

Statistical language model (SLM)

- Content today:
 1. Basic SLM methods (n-grams)
 1. Maximum likelihood estimation
 2. Smoothing methods
 3. Class-based methods
 2. Advanced SLM methods
 1. Maximum entropy methods
 2. Continuous vector space methods
 3. Introduction to Neural LMs

Goals of today

1. Learn how to model language by statistical methods
2. Learn basic idea of neural language modeling
3. Know some typical SLM **methods and applications**

About scores, points and grades in 2023

- Max score in home exercises was 161 => 50p
- Max score in lecture activity was 25 => 10p
- Exam points could substitute max 20p of missed points
- In 2023 the points corresponded to non-rounded grades like this:
 - ~ 60p gave 5.6
 - ~ 53p gave 4.6
 - ~ 46p gave 3.6
 - ~ 38p gave 2.5
 - ~ 31p gave 1.5
 - ~ 24p gave 0.6
 - ~ 20p or less gave 0
- **The final grade** is the average of this (60%) and the project (40%) grade

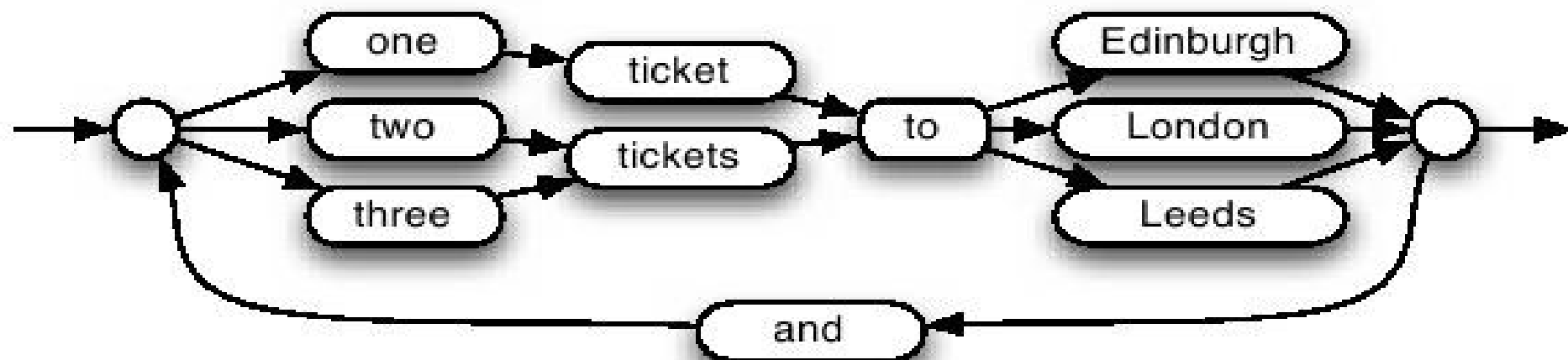
Statistical Language Model

- Model of a *natural language* that predicts the *probability distribution* of words and sentences in a text
- Often used to determine which is the most probable word or sentence in given conditions or context
- Estimated by counting word frequencies and dependencies in large text corpora
- Has to deal with: *big data, noisy data, sparse data, computational efficiency*

Some historical landmarks of SLMs

- Markov chains (Markov, 1913)
- N-grams (Shannon, 1948)
- Predicting unseen events (Good, 1953)
- Landmarks at *Aalto University* (Helsinki Univ. of Technology)
 - ~ Dynamically expanding context (Kohonen, 1986)
 - ~ Self-organizing semantic maps (Ritter and Kohonen, 1989)
 - ~ WEBSOM for organizing text collections (Kohonen, 1996)
 - ~ Morfessor for unsupervised analysis of words (Lagus. 2002)
 - ~ Varigram LM for sequences of words (Siivola, 2005)
 - ~ Unlimited vocabulary LMs for speech recognition (Hirsimäki, 2006)
 - ~ Class n-gram models for very large vocabulary speech recognition of Finnish and Estonian (Varjokallio, 2016)

A simple statistical language model



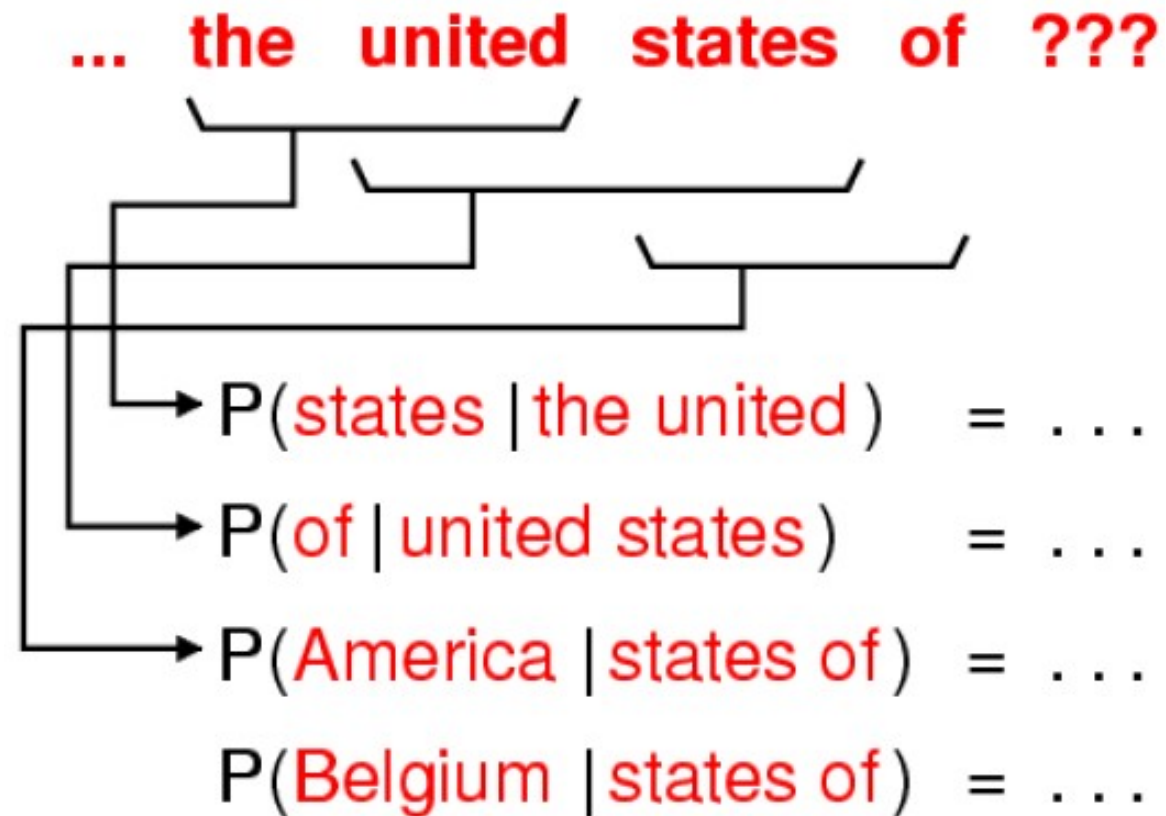
- Limited domain models, constructed by hand
- Transition probabilities can be estimated statistically
- Only a very limited set of sentences are recognized

