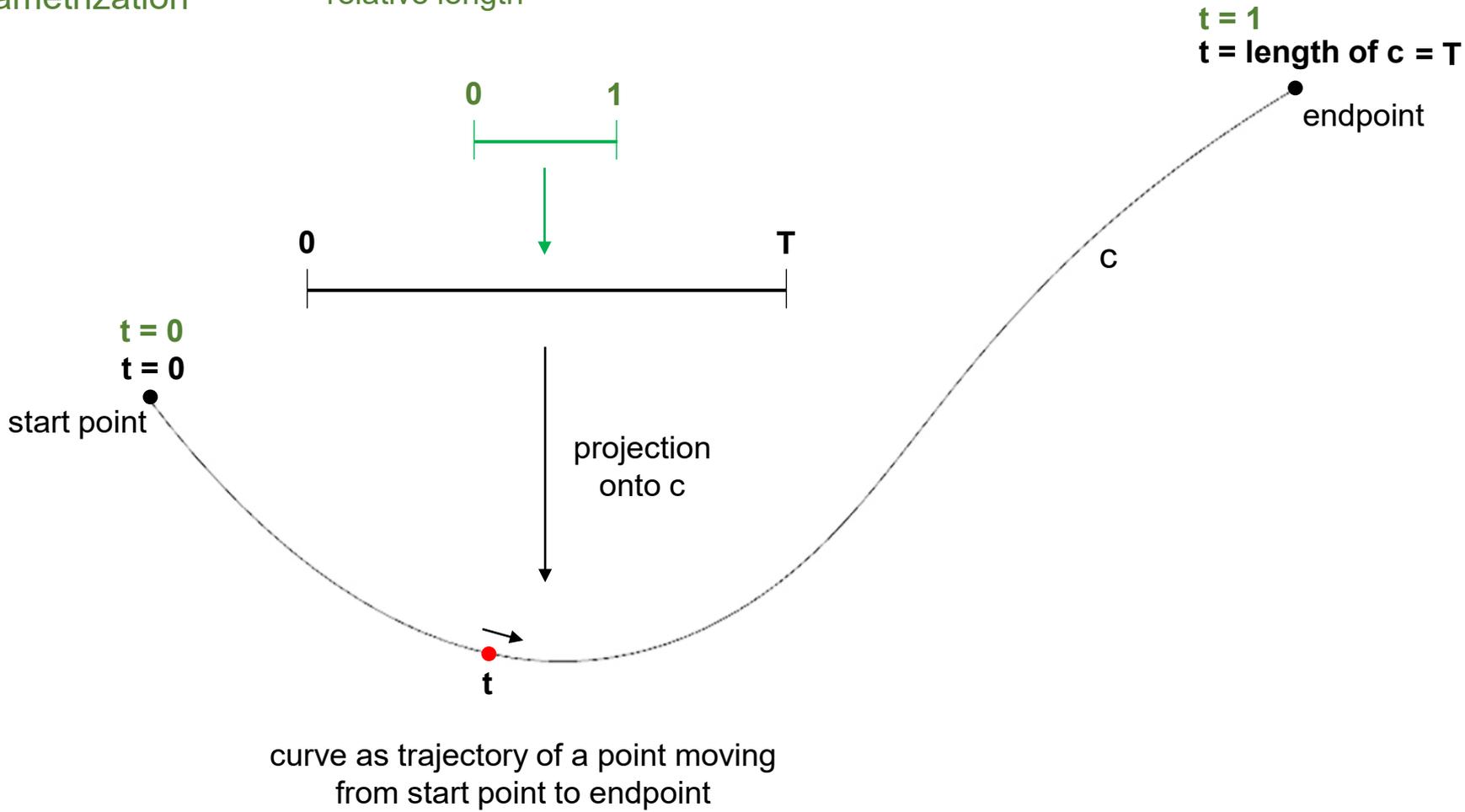
geometric information: curvature



# Parametrization

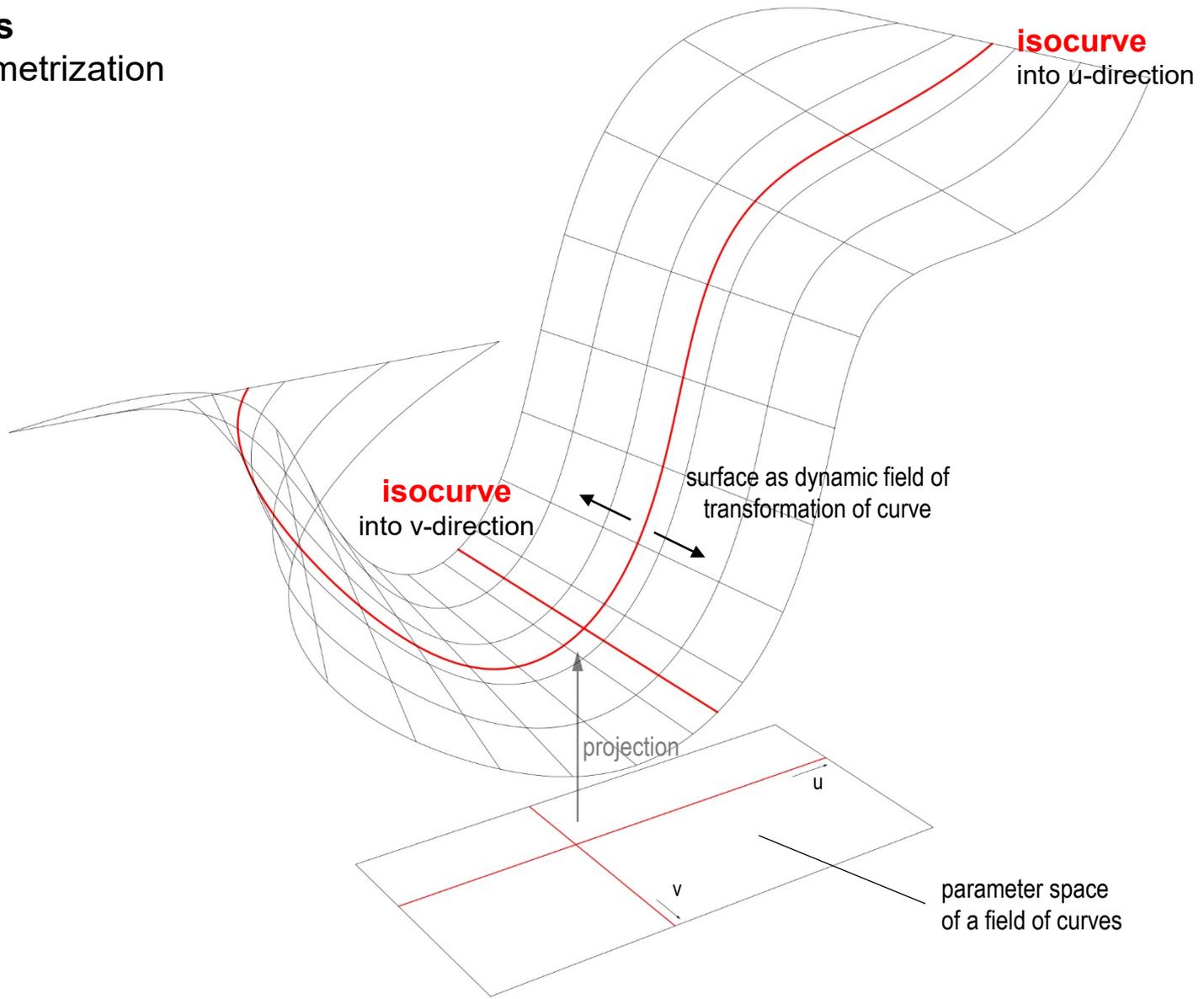
## Re-Parametrization

absolute length  
relative length



