ECON-L1350 - Empirical Industrial Organization PhD I: Static Models

Lecture 5

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About today's lecture

- Today's lecture is on modeling the supply side. We discuss
  1. A simple linear demand constant MC monopoly model to get going
  2. conduct parameter estimation within that model
  3. modeling and estimation of supply with single product firms in the discrete choice framework
  4. modeling and estimation of supply with multiproduct firms

Why model the supply side?

- Say you are interested in merger control and want to conduct a merger analysis: Then need estimates of marginal cost.
- You may need cost side instruments for demand estimation; then need to understand supply to find those.
- As will become clear, even if no direct interest in the supply side, it helps in identification of demand parameters.
- Cost of using supply side: need a behavioral assumption. This may bias also demand parameter estimates.

What determines supply?

- Supply depends on
  1. demand (elasticity)
  2. competition
  3. production technology (= marginal cost)
- Supply is the decision of the firm(s).

1. Linear monopoly model

- Demand (Q: how does one get linear demand from utility maximization?)

$$Q = a_0 + a_1 z - bP + e \tag{1}$$

- Q, P, z are data, a_0, a_1, b are unknown parameters and e is an unobservable.
- Marginal cost

$$c = c0 + c_1 w + v. \tag{2}$$

- Unknown cost parameters c_0, c_1, observable cost shifter (=data) w and unobservable cost component v. Let us assume $cov(e, v) = 0$.
- Think of observing data from many markets t.

Linear monopoly model: equilibrium Q^* and P^*

- Solving for equilibrium quantity yields

$$Q^* = \frac{a_0}{2} + \frac{a_1}{2}z - \frac{bc_0}{2} - \frac{bc_1}{2}w - \frac{b}{2}v + \frac{1}{2}e \tag{3}$$

- and for equilibrium price

$$P^* = \frac{a_0}{2b} + \frac{a_1}{2b}z + \frac{c_0}{2} + \frac{c_1}{2}w + \frac{1}{2}v + \frac{1}{2b}e \tag{4}$$

- Notice how both Q^* and P^* are functions of observables $\{z, w\}$ and unobservables $\{e, v\}$.

Linear monopoly model: demand

$$Q = a_0 + a_1 z - bP + e$$

- Q, P, z are data, a_0, a_1, b are unknown parameters and e is an unobservable.
- Endogeneity problem: $\mathbb{E}[P|e] \neq 0$ (recall equation for P^*).
- Estimation of demand: IV with w as instrument.
- Identifying assumption $\mathbb{E}[e|w] = 0$.

Linear monopoly model: supply

- Useful to recall inverse demand function:

$$P = \frac{a_0 + a_1 z}{b} - \frac{Q}{b} + \frac{e}{b}$$

- whence

$$P + \frac{Q}{b} = \frac{a_0 + a_1 z}{b} + \frac{e}{b}$$

- Supply decision given by the monopolist's FOC (after division by $-b$):

$$\begin{aligned} \frac{\partial \pi}{\partial P} &= \underbrace{-\frac{a_0 + a_1 z}{b} + 2P - \frac{e}{b}}_{MR} - \underbrace{c}_{MC} = 0 \\ &= \underbrace{P - \frac{Q}{b}}_{MR} - \underbrace{c}_{MC} = 0 \end{aligned} \tag{5}$$

Linear monopoly model: supply

- Reorganize equation (5):

$$P = c_0 + c_1 w + \frac{1}{b} Q + v \tag{6}$$

- Endogeneity problem: $\mathbb{E}[Q|v] \neq 0$ (recall equation for Q^*).
- Estimation by IV with z as instrument.
- Identifying assumption $\mathbb{E}[v|z] = 0$.

Linear monopoly model: supply

- Notice that we could identify the slope of the demand function b both from
  1. the demand equation (eqn. (1)) and
  2. the supply equation (eq. (6)).

- This is due to our functional form assumptions. Check what happens if $c = c0 + c_1 w + c_2 Q + v$

- In any case, gain efficiency from estimating the equations jointly.

- Notice the role of the behavioral assumption (profit-maximizing monopolist): It helps in identification.

- Another way of saying the same thing: The model is over-identified. We return to this at the end of the lectures.

- This approach has been known at least since Rosse, 1970.