N-gram language model

- Stochastic model of the **relations between words**
 - Which words often occur close to each other?
- The model **predicts the probability distribution of the next word** given the previous ones
- A conditional probability of word given its **context**
- Estimated from a large text corpus (count the contexts!)
- Smoothing and pruning required to learn compact long-span models from **sparse training data**

N-gram models

- E.g. trigram = 3-gram:
- Word occurrence depends only on its immediate short context
- A conditional probability of word given its context
- Estimated from a large text corpus (count the contexts!)



Estimation of N-gram model

$$P(w_i | w_j) = \frac{c(w_j, w_i)}{c(w_j)} \quad \frac{c(\text{"eggplant stew"})}{c(\text{"eggplant"})}$$

- Bigram example:

- ~ Start from a **maximum likelihood estimate**

- ~ probability of $P(\text{"stew"} | \text{"eggplant"})$ is computed from **counts** of *"eggplant stew"* and *"eggplant"*

| | I | want | to | eat | Chinese | food | lunch |
|---------|----|-------------|-----|-----|---------|------|-------|
| I | 8 | 1087 | 0 | 13 | 0 | 0 | 0 |
| want | 3 | 0 | 786 | 0 | 6 | 8 | 6 |
| to | 3 | 0 | 10 | 860 | 3 | 0 | 12 |
| eat | 0 | 0 | 2 | 0 | 19 | 2 | 52 |
| Chinese | 2 | 0 | 0 | 0 | 0 | 120 | 1 |
| food | 19 | 0 | 17 | 0 | 0 | 0 | 0 |
| lunch | 4 | 0 | 0 | 0 | 0 | 1 | 0 |

Uni-gram counts

Data from Berkeley restaurant corpus (Jurafsky & Martin, "Speech and language processing").

Calculate missing bi-gram probabilities

| | I | want | to | eat | Chinese | food | lunch |
|---------|--------|----------|-------|-------|----------|-------|----------|
| I | .0023 | X | 0 | .0038 | 0 | 0 | 0 |
| want | .0025 | 0 | .65 | 0 | .0049 | .0066 | X |
| to | .00092 | 0 | .0031 | .26 | X | 0 | .0037 |
| eat | 0 | 0 | .0021 | 0 | .020 | .0021 | .055 |
| Chinese | .0094 | 0 | 0 | 0 | 0 | .056 | .0047 |
| food | .013 | 0 | .011 | 0 | 0 | 0 | 0 |
| lunch | .0087 | 0 | 0 | 0 | 0 | .0022 | 0 |

| | |
|---------|-------------|
| I | 3437 |
| want | 1215 |
| to | 3256 |
| eat | 938 |
| Chinese | 213 |
| food | 1506 |
| lunch | 459 |

Data from Berkeley restaurant corpus (Jurafsky & Martin, "Speech and language processing").

| | I | want | to | eat | Chinese | food | lunch |
|---------|----|-------------|-----|-----|---------|------|-------|
| I | 8 | 1087 | 0 | 13 | 0 | 0 | 0 |
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| eat | 0 | 0 | 2 | 0 | 19 | 2 | 52 |
| Chinese | 2 | 0 | 0 | 0 | 0 | 120 | 1 |
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Uni-gram counts

$$1087 / 3437 = .32$$

Calculate missing bi-gram probabilities

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| eat | 0 | 0 | 2 | 0 | 19 | 2 | 52 |
| Chinese | 2 | 0 | 0 | 0 | 0 | 120 | 1 |
| food | 19 | 0 | 17 | 0 | 0 | 0 | 0 |
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Uni-gram counts

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Calculate missing bi-gram probabilities

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| eat | 0 | 0 | .0021 | 0 | .020 | .0021 | .055 |
| Chinese | .0094 | 0 | 0 | 0 | 0 | .056 | .0047 |
| food | .013 | 0 | .011 | 0 | 0 | 0 | 0 |
| lunch | .0087 | 0 | 0 | 0 | 0 | .0022 | 0 |

$$3 / 3256 = .00092$$

| | |
|---------|-------------|
| I | 3437 |
| want | 1215 |
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| | I | want | to | eat | Chinese | food | lunch |
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| Chinese | 2 | 0 | 0 | 0 | 0 | 120 | 1 |
| food | 19 | 0 | 17 | 0 | 0 | 0 | 0 |
| lunch | 4 | 0 | 0 | 0 | 0 | 1 | 0 |

Uni-gram counts

$$1087 / 3437 = .32$$

Calculate missing bi-gram probabilities

$$3 / 3256 = .00092$$

| | I | want | to | eat | Chinese | food | lunch |
|---------|--------|------------|-------|-------|---------------|-------|--------------|
| I | .0023 | .32 | 0 | .0038 | 0 | 0 | 0 |
| want | .0025 | 0 | .65 | 0 | .0049 | .0066 | .0049 |
| to | .00092 | 0 | .0031 | .26 | .00092 | 0 | .0037 |
| eat | 0 | 0 | .0021 | 0 | .020 | .0021 | .055 |
| Chinese | .0094 | 0 | 0 | 0 | 0 | .056 | .0047 |
| food | .013 | 0 | .011 | 0 | 0 | 0 | 0 |
| lunch | .0087 | 0 | 0 | 0 | 0 | .0022 | 0 |

| | |
|---------|-------------|
| I | 3437 |
| want | 1215 |
| to | 3256 |
| eat | 938 |
| Chinese | 213 |
| food | 1506 |
| lunch | 459 |

$$6 / 1215 = .0049$$

Estimation of N-gram model

$$P(w_i | w_j) = \frac{c(w_j, w_i)}{c(w_j)} \quad \frac{c(\text{"eggplant stew"})}{c(\text{"eggplant"})}$$

- Bigram example:

~ Start from a **maximum likelihood estimate**

~ probability of $P(\text{"stew"} | \text{"eggplant"})$ is computed from **counts** of *"eggplant stew"* and *"eggplant"*

~ works well for frequent bigrams

$$P(\text{"want"} | \text{"I"}) = 1087 / 3437 = 0.32$$

~ why not for rare bigrams?

$$P(\text{"Chinese"} | \text{"to"}) = 3 / 3256 = 0.00092$$

Where do we need SLMs?