# Surfaces

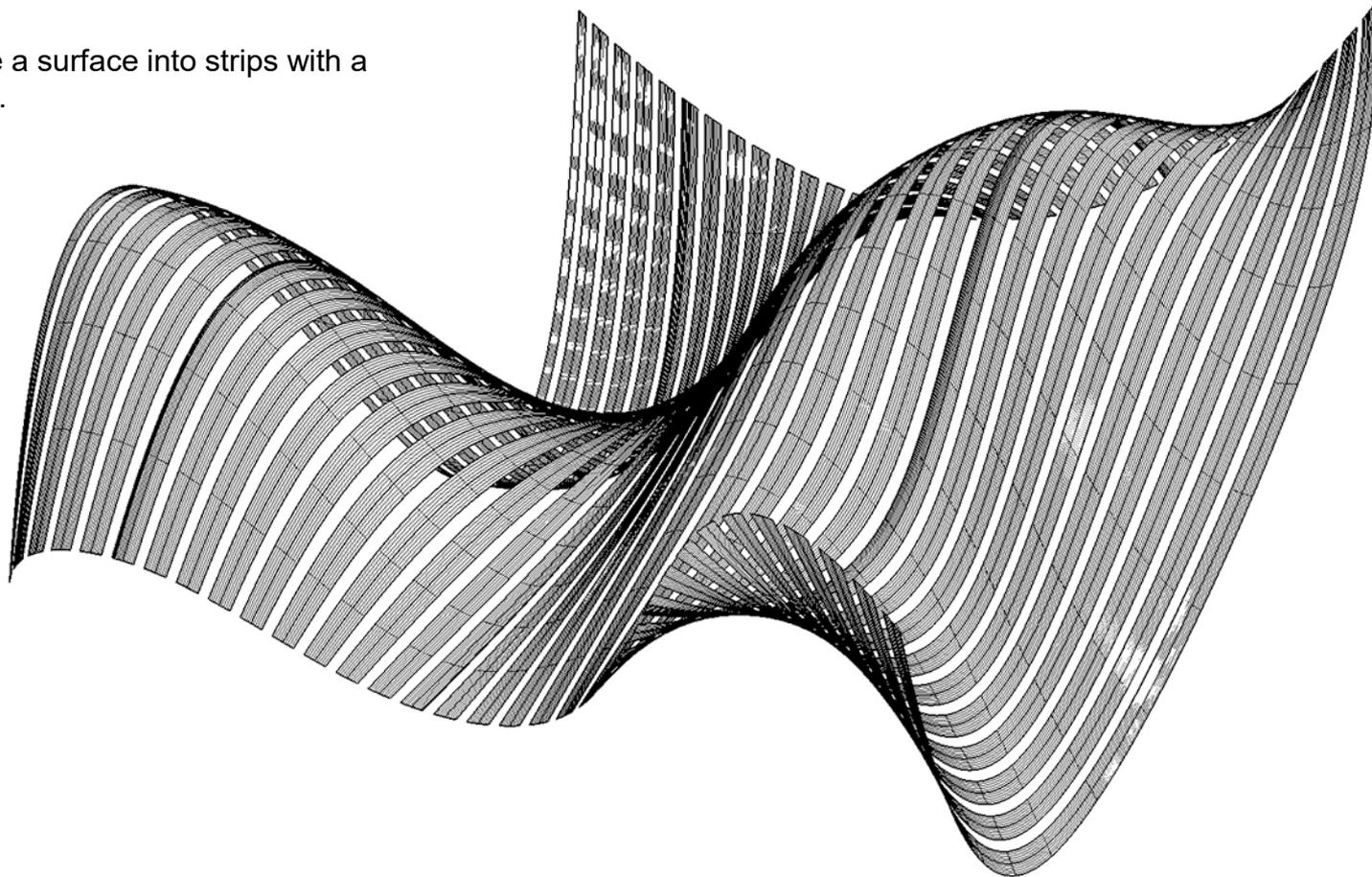
## uv-Parametrization



# Surfaces

## uv-Parametrization

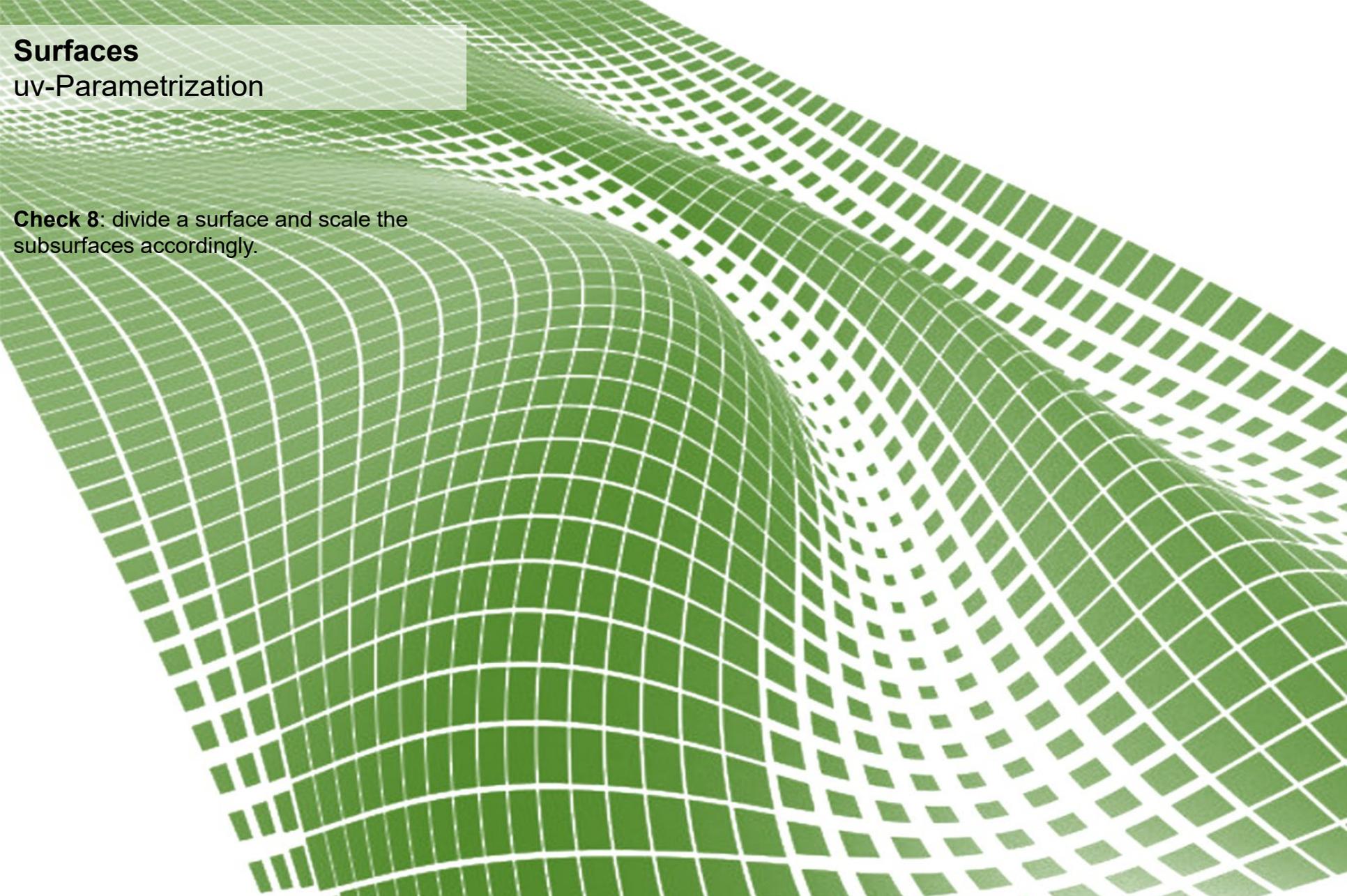
**Check 7:** divide a surface into strips with a gap in-between.

