Mathematical Methods for Economics Research: Optimization
Spring 2024

Optimization lies at the heart of economic modelling and analysis. A consumer's consumption and savings decisions, a firm's production decisions, a government's design of a tax scheme; almost all models of economic decision making involve optimization. In this course, we'll develop some of the fundamental tools of non-linear optimization and explore their applications to the analysis of models in both micro and macroeconomics. This course places special emphasis on developing techniques for optimization in dynamic settings and presenting the tools that will be used in the Research track MSc coursework.

Instructor
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Teaching Assistant
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Prerequisites:
This course is intended for third year bachelor's students who have completed the second year coursework; Intermediate Microeconomics, Intermediate Macroeconomics and Mathematics for Economists. A second year student can take this course simultaneously with Intermediate Macroeconomics 2.

Familiarity with the material from "Mathematics for Economists" is required -- in particular multivariate calculus, matrix algebra, and optimization (e.g. first and second order conditions, the extreme value theorem, Lagrange multipliers, convex optimization…). Some familiarity with probability at the level needed for Econometrics I and II is encouraged but not necessary.

Schedule:

We meet Tuesday, Thursday from 2:15-4pm, and Friday from 12:15-2.

Assessment:

The course grade consists of two components. There will be 4 problem sets and a final exam. The Problem Sets count for 40% of the grade, the exam counts for 60%.

Textbook:

The recommended textbook is:
- A.K. Dixit "Optimization in Economic Theory (Second Edition)" (Oxford University Press)

The textbook for math for economists
- Simon and Bloom "Mathematics for Economists"

A more advanced resource on dynamic optimization:
- Stokey and Lucas "Recursive Methods in Economic Dynamics"

Outline of the Course

1. Constrained Optimization
    a. Equality Constraints (Lagrange Multipliers)
    b. Inequality Constraints (Karush-Kuhn-Tucker Conditions)
    c. Economic Application: The Consumer Problem
2. Concave Analysis
    a. Concave functions
    b. Quasi-concave functions
    c. Separating Hyperplanes
    d. Economic Application: Linear Production
    e. Economic Application: Risk and Portfolio Choice
3. The Value Function
    a. Continuity and other Properties of the Value Function
    b. The Envelope Theorem
    c. Economic Applications: Some Comparative Statics
    d. Economic Application: Duality in the Consumer Problem
4. Dynamic Optimization: The Principle of Optimality
    a. Some Analysis
    b. The Principle of Optimality
    c. Bellman Equations
    d. Dynamic Optimization with Uncertainty (in Discrete Time)
    e. The HJB equation (without Uncertainty)
    f. Economic Application: Search
    g. Economic Application (time permitting): Bandit Problems and the Gittins Index
5. Dynamic Optimization: Pontryagin's Maximum Principle
    a. Hamiltonians
    b. A Discrete Time Maximum Principle
    c. A Continuous Time Maximum Principle
    d. Phase Diagrams
    e. Economic Application: Consumption/Savings
    f. Economic Application: Growth
    g. Economic Application (time permitting): Independent Variables that aren’t Time