- List tasks **where you need the probability** or to find the most probable **word or sentence** given some background information!

Some applications of SLMs

1. Spelling correction, text input
2. Optical character recognition, e.g. scanning old books
3. Automatic speech recognition
4. Statistical machine translation
5. Text-to-speech
6. Automatic question answering
7. Chatbots

Data sparsity

- Words and many other linguistic units follow a **power-law distribution**:
 - ~ Zipf's law: k th frequent word occurs $\propto 1/k$
 - ~ “Long tail”: few frequent words, lots of very rare words
- E.g. within the first 1.5 million words 23% subsequent trigrams were previously unseen (IBM laser patent text corpus)
- Maximum likelihood estimate overestimates frequencies of n -gram that occurred rarely, and underestimates those that did not occur at all. (why?)
- One needs a systematic approach to assign some non-zero probability to unseen words and sequences. This is called **smoothing**.

Zero probability problem

- If an N-gram is not seen in the corpus, it will get probability = 0
- The higher N, the sparser data, and the more zero counts there will be
- 20K words => 400M 2-grams => 8000G 3-grams, so even the largest corpora have MANY zero counts!
- **Solutions:**
- **Equivalence classes:** Cluster several similar n-grams together to reach higher counts
- **Smoothing:** Redistribute some probability mass from seen N-grams to unseen ones

Equivalence classes

- Divide features (e.g. words) into equivalence classes a.k.a. **bins**
- Assume equal statistical properties within a bin
- Estimate a SLM for the bin as a whole
- The more bins, the more data is needed for model estimation
- The fewer bins, the lower prediction accuracy, because the model becomes too general

Ways to form classes

- Transforming **inflected word forms** into the baseform: 'saunan', 'saunalle', 'saunojemme', etc. → 'sauna'
- Grouping by **part-of-speech** tags (the same syntactic role: *noun, verb, etc*)
- Grouping by **semantics** (a similar meaning)
- Important is that the words in a bin should really behave similarly! E.g. *february, may, august*

Ways to use classes

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research

- using equivalence classes only for previous words (Virpioja and Kurimo, 2006):
- $p(w_i | w_{i-2}, w_{i-1}) = p(w_i | t(w_{i-2}, w_{i-1}))$
- using class-based n-gram models:
- $p(w_i | w_{i-2}, w_{i-1}) = p(t(w_i) | t(w_{i-2}, w_{i-1}))$
- $\times p(w_i | t(w_i), \dots)$

Combining estimators

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- So far, the probability was estimated for all n-grams of a particular length
- How about improving the estimate using shorter sequences that are more frequent?
- The motivation is further smoothing of the estimates by **combining different information sources.**
- The **additional models** could also be other n-grams trained on **different data**, e.g. background models vs topical models
- **determine bin-specific interpolation weights for model combination (Broman and Kurimo, 2005)**

Backing-off

- **In principle:** Look for the most specific model that gives sufficient information from the current context
- **In practice:** Back off from using (too) long contexts to shorter ones that have more samples in the corpus.

Some smoothing methods

1. **Add-one**: Add 1 to each count and normalize => gives too much probability to unseen N-grams
2. **(Absolute) discounting**: Subtract a constant from all counts and redistribute this to unseen ones using N-1 gram probs and back-off (normalization) weights
3. **Witten-Bell smoothing**: Use the count of things seen once to help to estimate the count of unseen things
4. **Good Turing smoothing**: Estimate the rare n-grams based on counts of more frequent counts
5. Best: **Kneser-Ney smoothing**: Instead of the number of occurrences, weigh the back-offs by the **number of contexts** the word appears in
6. Instead of only back-off cases, **interpolate** all N-gram counts with N-1 counts

Add-1 smoothing

$$c_i^* = (c_i + 1) \frac{N}{N + V}$$

Probability $p = c / N$:

$$p_i^* = \frac{c_i + 1}{N + V}$$

C_i^* : new count

C_i : original count

N : Num of tokens

V : Total vocab size

| | I | want | to | eat | Chinese | food | lunch |
|---------|----|------|-----|-----|---------|------|-------|
| I | 9 | 1088 | 1 | 14 | 1 | 1 | 1 |
| want | 4 | 1 | 787 | 1 | 7 | 9 | 7 |
| to | 4 | 1 | 11 | 861 | 4 | 1 | 13 |
| eat | 1 | 1 | 3 | 1 | 20 | 3 | 53 |
| Chinese | 3 | 1 | 1 | 1 | 1 | 121 | 2 |
| food | 20 | 1 | 18 | 1 | 1 | 1 | 1 |
| lunch | 5 | 1 | 1 | 1 | 1 | 2 | 1 |

Figure 6.6 Add-one Smoothed Bigram counts for 7 of the words (out of 1616 total word types) in the Berkeley Restaurant Project corpus of ~10,000 sentences.

$$c_i^* = (c_i + 1) \frac{N}{N + V}$$

Probability $p = c / N$:

$$p_i^* = \frac{c_i + 1}{N + V}$$

N : Num of tokens

T : Num of types (seen)

Z : Num of types (unseen)