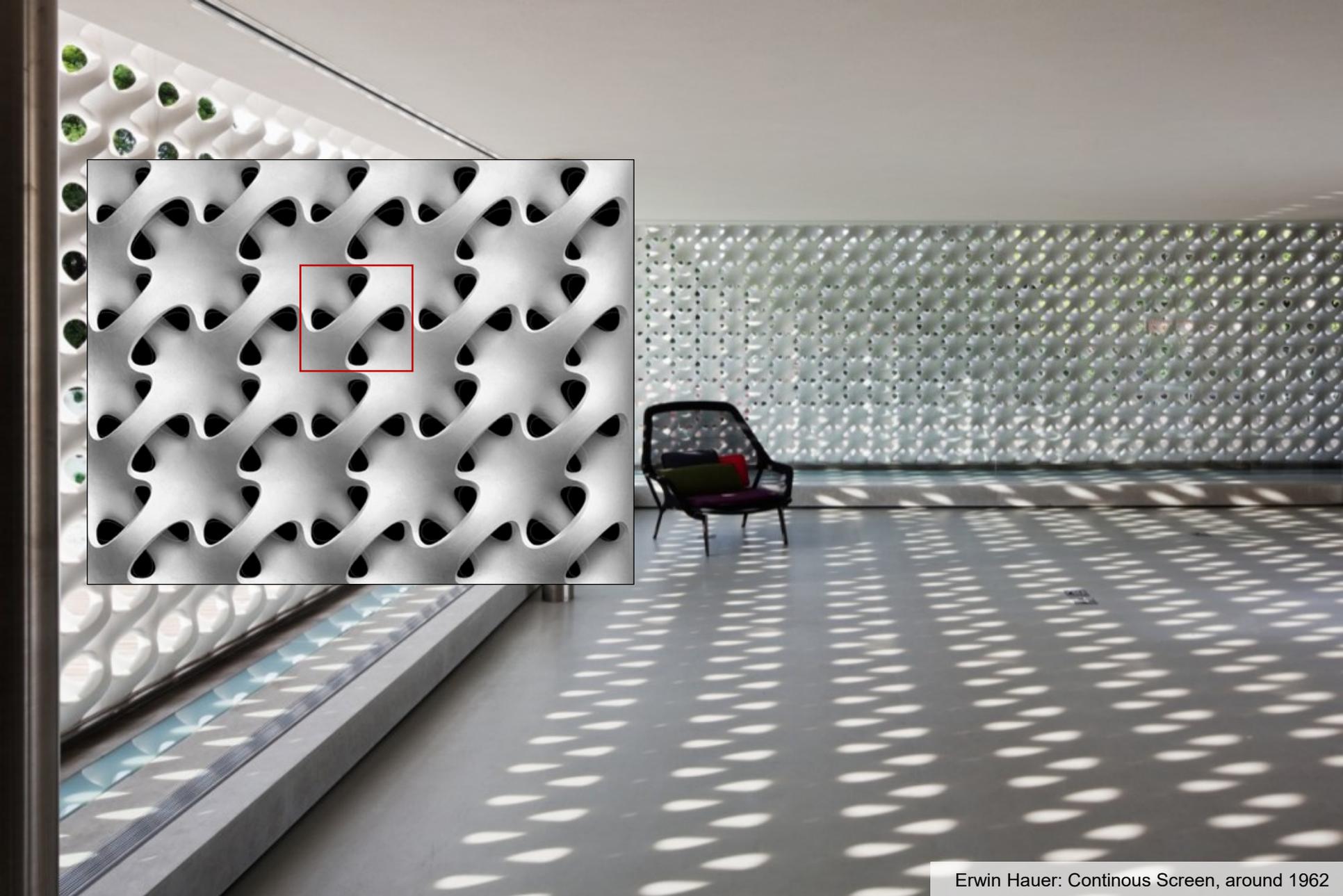


# Surfaces

## uv-Parametrization

**Check 8:** divide a surface and scale the subsurfaces accordingly.

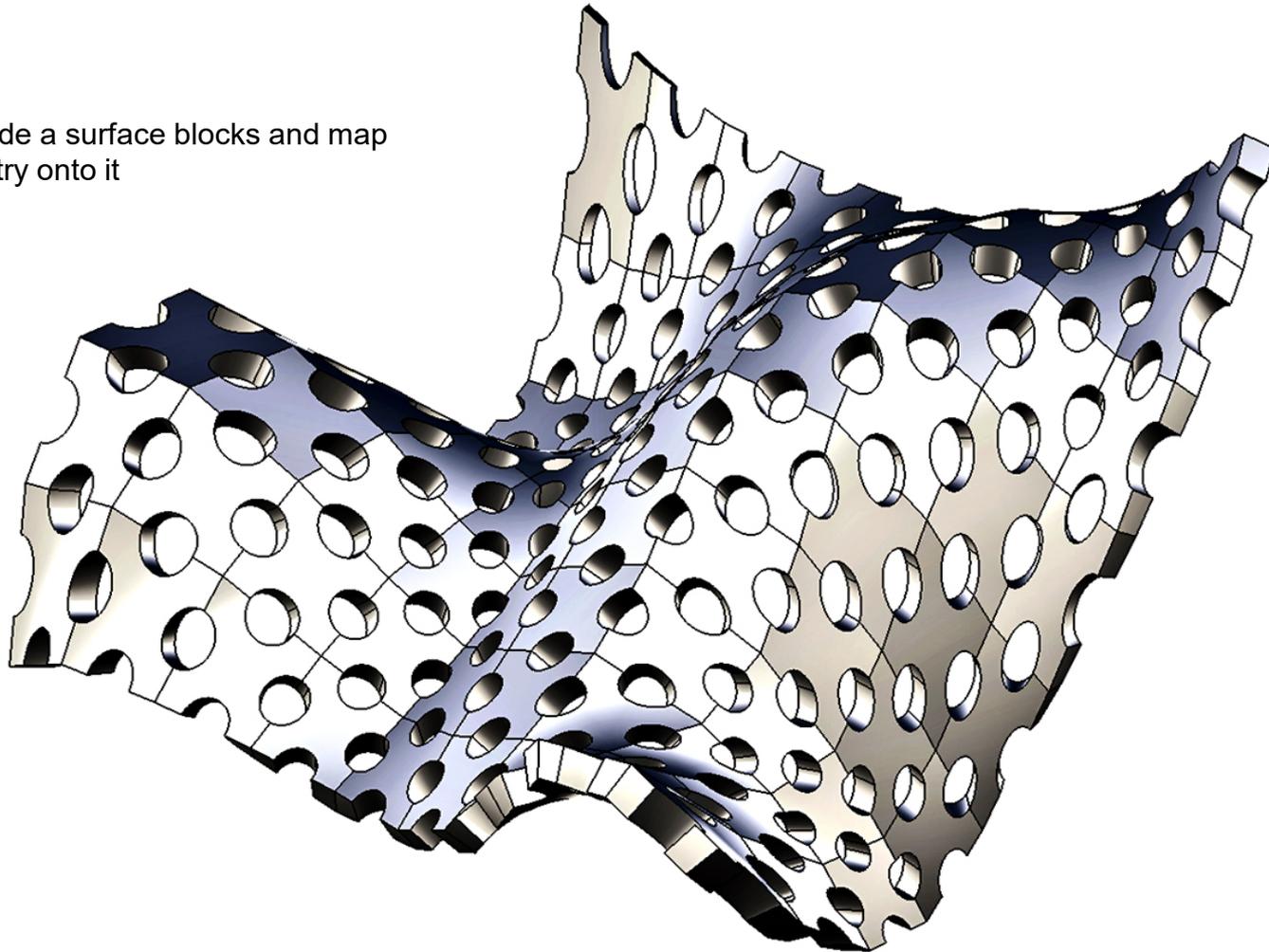




Erwin Hauer: Continuous Screen, around 1962

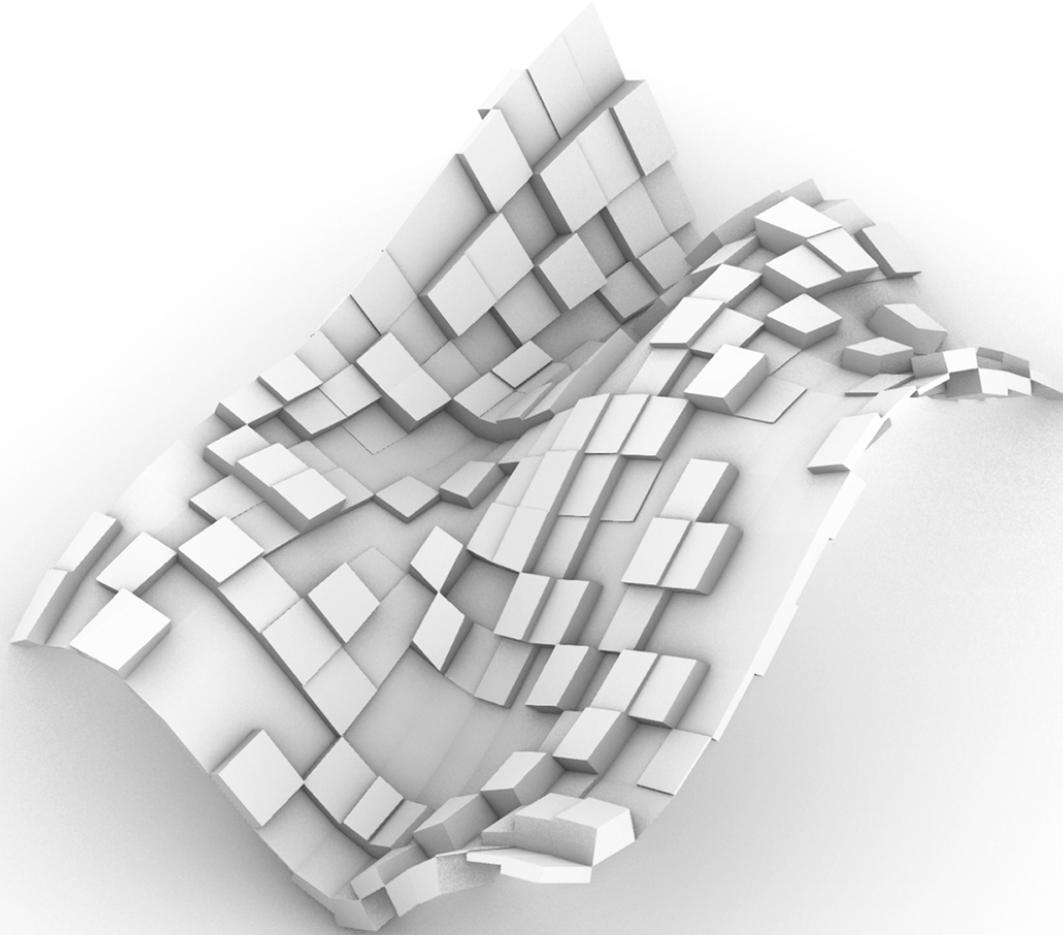
# Component Mapping

**Check 9:** divide a surface blocks and map some geometry onto it



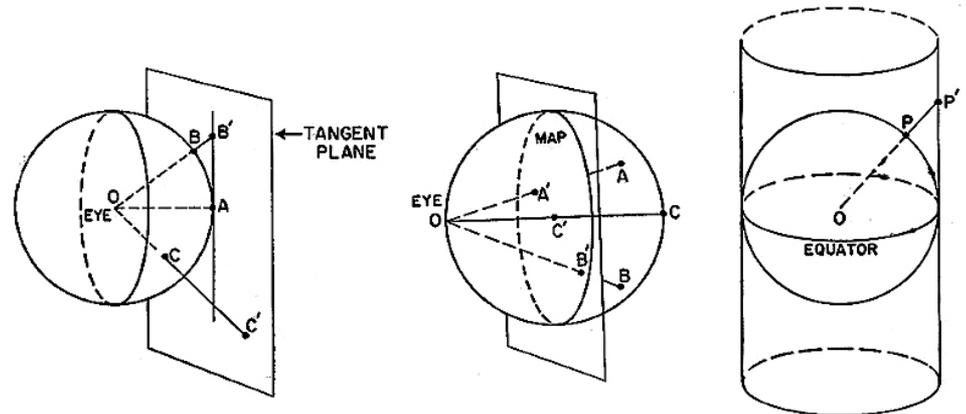
# Component Mapping

**Exercise 4:** divide a surface into blocks of arbitrary height

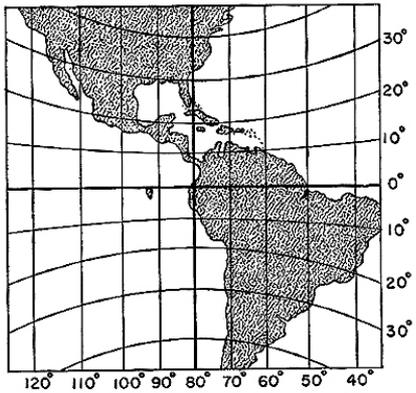


# Mapping

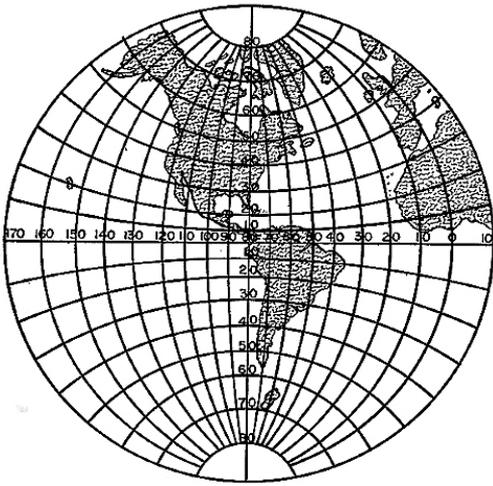
Mathematically, the problem of making a map is related to projecting figures from a sphere into the flat. The major difficulty arises out of the fact that a sphere cannot be laid out flat without distorting geometric properties like distance, angle, or area. Because of this it is impossible to represent exactly the geometric properties that exist on a sphere.