Linear monopoly model: known MC

- Assume we have a direct measure of c, as in e.g. Genesove and Mullin, 1998.
- Notice that then, after having estimated demand, we already know all the structural parameters (a_0, a_1, b, c).
- $\rightarrow$ No need for estimating the supply.
- **But this is so only if we believe our behavioral assumption (=FOC) is correct**.
- $\rightarrow$ Could use the FOC to estimate a further parameter = conduct.

Digression into estimating conduct: homogenous goods case

- Profit function

$$\pi = Q(P - c)$$

.

- Introduce conduct parameter θ and write the FOC as

$$\frac{\partial \pi}{\partial P} = \frac{1}{\theta}\frac{\partial Q}{\partial P}(P - c) + Q = 0 \qquad (7)$$

- Note on notation: use θ here for the conduct parameter as that was the norm in the relevant literature.
- In the discrete choice demand literature, θ plays another role.

Estimating conduct

- Reorganize (divide by $(1/\theta)\partial Q/\partial P$) to get

$$\frac{\partial \pi}{\partial P} = (P - c) + \theta \frac{\partial P}{\partial Q} Q = 0 \tag{8}$$

- Equation (8) is the famous behavioral equation in **N**ew **I**ndustrial **O**rganization (NEIO; see Bresnahan, 1989).
- A more straightforward way of arriving at the same place is to maximize w.r.t Q (or q_j, think Cournot).

Estimating conduct with cost information

- Reorganize equation (8) to get

$$\theta = \frac{\partial Q/Q}{\partial P/P}\frac{P-c}{P} = \epsilon\frac{P-c}{P} \tag{9}$$

- where ϵ = price elasticity of demand.
- Note:
  1. $\theta = 1$ = monopoly.
  2. $\theta = 0$ = perfect competition.
  3. $\theta = 1/N$ = symmetric Cournot (N = number of firms).
- If you have cost data, this is what you can do (see Genesove and Mullin, 1998).

Estimation of conduct

- Conclusion: with information on cost + demand estimates you can infer conduct.
- Note #1: without cost information, generally need more (”rotations of demand”, see Bresnahan, 1989). We do not go into detail here.
  - Note though that with the above functional form assumptions, the conduct parameter θ is identified. This is not generally the case.
- Note #2: lengthy tradition of estimating conduct that got into disrepute in the late 1990s, especially after Corts, 1999.
  - Key argument: most values of θ not consistent with theory.
- Modern approach: As in Nevo, 2001: test which conduct assumption(s) in line with data.

Estimation of conduct

- Berry and Haile, 2018 discuss how to approach questions related to firm conduct in discrete choice models.
- Key idea: different assumptions of conduct may be falsified by the data.
- In our context, we could e.g. test $\theta = 1$ using equation (9).
- Alternatively, if in different markets with same P, MC we find different ϵ, then conduct not the same in those two markets.

2. Discrete choice demand

- To model supply, need to fix the demand side first.
- Utility specification: $u_{ijt} = x_{jt}\beta_{it} - \alpha p_{jt} + \xi_{jt} + e_{ijt}$
  - exogenous product characteristics x_{jt} (uncorrelated with ξ_{jt})
  - endogenous price p_{jt}
    - firms know ξ_{jt} when setting prices.
    - each price depends on $\xi_t = (\xi_{1t}, ..., \xi_{Jt})$.
- For simplicity, let's mostly assume simple Logit.
- This makes essentially no difference in terms of modeling supply.

Demand

- If M_t is a measure of the total number of potential consumers in market t, the total demand for product j is in market t:

$$q_{jt} = M_t \times s_j(\delta_t, x_t, \theta_2) \tag{10}$$

- and for the outside good:

$$q_{0t} = M_t - \sum_{j=1}^{J} M_t \times s_j(\delta_t, x_t, \theta_2) \tag{11}$$

3. Single product firms

- Profit for firm j (=firm owning product j) is given by (dropping market index t):

$$\pi_j = q_j(p_j - MC_j) = Ms_j(p_j - MC_j) \tag{12}$$

- MC_j = marginal cost of firm j.
- Assuming Bertrand-Nash behavior FOC is:

$$\frac{\partial \pi_j}{\partial p_j} = s_j + (p_j - MC_j)\frac{\partial s_j}{\partial p_j} = 0 \tag{13}$$

- Plugging in our (simple) Logit demand we get:

$$\frac{\partial \pi_j}{\partial p_j} = s_j + (p_j - MC_j)[-\alpha s_j(1 - s_j)] = 0 \tag{14}$$