V : Total vocab size

$$c_i^* = \begin{cases} \frac{T}{Z} \frac{N}{N+T}, & \text{if } c_i = 0 \\ c_i \frac{N}{N+T}, & \text{if } c_i > 0 \end{cases}$$

| | I | want | to | eat | Chinese | food | lunch |
|---------|----|------|-----|-----|---------|------|-------|
| I | 9 | 1088 | 1 | 14 | 1 | 1 | 1 |
| want | 4 | 1 | 787 | 1 | 7 | 9 | 7 |
| to | 4 | 1 | 11 | 861 | 4 | 1 | 13 |
| eat | 1 | 1 | 3 | 1 | 20 | 3 | 53 |
| Chinese | 3 | 1 | 1 | 1 | 1 | 121 | 2 |
| food | 20 | 1 | 18 | 1 | 1 | 1 | 1 |
| lunch | 5 | 1 | 1 | 1 | 1 | 2 | 1 |

Figure 6.6 Add-one Smoothed Bigram counts for 7 of the words (out of 1616 total word types) in the Berkeley Restaurant Project corpus of ~10,000 sentences.

| | I | want | to | eat | Chinese | food | lunch |
|---------|------|------|------|------|---------|------|-------|
| I | 8 | 1060 | .062 | 13 | .062 | .062 | .062 |
| want | 3 | .046 | 740 | .046 | 6 | 8 | 6 |
| to | 3 | .085 | 10 | 827 | 3 | .085 | 12 |
| eat | .075 | .075 | 2 | .075 | 17 | 2 | 46 |
| Chinese | 2 | .012 | .012 | .012 | .012 | 109 | 1 |
| food | 18 | .059 | 16 | .059 | .059 | .059 | .059 |
| lunch | 4 | .026 | .026 | .026 | .026 | 1 | .026 |

Figure 6.9 Witten-Bell smoothed bigram counts for 7 of the words (out of 1616 total word types) in the Berkeley Restaurant Project corpus of ~10,000 sentences.

Good-Turing smoothing

- How to compute the **probability of an unseen event**, e.g. an out-of-vocabulary word?
- Idea invented by Alan Turing during World War 2 when he was working to break German cipher
- Published later by his student (Good, 1953)
- Set: N = Num of words
 - N_1 = Num of words that occur only once
 - N_c = Num of words that occur c -times (freq. of freq.)
- Estimate prob of unseen things = N_1/N
- Estimate count of things seen once = $2*N_2/N_1$
- Smoothed count c^* for all c :

$$c^* = (c + 1) \frac{N_{c+1}}{N_c}$$

Exercise 2: Good-Turing smoothing

- **Watch a video** where Prof. Jurafsky (Stanford) explains Good-Turing smoothing (between 02:00 – 08:45)
 - ~ Click: <http://www.youtube.com/watch?v=GwP8gKa-ij8>
 - ~ Or search: "Good Turing video Jurafsky"
- Work in groups and submit answers for these 3 questions in **MyCourses > Lectures > Lecture 2 exercise return box:**
 1. Estimate the prob. of catching next any new fish species, if you already got: 5 perch, 2 pike, 1 trout, 1 zander and 1 salmon?
 2. Estimate the prob. of catching next a salmon?
 3. What may cause practical problems when applying Good-Turing smoothing for rare words in large text corpora?

Hints for solving the exercise

1. Estimate the prob of unseen things using the prob of things seen only once N_1/N
2. The counts must be smoothed. The new count for things seen once is $(c+1)*N_2/N_1$
3. What if $N_c = 0$ for some c ?

Estimation of N-gram model

$$P(w_i | w_j) = \frac{c(w_j, w_i)}{c(w_j)} \quad \frac{c(\text{"eggplant stew"})}{c(\text{"eggplant"})}$$

- Bigram example:

~ Start from a **maximum likelihood estimate**

~ probability of $P(\text{"stew"} | \text{"eggplant"})$ is computed from **counts** of *"eggplant stew"* and *"eggplant"*

~ works well for frequent bigrams

Backing off

$$P(w_i | w_j) = \frac{c(w_j, w_i)}{c(w_j)} \quad \text{if } c(w_j, w_i) > c$$
$$= P(w_i) b_{w_j} \quad \text{otherwise}$$

- Divide the room of rare bigrams, e.g. “*eggplant francisco*”, in proportion to the unigram $P(\text{“francisco”})$
- The sum of all these rare bigrams “*eggplant [word j]*” is $b(\text{“eggplant”})$ which is called the **back-off weight**

Absolute discounting and backing off

$$P(w_i | w_j) = \frac{c(w_j, w_i) - D}{c(w_j)} \quad \text{if } c(w_j, w_i) > c$$
$$= P(w_i) b_{w_j} \quad \text{otherwise}$$

- If bigram is common: Subtract constant D from the count
- If not: Back off to the unigram probability normalized by the back-off weight
- Similarly back off all rare N-grams to N-1 grams

Kneser-Ney smoothing

$$P(w_i | w_j) = \frac{c(w_j, w_i) - D}{c(w_j)} \quad \text{if } c(w_j, w_i) > c$$
$$= \mathbf{V}(w_i) b_{w_j} \quad \text{otherwise}$$

- Instead of the number of occurrences, weigh the back-offs by the **number of contexts** $\mathbf{V}(\text{word})$ the word appears in:
 - ~ In this case the context is the previous word, thus, how many different previous words the corpus has for that word
 - ~ E.g. $P(\text{Stew} | \text{EggPlant})$ is high, because stew occurs in many contexts
 - ~ But $P(\text{Francisco} | \text{EggPlant})$ is low, because Francisco is common, but only in "San Francisco"

Smoothing by interpolation

$$P(w_i | w_j) = \frac{c(w_j, w_i) - D}{c(w_j)} + P(w_i)b_{w_j}$$

- Like backing off, but always compute the probability as a **linear combination** (weighted average) with lower order (N-1)gram probabilities
- Improves the probabilities of rare N-grams
- Discounts (D) (and interpolation weights) can be separately optimized for each N using a held-out data

N-gram example

| eggplant X) | 1G freq | 1G prob | 2G freq | 2G prob |
|-------------|---------|---------|---------|---------|
| X = stew | 10 | 0.1 | 0 | 0 |
| sue | 20 | 0.2 | 0 | 0 |
| san | 40 | 0.4 | 0 | 0 |
| francisco | 30 | 0.3 | 0 | 0 |
| SUM | 100 | 1 | 0 | 0 |