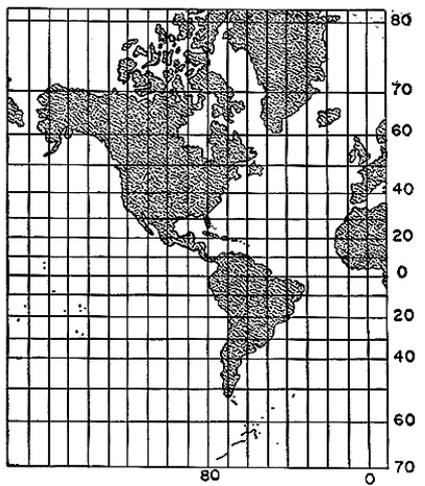
gnomonic projection



stereographic projection



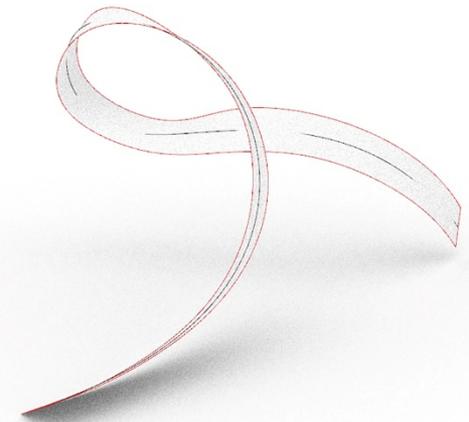
perspective cylindrical projection



# Mapping architecture as cartographic problem



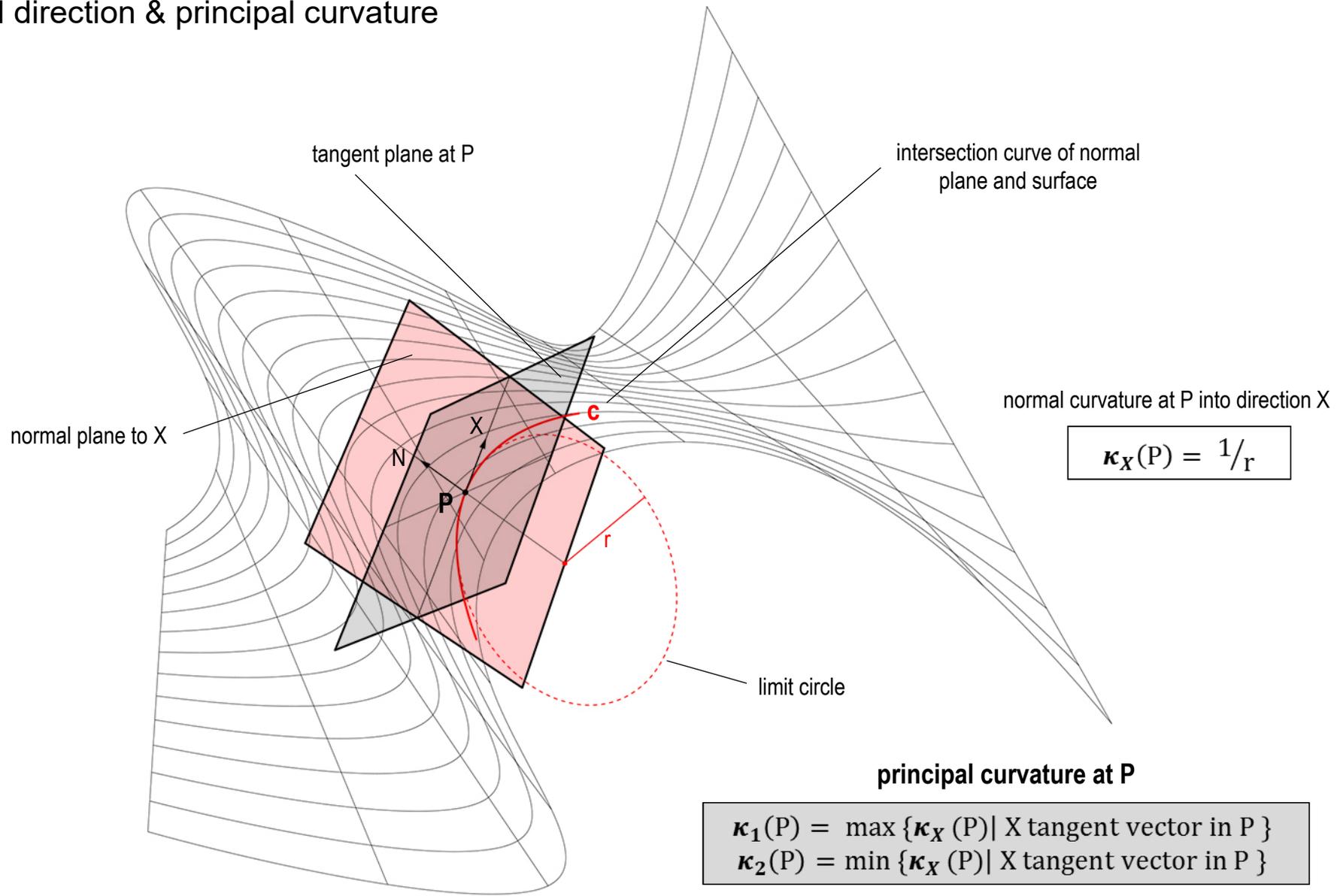
The cartographic problem of distortion is immanent to the construction of curvilinear architecture. Most building material is planar. This means only surfaces that can be mapped without any distortion – so called developable surfaces - can be fabricated and assembled relatively easy.