Price-cost margin

- We can derive the **markup** (which some call the **P**rice-**C**ost **M**argin (PCM)) of firm j as

$$p_j - MC_j = \frac{\partial p_j}{\partial s_j} s_j = -\frac{1}{-\alpha s_j(1-s_j)} s_j = \frac{1}{\alpha(1-s_j)} = 0 \tag{15}$$

- Notice how the **Lerner**-index becomes

$$\frac{p_j - MC_j}{p_j} = \frac{\partial p_j}{\partial s_j}\frac{s_j}{p_j} = \frac{1}{\epsilon_j} \tag{16}$$

- Note: the literature is inconsistent in its use of PCM and markup, so need to be careful. Both sometimes mean (p-MC)/p.

Price-cost margin

- Notice how the markup

$$p_j - MC_j = \frac{1}{\alpha(1 - s_j)}$$

  is only a function of the price elasticity and the market share of good j.

- As discussed in lectures 1 and 2, this is one of the unappealing consequences of the Logit model: Two products with same market shares are forced to have same markups.

Modeling marginal cost

- To progress to estimation of MC_j, we need to specify the production technology.
- A couple of points to consider:
  1. Do you want to restrict $MC > 0$? If so, use log-specification.
  2. Do you want to allow for economies of scale? If so, make MC a function of quantity.
  3. What would be natural cost shifters in your application?
- Let's follow Conlon and Gortmaker, 2020 and assume that

$$MC_j = x_j\gamma_1 + w_j\gamma_2 + \omega_j \tag{17}$$

- where x_j are product characteristics (that enter demand) and w_j are cost shifters that by assumption do not enter demand.

Estimation of marginal cost

- Let us assume you have estimated demand.
- $\rightarrow$ you know all the demand parameters.
- We use the FOC as

$$p_j - (p_j - MC_j) = p_j - \frac{1}{\alpha(1 - s_j)} = x_j\gamma_1 + w_j\gamma_2 + \omega_j \tag{18}$$

- Notice that the LHS of equation (18) is observable to you at this stage. Alternatively you can form a moment condition by rearranging so that only ω_j on the LHS.
- Question: How would the above equation change if you assumed the following MC function?

$$\ln MC_j = x_j\gamma_1 + \ln w_j\gamma_2 + \omega_j$$

Estimation of marginal cost

- More generally, let's write

$$\boldsymbol{p} - \boldsymbol{\Delta}(\boldsymbol{p})^{-1}\boldsymbol{s} = \boldsymbol{x}\gamma_1 + \boldsymbol{w}\gamma_2 + \boldsymbol{\omega} \tag{19}$$

- where $\boldsymbol{\Delta}(\boldsymbol{p})$ = the matrix of price derivatives of demand $\partial \boldsymbol{s}/\partial \boldsymbol{p}$.
- In other words, you can estimate the marginal costs of all firms' products in one go.
- Why include x_j into the *MC* function? Allows for variation in *MC* as a function of quality.

4. Multiproduct firms

- With multiproduct firms, firm f's profit function is written as

$$\pi_f = \sum_{k \in \mathcal{J}_f} (p_k - MC_k) - F \tag{20}$$

where $\mathcal{J}_f$ is the set of products owned by firm f that are available in the market and F is a fixed cost of production.

- Now firm f has $\mathcal{J}_f$ prices to consider.
- It takes the cross-price elasticities between **its own products** into account when deriving the FOC for a particular product j:

$$\frac{\partial \pi_f}{\partial p_j} = s_j + \sum_{k \in \mathcal{J}_f} (p_k - MC_k) \frac{\partial s_k}{\partial p_j} = 0 \tag{21}$$

Markup

- We can write now

$$p_j - MC_j = \left[\frac{\partial s_j}{\partial p_j}\right]^{-1} s_j + \sum_{k \in \mathcal{J}_f; k \neq j} \left[\frac{\partial s_j}{\partial p_j}\right]^{-1} \left[(p_k - MC_k)\frac{\partial s_k}{\partial p_j}\right] \tag{22}$$

- With substitutes $\partial s_k / \partial p_j > 0$ for $k \neq j$, and $P_k - MC_k > 0$.
- $\rightarrow PCM_j^{multi} > PCM_j^{single}$, meaning also that
- $p_j^{multi} > p_j^{single}$.