10/100

$$P(w_i | w_j) = \frac{c(w_j, w_i)}{c(w_j)}$$

Absolute discounting

| | 1G freq | 1G prob | 2G freq | 2G prob | D=0.50 discount |
|-------------|---------|---------|---------|---------|--------------------|
| eggplant X) | | | | | |
| X = stew | 10 | 0.1 | 0 | 0 | |
| sue | 20 | 0.2 | 0 | 0 | |
| san | 40 | 0.4 | 0 | 0 | |
| francisco | 30 | 0.3 | 0 | 0 | |
| SUM | 100 | 1 | 0 | 0 | 0.5 |

$$P(w_i | w_j) = \frac{c(w_j, w_i) - D}{c(w_j)} \quad \text{if } c(w_j, w_i) > c$$

(c=0, D=0.5 selected)

Back-off

D=0.50

| eggplant X) | 1G freq | 1G prob | 2G freq | 2G prob | discount | Abs back-off | normalize |
|-------------|---------|---------|---------|---------|----------|--------------|-----------|
| X = stew | 10 | 0.1 | 0 | 0 | | 0.1 | 0.05 |
| sue | 20 | 0.2 | 0 | 0 | | 0.2 | 0.1 |
| san | 40 | 0.4 | 0 | 0 | | 0.4 | 0.2 |
| francisco | 30 | 0.3 | 0 | 0 | | 0.3 | 0.15 |
| SUM | 100 | 1 | 0 | 0 | 0.5 | 1 | 0.5 |

$$P(w_i | w_j) = \frac{c(w_j, w_i) - D}{c(w_j)} \quad \text{if } c(w_j, w_i) > c$$

$$= P(w_i) b_{w_j} \quad \text{otherwise}$$

Back-off

D=0.50

| eggplant X) | 1G freq | 1G prob | 2G freq | 2G prob | discount | Abs back-off | normalize |
|-------------|---------|---------|---------|---------|----------|--------------|-----------|
| X = stew | 10 | 0.1 | 0 | 0 | | 0.1 | 0.05 |
| sue | 20 | 0.2 | 0 | 0 | | 0.2 | 0.1 |
| san | 40 | 0.4 | 0 | 0 | | 0.4 | 0.2 |
| francisco | 30 | 0.3 | 0 | 0 | | 0.3 | 0.15 |
| SUM | 100 | 1 | 0 | 0 | 0.5 | 1 | 0.5 |

$$P(w_i | w_j) = \frac{c(w_j, w_i) - D}{c(w_j)} \quad \text{if } c(w_j, w_i) > c$$

$$= P(w_i) b_{w_j} \quad \text{otherwise}$$

0.1/1.0*0.5

Absolute discounting and back-off

| (eggplant X) | 1G freq | 2G freq | Abs back-off | normalize |
|--------------|---------|---------|--------------|-----------|
| X = stew | 10 | 0 | 0.1 | 0 |
| sue | 20 | 0 | 0.2 | 0 |
| san | 40 | 0 | 0.4 | 0 |
| francisco | 30 | 0 | 0.3 | 0 |
| SUM | 100 | 0 | 1 | 0 |

$$P(w_i | w_j) = \frac{c(w_j, w_i) - D}{c(w_j)} \quad \text{if } c(w_j, w_i) > c$$

$$= P(w_i) b_{w_j} \quad \text{otherwise} \quad (\text{c=0, D=0.5 selected})$$

Kneser-Ney smoothing

| (eggplant X) | 1G freq | 2G freq | Abs back-off | normalize | #contexts |
|--------------|---------|---------|--------------|-----------|-----------|
| X = stew | 10 | 0 | 0.1 | 0 | 10 |
| sue | 20 | 0 | 0.2 | 0 | 5 |
| san | 40 | 0 | 0.4 | 0 | 3 |
| francisco | 30 | 0 | 0.3 | 0 | 1 |
| SUM | 100 | 0 | 1 | 0 | 19 |

$$P(w_i | w_j) = \frac{c(w_j, w_i) - D}{c(w_j)} \quad \text{if } c(w_j, w_i) > c$$

$$= \mathbf{V}(w_i) b_{w_j} \quad \text{otherwise} \quad (\text{c=0, D=0.5 selected})$$

Kneser-Ney smoothing

| (eggplant X) | 1G freq | 2G freq | Abs back-off | normalize | #contexts | KN back-off |
|--------------|---------|---------|--------------|-----------|-----------|-------------|
| X = stew | 10 | 0 | 0.1 | 0.05 | 10 | 0.26 |
| sue | 20 | 0 | 0.2 | 0.1 | 5 | 0.13 |
| san | 40 | 0 | 0.4 | 0.2 | 3 | 0.08 |
| francisco | 30 | 0 | 0.3 | 0.15 | 1 | 0.03 |
| SUM | 100 | 0 | 1 | 0.5 | 19 | 0.5 |

$$P(w_i | w_j) = \frac{c(w_j, w_i) - D}{c(w_j)} \quad \text{if } c(w_j, w_i) > c$$

$$= \mathbf{V}(w_i) b_{w_j} \quad \text{otherwise}$$

(c=0, D=0.5 selected)
10/19*0.5

Weaknesses of N-grams

- Skips long-span dependencies:
 - ~ “The **girl** that I met in the train **was** ...”
- Too dependent on word order:
 - ~ “dog chased **cat**”: “koira jahtasi **kissaa**” ~ “**kissaa** koira jahtasi”
- Dependencies directly between words, instead of latent variables, e.g. word categories

Some model variants

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- Variable-length n-gram, aka. **Varigram**:
 - ~ Span depends on particular context, optimized for the data, e.g. [Siivola, 2007]
 - ~ Especially useful for short units (letters, morphemes)
- **Class-based** n-gram, e.g. [Brown, 1992]:
 - ~ Cluster words into classes, find class sequences
 - ~ Reduces sparsity, model size, and accuracy
- **Bayesian** n-gram:
 - ~ Computationally demanding
 - ~ Kneser-Ney smoothing approximates hierarchical Pitman-Yor process model [Goldwater, 2006; Teh, 2006]

Sources and further reading

- Manning, C. D. and Schütze, H. (1999). Foundations of Statistical Natural Language Processing. The MIT Press. (Chapter 6)
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Other language modeling approaches