Frank Gehry: Walt Disney Concert Hall  
Los Angeles, USA, 1999-2003

# Surfaces

normal direction & principal curvature



$$\kappa_X(P) = 1/r$$

**principal curvature at P**

$$\kappa_1(P) = \max \{ \kappa_X(P) \mid X \text{ tangent vector in } P \}$$

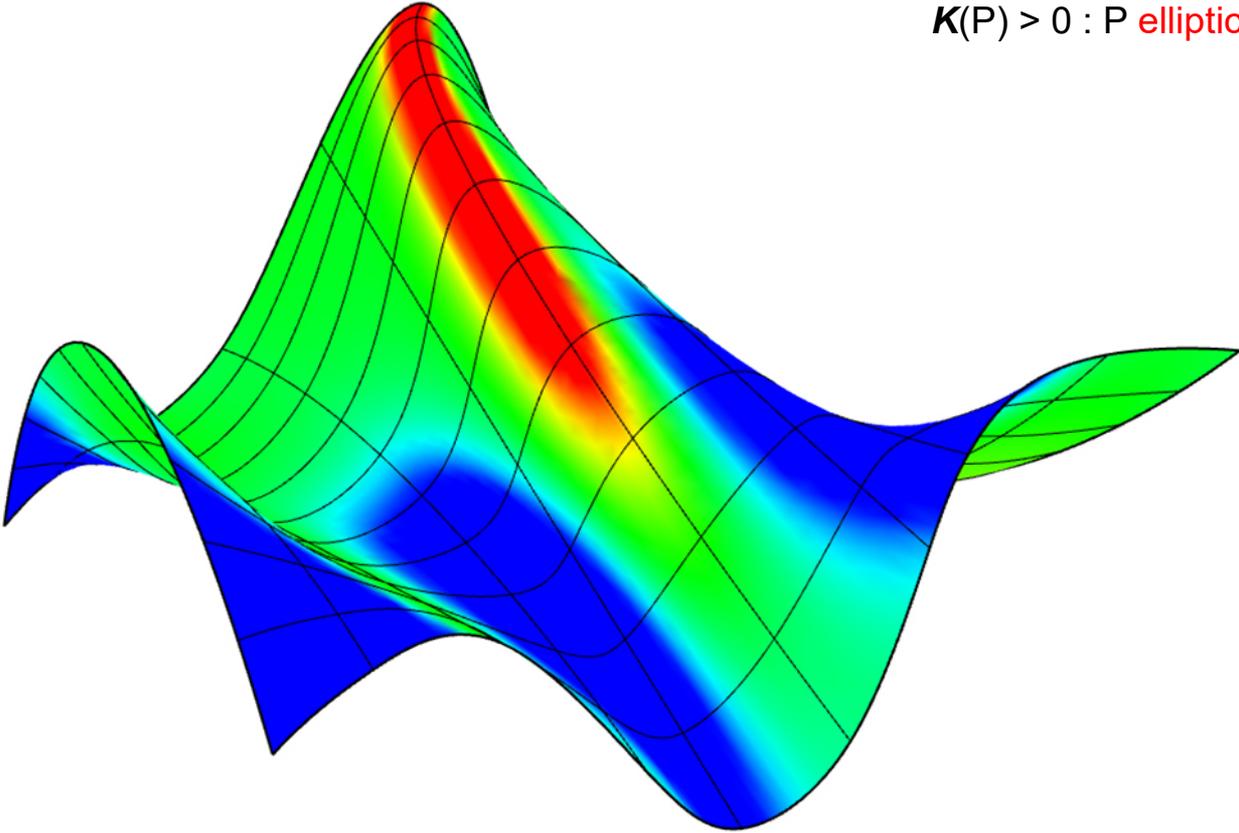
$$\kappa_2(P) = \min \{ \kappa_X(P) \mid X \text{ tangent vector in } P \}$$

# Surfaces

## Gaussian curvature

$$K = \kappa_1 \kappa_2$$

- $K(P) < 0$  : P hyperbolic point
- $K(P) = 0$  : P parabolic point
- $K(P) > 0$  : P elliptic point



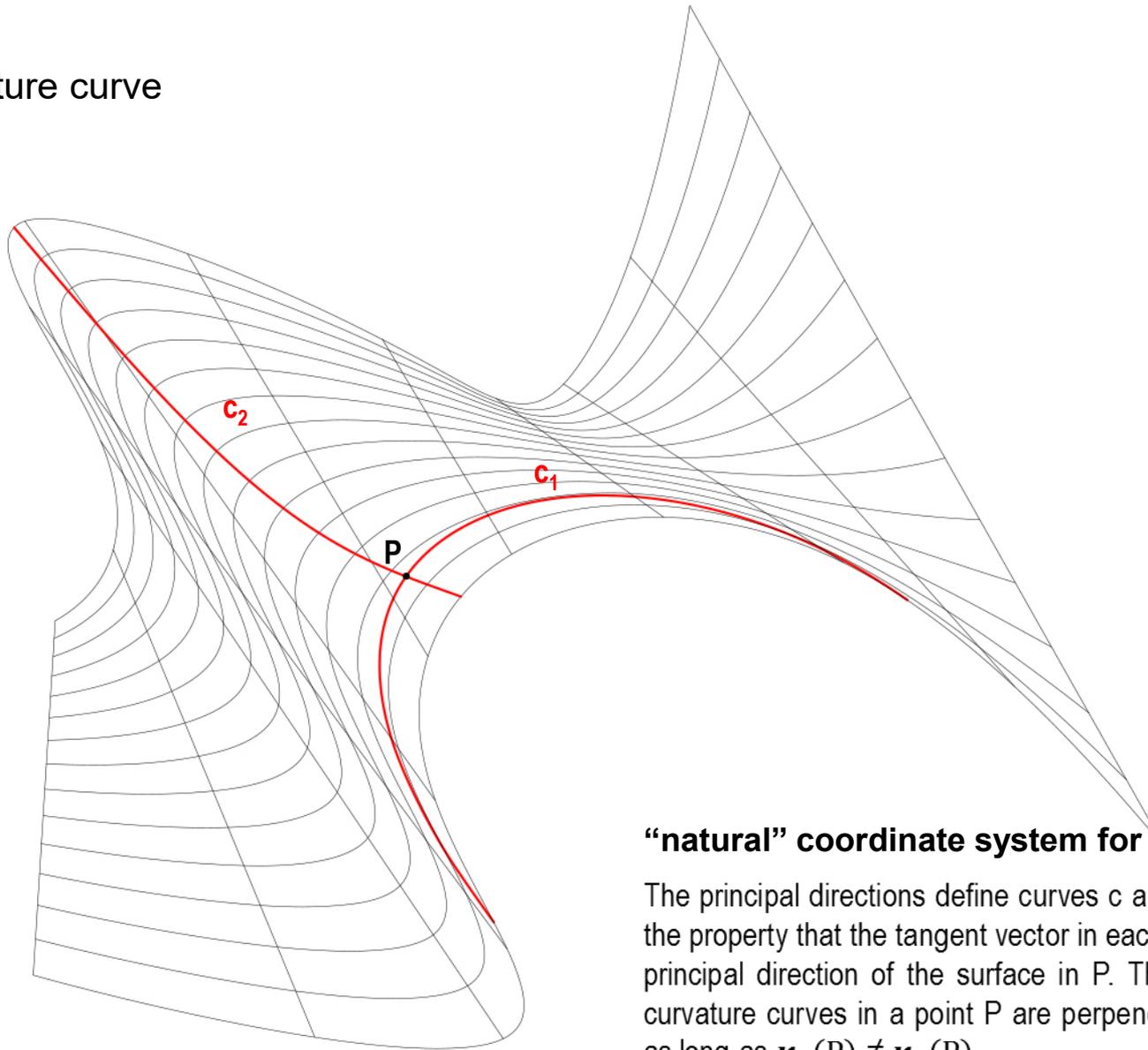
a developable surface is a surface where every point is a parabolic point, e.g. a paperstrip



Frank Gehry: Walt Disney Concert Hall  
Los Angeles, USA, 1999-2003

# Surfaces

## principal curvature curve



### “natural” coordinate system for surfaces

The principal directions define curves  $c$  along the surface with the property that the tangent vector in each point  $P = c(t)$  is the principal direction of the surface in  $P$ . The resulting principal curvature curves in a point  $P$  are perpendicular to each other as long as  $\kappa_1(P) \neq \kappa_2(P)$ .