Markups and prices: Logit

- With Logit demand, the markup for product j is now given by

$$p_j - MC_j = \frac{1}{\alpha} + \sum_{k \in \mathcal{J}_f} (p_k - MC_k) s_k \tag{23}$$

- Sidenote: with Logit demand, a multiproduct firm sets the same markup for all products. This is not true for more general demand functions.

5. General setup for multiproduct firms

- We can break up the parameter space into three parts:
  1. θ_1: linear exogenous demand parameters with dimension K_1
  2. θ_2: parameters including price and random coefficients (endogenous / nonlinear) with dimension K_2
     - $\theta_2 = [\alpha, \widetilde{\theta}_2]$
  3. θ_3: linear exogenous supply parameters with dimension K_3
- $N = \sum_t \dim(\mathcal{J}_t)$ observations.

Supply side

- Consider the multi-product Bertrand FOCs:

$$\max_{p_{jt}:j\in\mathcal{J}_{ft}} \sum_{j\in\mathcal{J}_{ft}} s_{jt}\left(\boldsymbol{p}_t\right)\cdot\left(p_{jt}-c_{jt}\right)$$
$$s_{jt}\left(\boldsymbol{p}_t\right)+\sum_{k\in\mathcal{J}_{ft}} \frac{\partial s_{kt}}{\partial p_{jt}}\left(\boldsymbol{p}_t\right)\cdot\left(p_{kt}-c_{kt}\right)=0$$

Supply side

- It is helpful to define the multiproduct oligopoly ownership matrix $\mathcal{H}_t$ as having 1's if (j, k) have the same owner, and 0's otherwise. We can re-write the FOC in matrix form where $\odot$ denotes Hadamard product:

$$\Delta_t(\boldsymbol{p}_t) \equiv -\mathcal{H}_t \odot \frac{\partial \boldsymbol{s}_t}{\partial \boldsymbol{p}_t}(\boldsymbol{p}_t)$$

$$s_t(\boldsymbol{p}_t) = \Delta_t(\boldsymbol{p}_t) \cdot (\boldsymbol{p}_t - \boldsymbol{c}_t)$$

$$\underbrace{\Delta_t(\boldsymbol{p}_t)^{-1} \boldsymbol{s}_t(\boldsymbol{p}_t)}_{\eta_t(\boldsymbol{p}_t, \boldsymbol{s}_t, \theta_2)} = \boldsymbol{p}_t - \boldsymbol{c}_t$$

Hadamard product = element-by-element multiplication. Notice that $\mathcal{H}_t$ is an identity matrix in the case of single-product firms.

Recovering marginal costs

- Recover implied markups/ marginal costs, and assume a functional form for $mc_{jt}(x_{jt}, w_{jt})$.

$$\widehat{\mathbf{mc_t}}(\theta_2) = \mathbf{p_t} - \boldsymbol{\eta_t}(\mathbf{p_t}, \mathbf{s_t}, \theta_2)$$
$$f(mc_{jt}) = h_s(x_{jt}, w_{jt}, \theta_3) + \omega_{jt}$$

We can solve these for ω_{jt}:

$$\omega_{jt} = f(\mathbf{p_t} - \boldsymbol{\eta_t}(\mathbf{p}, \mathbf{s}, \theta_2)) - h_s(x_{jt}, w_{jt}, \theta_3)$$

- $f(\cdot)$ is usually $\log(\cdot)$ or identity, depending on the assumptions about the MC function.
- $h_s(x_{jt}, w_{jt}, \theta_3) = [x_{jt},\ w_{jt}]\gamma$ is usually linear.
- Use this to form additional moments: $E[\omega'_{jt} Z^s_{jt}] = 0$

Additional details Conlon and Gortmaker, 2020

- If everything is linear:

$$y_{jt}^{D} := \widehat{\delta}_{jt}(\theta_2) + \alpha p_{jt} = (x_{jt}\ v_{jt})'\beta + \xi_t =: x_{jt}^{D\prime}\beta + \xi_{jt}$$
$$y_{jt}^{S} := f(\widehat{mc}_{jt}(\theta_2)) \quad = (x_{jt}\ w_{jt})'\gamma + \omega_t =: x_{jt}^{S\prime}\gamma + \omega_{jt}$$

- Stacking the system across observations yields:

$$\underbrace{\begin{bmatrix} y_D \\ y_S \end{bmatrix}}_{2N\times 1} = \underbrace{\begin{bmatrix} X_D & 0 \\ 0 & X_S \end{bmatrix}}_{2N\times(K_1+K_3)} \underbrace{\begin{bmatrix} \beta \\ \gamma \end{bmatrix}}_{(K_1+K_3)\times 1} + \underbrace{\begin{bmatrix} \xi \\ \omega \end{bmatrix}}_{2N\times 1}$$

- Note: we cannot perform independent regressions unless we are willing to assume that $Cov(\xi_{jt}, \omega_{jt}) = 0$.