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- **Maximum-entropy** LM (Rosenfeld, 2007)
 - ~ Combines different knowledge sources into a single model
 - ~ Good for adaptation (Alumäe and Kurimo, 2010)
- **Continuous-space** LM (a.k.a. Neural Network LM (NNLM))
 - ~ Map words to continuous-valued vectors and models them using DNN (Bengio et al, 2003; Siivola and Honkela, 2003)
 - ~ State-space models can use indefinitely long contexts, such as in Recurrent Neural Networks (Mikolov et al, 2010)
- **Cache** models and **Topic** models

Maximum entropy LMs

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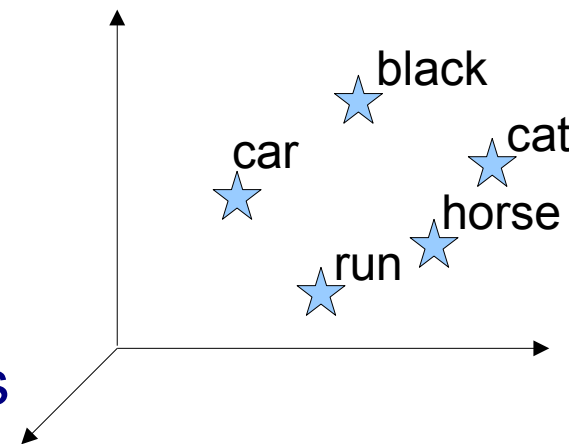
research

- Represents dependency information
- by a weighted sum of features $f(x,h)$
- Features can be e.g. n-gram counts
- Alleviates the data sparsity problem by smoothing the feature weights (lambda) towards zero
- The weights can be adapted in more flexible ways than n-grams
 - ~ Adapting only those weights that significantly differ from a large background model (Alumäe and Kurimo, 2010)
- Normalization is computationally hard, but can be approximated effectively

$$P(x|h) = \frac{e^{\sum_i \lambda_i f_i(x,h)}}{\sum_{x'} e^{\sum_j \lambda_j f_j(x',h)}}$$

Mapping words into continuous space

- Map words into a continuous vector space
- to learn a distributed representation known as *word embedding*
- The goal is to use a vector space that keeps similarly behaving words near each other
- Words can be clustered by context, e.g. n-gram probabilities
 - ~ *word2vec* (Mikolov, 2013) is one widely used option
 - ~ Other embeddings to reflect various contextual properties
- Set of words can be represented by a sum of the vectors
- N-gram can be represented by a sequence of vectors

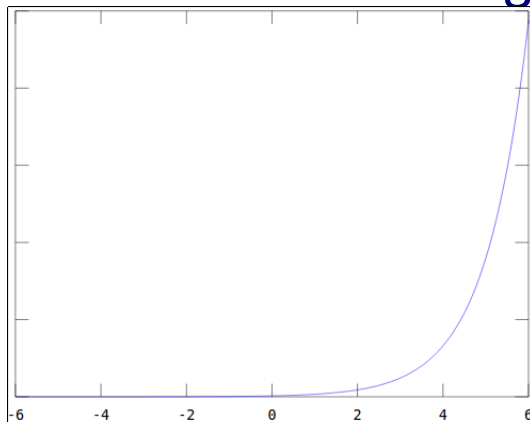


Continuous space LMs

- Alleviates the data sparsity problem by representing words in a distributed way
- Various algorithms can be used to learn the most efficient and discriminative representations and classifiers
- The most popular family of algorithm is called (Deep) **Neural Networks (NN)**
 - ~ can learn very complex functions by combining simple computation units in a hierarchy of non-linear layers
 - ~ Fast in action, but training takes a lot of time and labeled training data
- Can be seen as a non-linear multilayer generalization of the maximum entropy model

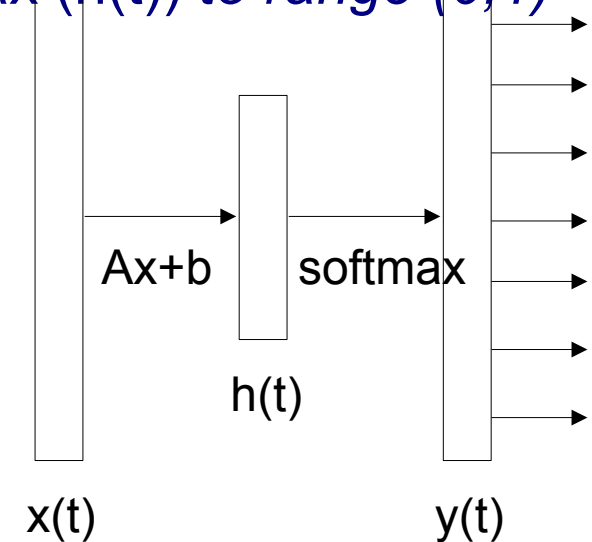
A simple bigram NN LM

- Outputs the *probability of next word* $y(t)$ given the previous word $x(t)$
- **Input layer** maps the previous word as a vector $x(t)$
- **Hidden layer** has a linear transform $h(t) = Ax(t) + b$ to compute a representation of *linear distributional features*
- **Output layer** maps the values by $y(t) = \text{softmax}(h(t))$ to range $(0, 1)$ that add up to 1
- Resembles a bigram Maximum entropy LM



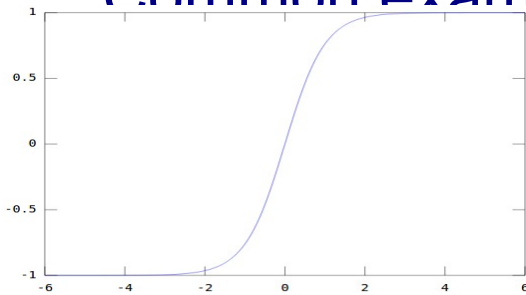
Softmax:

$$\sigma(\mathbf{z})_j = \frac{e^{z_j}}{\sum_{k=1}^K e^{z_k}} \quad \text{for } j = 1, \dots, K.$$

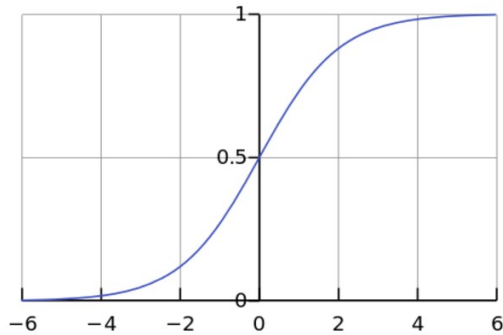


A non-linear bigram NN LM

- The only difference to the simple NN LM is that the hidden layer $h(t)$ now includes a non-linear function $h(t) = U(Ax(t) + b)$
- Can learn more complex feature representations
- Common examples of non-linear functions U :