# Principal Curvature Curve



Peter Cook & Colin Fournier: Art Museum  
Graz, Austria, 2002-03

# Principal Curvature Curve



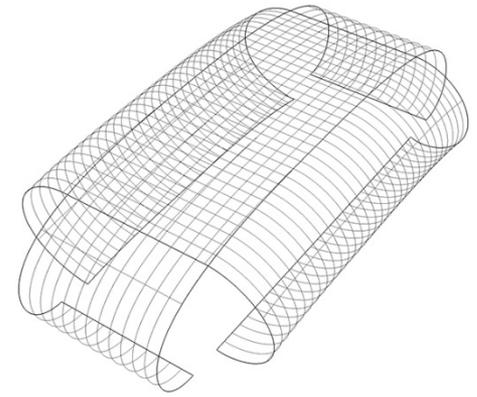
Peter Cook & Colin Fournier: Art Museum  
Graz, Austria, 2002-03

# Principal Curvature Curve

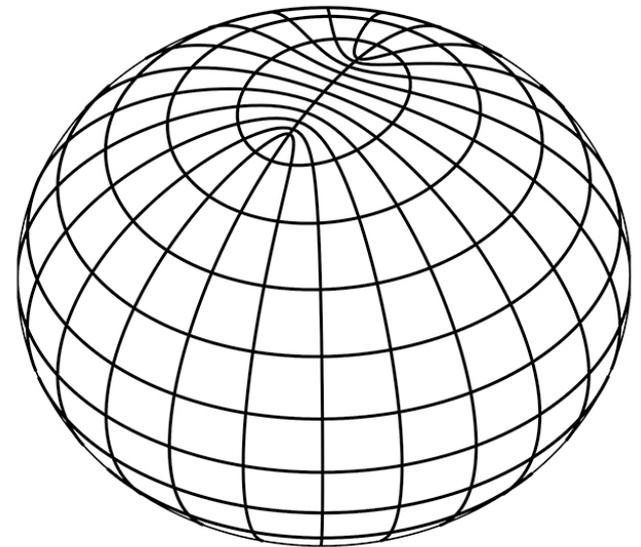


Peter Cook & Colin Fournier: Art Museum  
Graz, Austria, 2002-03

# Principal Curvature Curve panel layout



subdivision with respect to coordinate axis  
results in unresolved corner



principal curvature curves for an ellipsoid

Peter Cook & Colin Fournier: Art Museum  
Graz, Austria, 2002-03

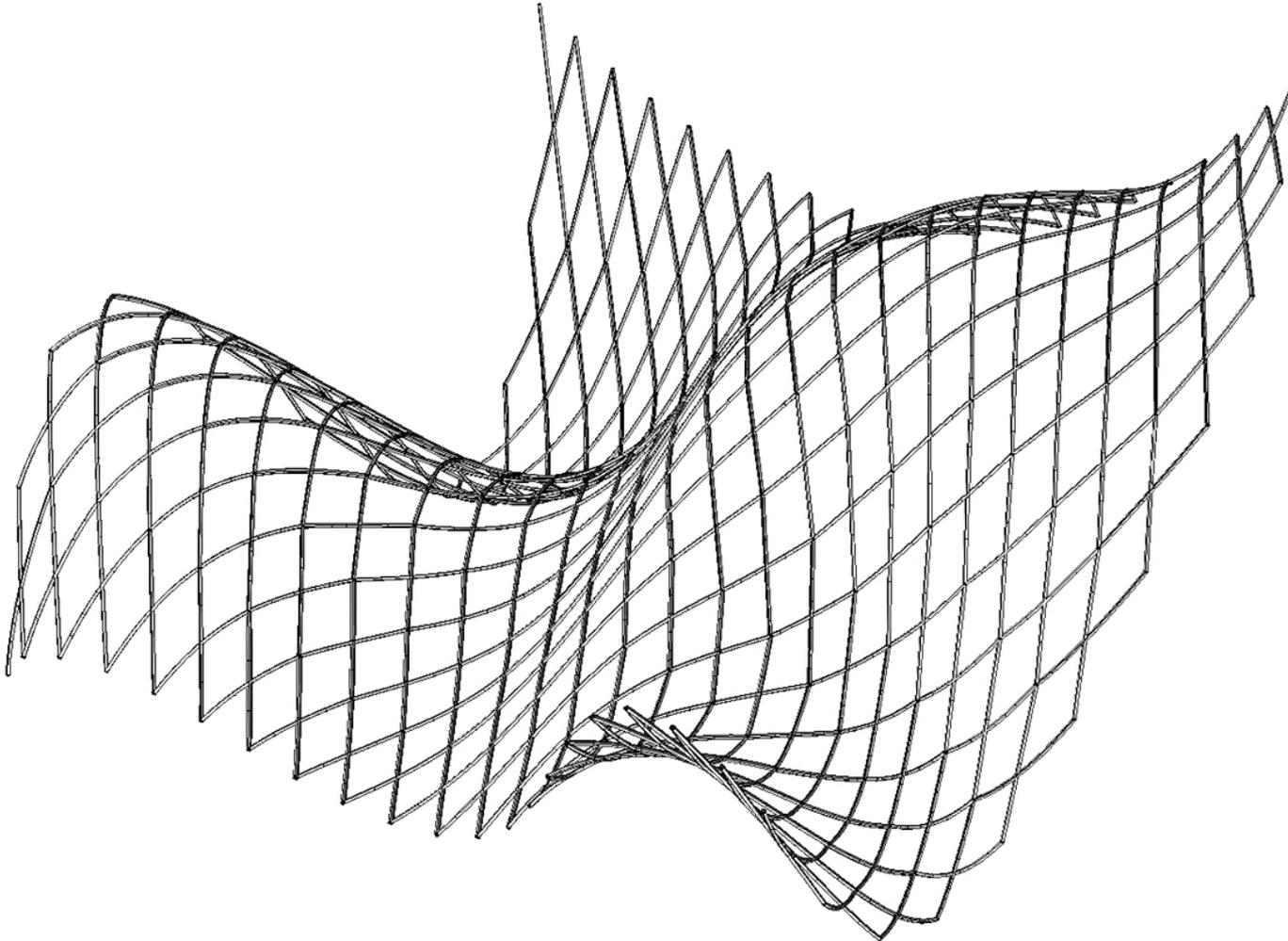
diagrid



Foster + Partner: Smithsonian Institute  
Washington, USA, 2004-07

# diagrid

Check 10: divide a surface into a diagrid



diagrid

