$\underline{\underline{1.}}$ a) $x(t) = R\cos t \Rightarrow x'(t) = -R\sin t \Rightarrow x''(t) = -R\cos t$

$y(t) = R\sin t \Rightarrow y'(t) = R\cos t \Rightarrow y''(t) = -R\sin t$

$\Rightarrow x'(t)^2 + y'(t)^2 = R^2(\sin^2 t + \cos^2 t) = R^2$

$$\Rightarrow K(t) = \frac{(-R\sin t)(-R\sin t) - (-R\cos t)\cdot R\cos t}{(R^2)^{3/2}} = \frac{R^2}{R^3} = \frac{1}{R}$$

$\Rightarrow R(t) = \underline{\underline{R = \text{VAKIO}}}$

b) $x(t) = t$, $x'(t) = 1$, $x''(t) = 0$

$y(t) = t^2$, $y'(t) = 2t$, $y''(t) = 2$

$x'(0)^2 + y'(0)^2 = 1$

$$K(0) = \frac{1\cdot 2 - 0\cdot 2t}{1^{3/2}} = 2 \Rightarrow R(0) = \underline{\underline{\frac{1}{2}}}$$

$\underline{\underline{2.}}$ $x = 0,\ y = \frac{1}{2} \Rightarrow g(0, 1/2) = 1 - \frac{1}{4}\cdot\left(3 + \frac{2}{2}\right) = 0$ ok

$\nabla g = 4x(x^2-1)\bar{i} - (2y(3+2y) + 2y^2)\bar{j}$

$\nabla g(0, 1/2) = -\frac{9}{2}\bar{j}$ = PYSTYSUORA NORMAALI $\Rightarrow$ TANGENTTI VAAKASUORA

b) $\begin{cases} 4x(x^2-1) = 0 & \Rightarrow x = 0 \text{ TAI } x = \pm 1 \\ 6y + 6y^2 = 0 & \Rightarrow y = 0 \text{ TAI } y = -1 \\ g = 0 \end{cases}$

VAIN $\underline{\underline{(\pm 1, 0)}}$ JA $\underline{\underline{(0, -1)}}$ TOT. $g = 0$

3. $\nabla f = (2x + 2ty + 2tz)\bar{i} + (2y + 2tx + 2tz)\bar{j} + (2z + 2tx + 2ty)\bar{k}$

$= \bar{0}$, KUN $x = y = z = 0$. SIIS $(0,0,0)$ ON KRIITTINEN PISTE.

b)

$$H_f(x,y,z) = \begin{bmatrix} 2 & 2t & 2t \\ 2t & 2 & 2t \\ 2t & 2t & 2 \end{bmatrix} = H_f(0,0,0)$$

c) PAIKALLINEN MINIMI, JOS $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$

$\Rightarrow$ $1 + 2t > 0$ JA $1 - t > 0$

$\Rightarrow$ $t > -\frac{1}{2}$ JA $t < 1$

$\Rightarrow$ ARVOILLA $\underline{\underline{-\frac{1}{2} < t < 1}}$

(TARKEMPI TUTKIMUS: $-\frac{1}{2} \leq t \leq 1$)

EI VAADITA!

4. $\nabla g \neq \bar{0}$ AINA $\Rightarrow$ EI ERIKOISTAPAUKSIA

$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ f_z = \lambda g_z \\ g = 0 \end{cases} \Leftrightarrow \begin{cases} 2x = 2\lambda \\ 2y = \lambda \\ 2z = -\lambda \\ g = 0 \end{cases}$$

$x = 2y$

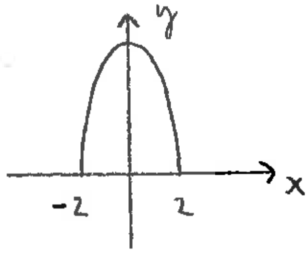
$z = -y$

SIJ. $\Rightarrow 4y + y + y = 12 \Rightarrow 6y = 12$

$\Rightarrow y = 2$, $x = 4$, $z = -2$ AINOA ÄÄRIARVOKOHTA, SELVÄSTI MINIMIKOHTA

$f(4,2,-2) = 24 \Rightarrow$ ETÄISYYS $= \sqrt{24} = \underline{\underline{2\sqrt{6}}}$

5. $A = \int_{-2}^{2}(4-x^2)\,dx = 2\int_{0}^{2}(4-x^2)\,dx = 2\Big/_{0}^{2}\left(4x-\frac{1}{3}x^3\right)$



$$= 2\left(8-\frac{8}{3}\right) = \underline{\underline{\frac{32}{3}}}$$

$$\iint_P y\,dA = \int_{-2}^{2}\int_{0}^{4-x^2} y\,dy\,dx = \int_{-2}^{2}\left(\Big/_{0}^{4-x^2}\frac{1}{2}y^2\right)dx = \frac{1}{2}\int_{-2}^{2}(4-x^2)^2\,dx$$

$$= \int_{0}^{2}(16-8x^2+x^4)\,dx = \Big/_{0}^{2}\left(16x-\frac{8}{3}x^3+\frac{1}{5}x^5\right)$$

$$= 32-\frac{64}{3}+\frac{32}{5} = \frac{256}{15} \Rightarrow \bar{y} = \frac{256/15}{32/3} = \underline{\underline{\frac{8}{5}}}$$

6. a) $EX = \frac{1}{\pi}\int_{0}^{2\pi}\int_{0}^{1} r\cdot\underbrace{r\,dr\,d\theta}_{dA} = \frac{2\pi}{\pi}\cdot\Big/_{0}^{1}\frac{1}{3}r^3 = \underline{\underline{\frac{2}{3}}}$

b) $EY = \frac{3}{4\pi}\int_{0}^{2\pi}\int_{0}^{\pi}\int_{0}^{1} r\cdot\underbrace{r^2\sin\varphi\,dr\,d\varphi\,d\theta}_{dV}$

$$= \frac{3}{4\pi}\cdot 2\pi\int_{0}^{\pi}\sin\varphi\,d\varphi\cdot\int_{0}^{1}r^3\,dr = \frac{3}{2}\cdot 2\cdot\frac{1}{4} = \underline{\underline{\frac{3}{4}}}$$