

Chapter 4

Exercises covered : 1 a), b), c), 3, 7 (case $\mathbb{Q}(\sqrt{-5})$), 8, 11

Hints :

1. • Recall $\mathbb{Z}[i]$ is a unique factorization domain,

- $N(a+ib) = a^2 + b^2$

- Assume $a+bi = \alpha\beta$ for some $\alpha, \beta \in \mathbb{Z}[i]$ and

take norms

OR recall that if the norm is a rational prime, then the element is irreducible.

7. • Show that 4 has two different factorizations into irreducibles (which are not associates).

- Recall $\mathcal{O}_K = \mathbb{Z}\left[\frac{1+\sqrt{-5}}{2}\right]$ (since $-5 \equiv 1 \pmod{4}$).

11. • Recall definition of a prime. $\sqrt{-5}$ is a prime in $\mathbb{Z}[\sqrt{-5}]$ if $\sqrt{-5} \mid xy$ for some $x, y \in \mathbb{Z}[\sqrt{-5}] \Rightarrow \sqrt{-5} \mid x$ or $\sqrt{-5} \mid y$.