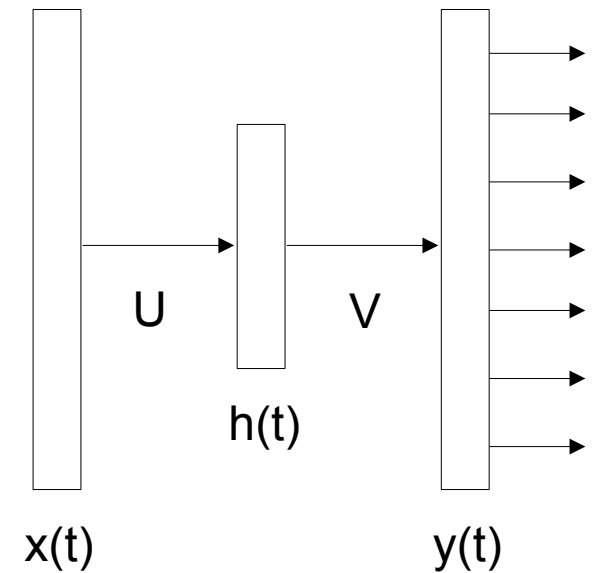


$$U(t) = \tanh(t)$$



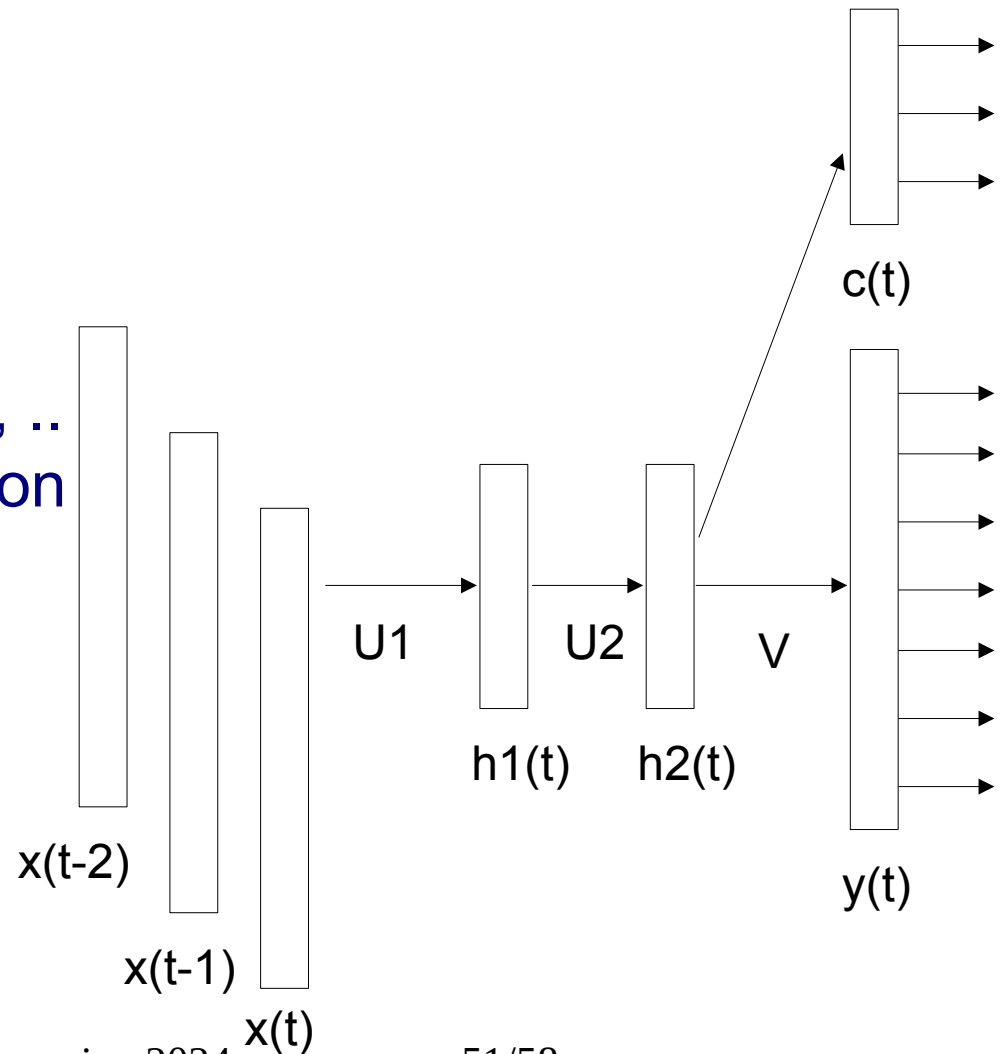
Sigmoid

$$U(t) = \frac{1}{1 + e^{-t}}$$



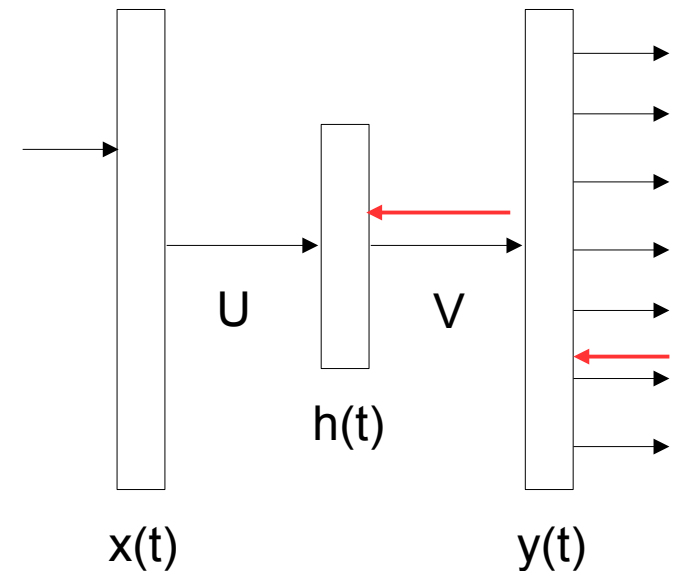
Common NN LM extensions

- **Input layer** is expanded over several previous words $x(t-1)$, $x(t-2)$, .. to learn richer representations
- Deep neural networks have several **hidden layers** $h_1, h_2, ..$ to learn to represent information at several hierarchical levels
- Can be scaled to a very large vocabulary by training also a **class-based output layer** $c(t)$