Simultaneous supply and demand (Conlon and Gortmaker, 2020)

a. For each market t: solve $\mathcal{S}_{jt} = s_{jt}(\delta_{\cdot t}, \theta_2)$ for $\widehat{\delta_{\cdot t}}(\theta_2)$.
b. For each market t: use $\widehat{\delta_{\cdot t}}(\theta_2)$ to construct $\eta_{\cdot t}(\mathbf{q_t}, \mathbf{p_t}, \widehat{\delta_{\cdot t}}(\theta_2), \theta_2)$
c. For each market t: Recover $\widehat{mc}_{jt}(\widehat{\delta_{\cdot t}}(\theta_2), \theta_2) = p_{jt} - \eta_{jt}(\widehat{\delta_{\cdot t}}(\theta_2), \theta_2)$
d. Stack up $\widehat{\delta_{\cdot t}}(\theta_2)$ and $\widehat{mc}_{jt}(\widehat{\delta_{\cdot t}}(\theta_2), \theta_2)$ and use linear IV-GMM to recover $[\widehat{\theta_1}(\theta_2), \widehat{\theta_3}(\theta_2)]$ following the recipe on previous slide
e. Construct the residuals:

$$\widehat{\xi_{jt}}(\theta_2) = \widehat{\delta_{jt}}(\theta_2) - [x_{jt}\ v_{jt}]\,\widehat{\beta}(\theta_2) + \alpha p_{jt}$$
$$\widehat{\omega_{jt}}(\theta_2) = f(\widehat{mc}_{jt}(\theta_2)) - [x_{jt}\ w_{jt}]\,\widehat{\gamma}(\theta_2)$$

f. Construct sample moments

$$g_n^D(\theta_2) = \frac{1}{N}\sum_{jt} Z_{jt}^{D\prime}\widehat{\xi_{jt}}(\theta_2)$$
$$g_n^S(\theta_2) = \frac{1}{N}\sum_{jt} Z_{jt}^{S\prime}\widehat{\omega_{jt}}(\theta_2)$$

g. Construct GMM objective $Q_n(\theta_2) = \begin{bmatrix} g_n^d(\theta_2) \\ g_n^s(\theta_2) \end{bmatrix}' W \begin{bmatrix} g_n^d(\theta_2) \\ g_n^s(\theta_2) \end{bmatrix}$

What's the point (Conlon and Gortmaker, 2020)

- A well-specified supply side can make it easier to estimate θ_2 parameters (price in particular).
- Imposing the supply side only helps if we have information about the marginal costs / production function that we would like to impose.
- May want to enforce some economic constraints: ($mc_{jt} > 0$ is a good one).

What's the point (Conlon and Gortmaker, 2020)

- Table 5 of Conlon and Gortmaker, 2020 reports Monte Carlo results for using different instruments and excluding (upper panel) and including (lower panel) the supply side:

What about mis-specification?

- Figure 2 of Conlon and Gortmaker, 2020 is another way to show the effect of including the supply side.
- Notice the big difference between the upper (well-specified supply) and lower (mis-specified supply) figure.

What about mis-specification?

Final notes on the supply side

- Assuming the wrong conduct ($=\mathcal{H}_t$) can lead to mis-specification.
- This opens also a way of testing for whether assumed conduct is correct as including it leads to an over-identified model (recall our discussion of the linear monopoly model and identification of the slope parameter b).
- Procedure:
  1. Estimate demand with supply side using your assumed conduct to get $\hat{\theta}$.
  2. Estimate demand without supply side to get $\hat{\theta}_D$.
  3. Calculate

$$LR = N\left[g(\hat{\theta})'Wg(\hat{\theta}) - g_D(\hat{\theta}_D)'W_D g_D(\hat{\theta})_D)\right] \sim \chi^2_{K-K_x}$$

- PyBLP supports this test.