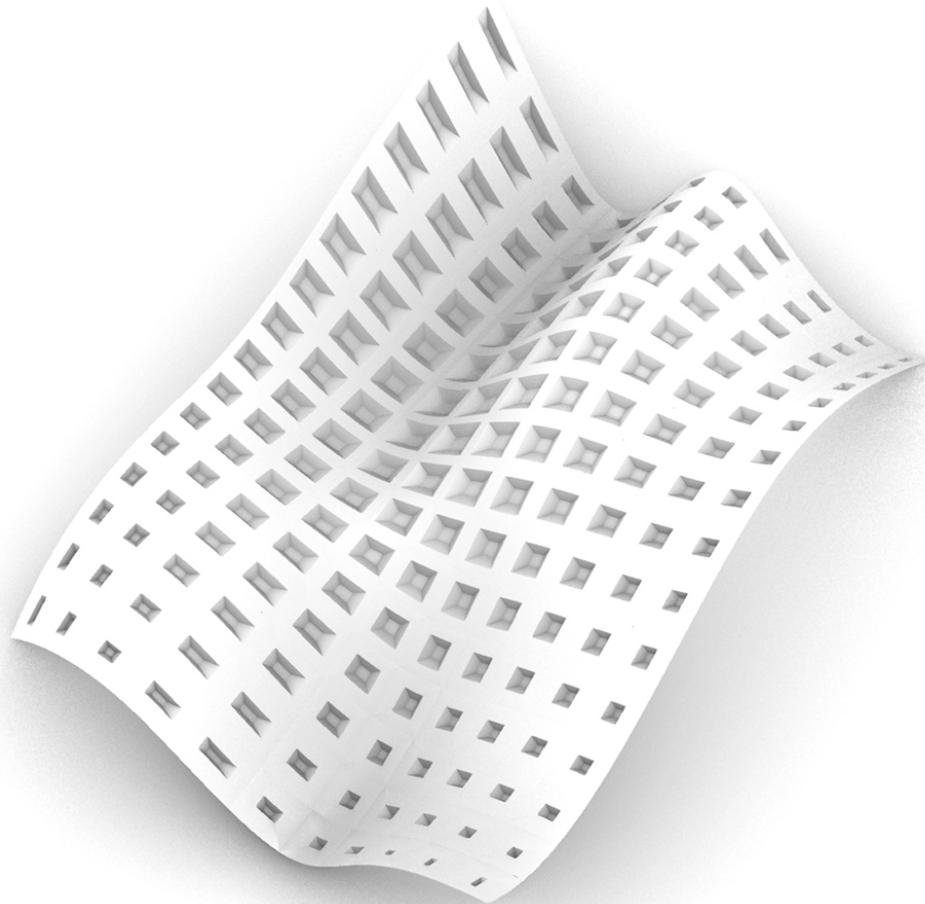


Foster + Partner: Smithsonian Institute  
Washington, USA, 2004-07



Foster + Partner: Smithsonian Institute  
Washington, USA, 2004-07

**Exercise 5:** construct a roof with varying size of opening with the variation controlled by the curvature of the surface.



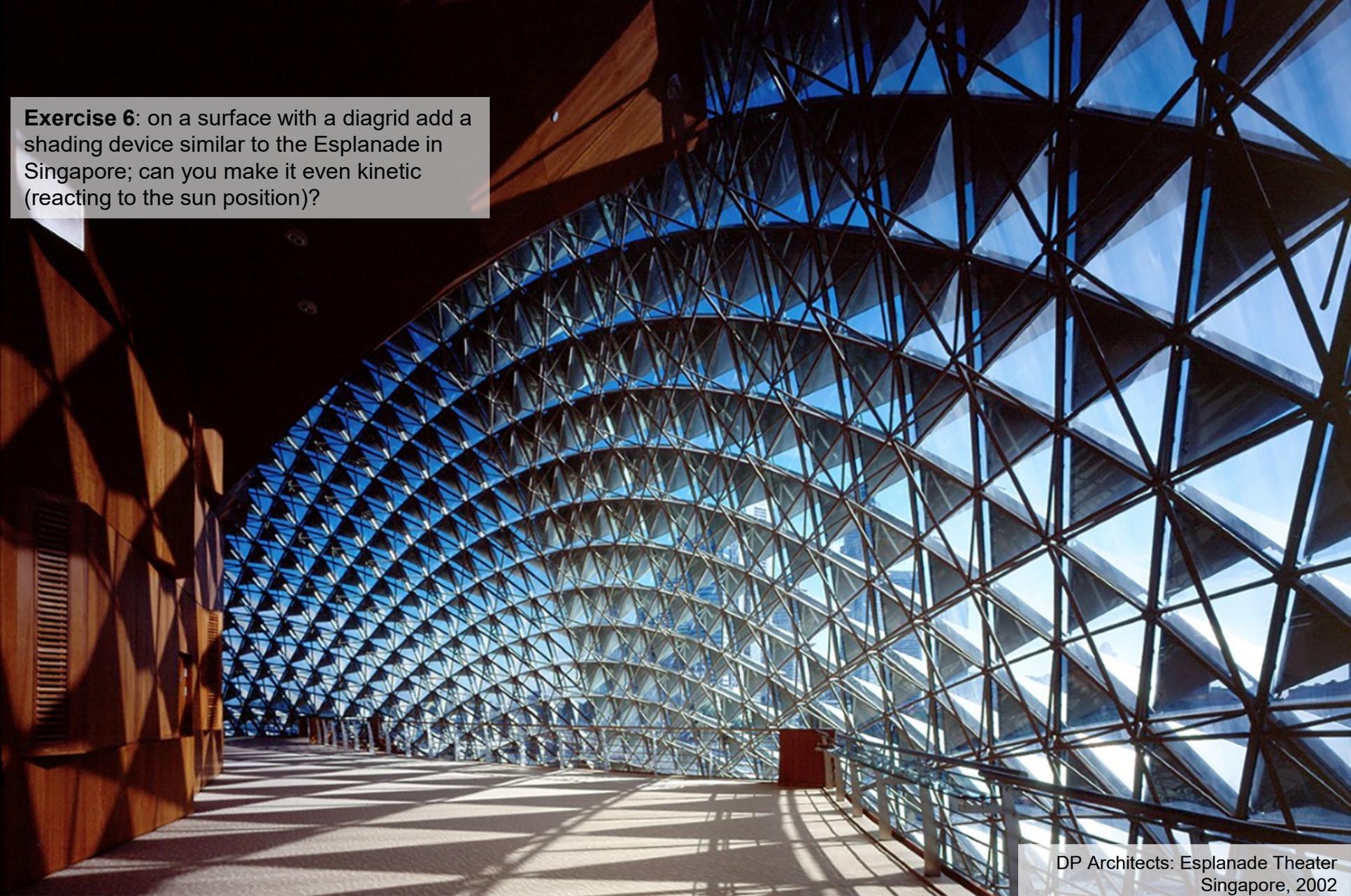


DP Architects: Esplanade Theater  
Singapore, 2002



DP Architects: Esplanade Theater  
Singapore, 2002

**Exercise 6:** on a surface with a diagrid add a shading device similar to the Esplanade in Singapore; can you make it even kinetic (reacting to the sun position)?



DP Architects: Esplanade Theater  
Singapore, 2002

# ARK-E2515 Parametric Design Surfaces

uv-coordinates

mapping onto surfaces

curvature & principal curve

