Aalto University - School of Science Department of Mathematics and System Analysis

$\operatorname{MS-A0311}$ - Differential and integral calculus 3

Rogovin/Brummet

Course exam and general exam 17.4.2024 klo 9.00–12.00.

Course exam: the five best problems will be included in the evaluation. General exam: Do all six problems.

All participants of the lecture course in the period IV/2024 can do all six problems. The evaluation is calculated with that option that gives the highest score: either "five best problems + exercise points" or "6 problems only".

No calculators or notes of any kind are allowed.

Each problem is worth 6 points.

1.

2.

3.

J.

4.

5.

6.

Formulas without explanation:

- $x^2/a^2 + y^2/b^2 = 1 \longleftrightarrow x = a \cos t, \ y = b \sin t$
- $\nabla \cdot (\nabla f) = \Delta f, \, \nabla \times \nabla f = \overline{0}, \, \nabla \cdot (\nabla \times \mathbf{F}) = 0$
- $\nabla \cdot (f\mathbf{F}) = \nabla f \cdot \mathbf{F} + f(\nabla \cdot \mathbf{F}), \ \nabla \times (f\mathbf{F}) = \nabla f \times \mathbf{F} + f(\nabla \times \mathbf{F})$
- $\oint_{\partial D} F_1 dx + F_2 dy = \iint_D \left(\frac{\partial F_2}{\partial x} \frac{\partial F_1}{\partial y}\right) dA$
- In the following formulas $\mathbf{n} = \text{unit normal.}$

•
$$\iiint_D \nabla \cdot \mathbf{F} \, dV = \iint_{\partial D} \mathbf{F} \cdot \mathbf{n} \, dS$$

•
$$\iint_{S} (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS = \oint_{\partial S} \mathbf{F} \cdot \mathbf{dr} = \oint_{\partial S} F_1 \, dx + F_2 \, dy + F_3 \, dz$$

- (r,φ,θ) : $x = r\sin(\varphi)\cos(\theta), \ y = r\sin(\varphi)\sin(\theta), \ z = r\cos(\varphi), \ dV = r^2\sin(\varphi) \ dr \ d\varphi \ d\theta$
- (ρ, θ, z) : $x = \rho \cos(\theta), y = \rho \sin(\theta), z = z, dV = \rho d\rho d\theta dz$
- $\sin(\pi/6) = \cos(\pi/3) = 1/2$, $\sin(\pi/3) = \cos(\pi/6) = \sqrt{3}/2$, $\sin 0 = \cos(\pi/2) = 0$, $\sin(\pi/2) = \cos 0 = 1$, $\sin \pi = 0$, $\cos \pi = -1$, $\sin^2 x + \cos^2 x = 1$, $\sin(2x) = 2\sin x \cos x$, $\cos(2x) = 2\cos^2 x 1 = 1 2\sin^2 x$.

Note 1: Answering the course feedback gives one course point!

Note 2: You can retake the course exam when there is a general exam at June. On that retake the exercise points of the course are valid. To retake the exam you have to enroll to the exam through Sisu.