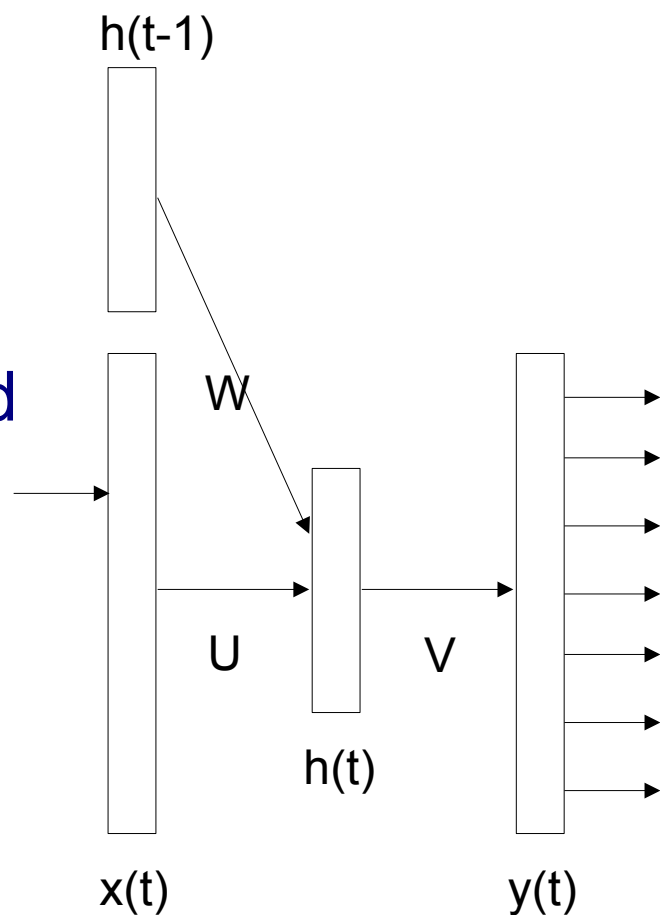
NN LM training

- Supervised training minimizes the **output errors** by training the weights for V by *stochastic gradient descend*
- Propagate the output **error to hidden layer** to train the weights for U
- In practice, a deep NN will require more complex training procedures, since the gradients *vanish* quickly



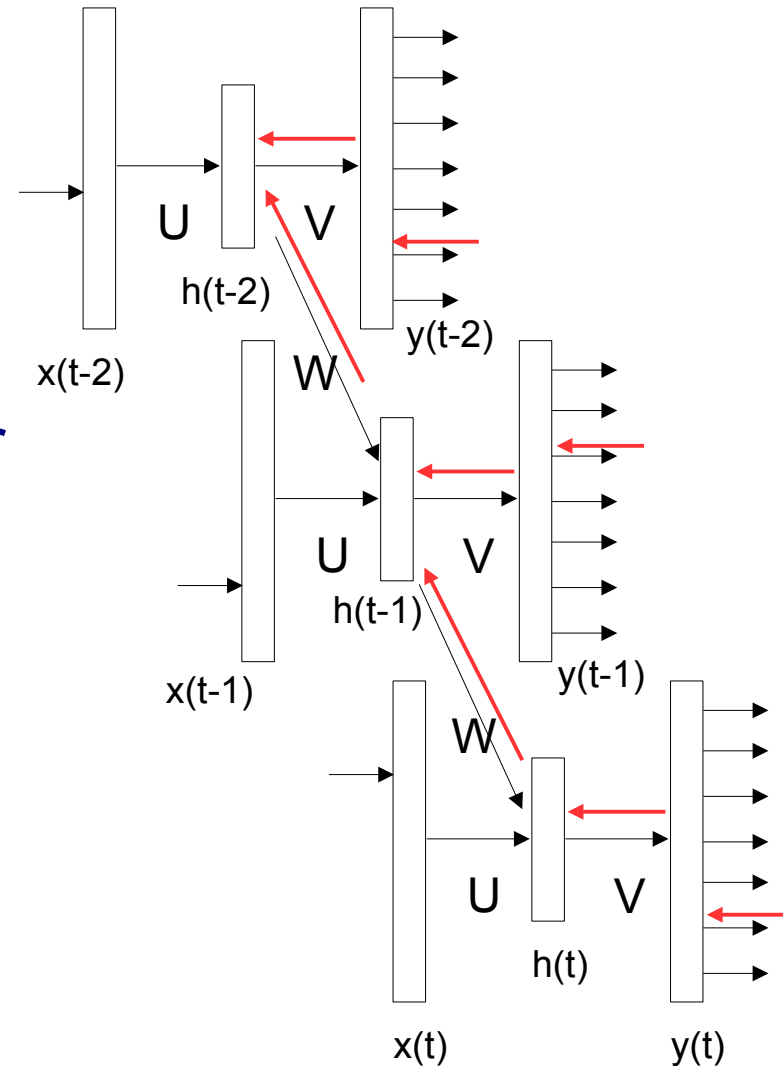
Recurrent Neural Network (RNN) LM

- **Looks like** a bigram NNLM
- **But**, takes an additional input from the hidden layer of the *previous time step*
- Hidden layer becomes a compressed representation of the word history
- Can learn to represent unlimited memory, in theory



RNN LM training

- Minimizes the output error by training the weights by *stochastic gradient descent*
- Propagates the output error to all *layers and time steps* (called *backpropagation through time*) to train the hidden layer
- **Looks now like** a very deep neural network with shared weights U and W



Feedback

Go to **MyCourses > Lectures > Feedback for Lecture 2** and fill in the form.

Feedback from last week:

+ Captions going on with the teacher's speaking worked surprisingly well!

+ The group discussion was interesting and insightful

+ Nice to finally have a "normal" course and to see people in real life

- I found it difficult to hear from the back rows, please use mic

- The speed was too slow

- Need a break in the middle

Thanks for all the valuable feedback!

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