Practical Quantum Computing

Week	**Tuesday** (3h)			**Wednesday** (3h)			**Deadlines**	
1. The Basics	Introduction	Gates	Circuit Identities	Qiskit	Cirq/Qualtran	Q&A		
	Programming Assignment 1: The basics of a quantum circuit simulator			**Programming Assignment 1:** The building blocks of a quantum circuit simulator				
2. Entanglement and its Applications	Teleportation	Superdense Coding	Quantum Key Distribution	PennyLane	Terminology of Projects	Q&A		
	Programming Assignment 2: The basics of a quantum circuit optimizer			**Programming Assignment 2:** The building blocks of a quantum circuit optimizer				
3. Computing	Phase Kickback and Toffoli	Distinguishing quantum states and The First Algorithms	Grover’s Algorithm	Invited TBA		Q&A		11 May 2024
4. Advanced Topics*	Arithmetic Circuits*	Fault-Tolerance*	QML*	Invited TBA	Crumble	Q&A	18 May 2024	

* not evaluated

Estimated Workload and ECTS points

Course (24h):

- 3h lecture x 4 weeks
- 3h q&a x 4 weeks

Programming (80h):

- first programming assignment 10h
- second programming assignment 20h
- project 50h

Independent study (30h)

24h + 80h + 30h = **134h → 5 ECTS**

Project list will be announced on 2nd May

Grading

The total number of achievable points: **100 points**

- Programming Assignment 1 – Quantum Circuit Simulator - **10 Points**
- Programming Assignment 2 – Quantum Circuit Optimizer - **20 Points**
- Project – **50 Points**
- Quiz (timed on MyCourses with tutorial questions, end of last week) – **20 Points**
- Feedback - **5 Points (bonus: add towards the maximum)**
  - each week there will be a feedback form - **1 point (4 weeks)**
  - final feedback at end of the course - **1 point**

Project list will be announced on 2nd May

Grading

Grade 0: 0 -19 points

Grade 1: 20 - 30 points

Grade 2: 31 - 40 points

Grade 3: 41 - 50 points

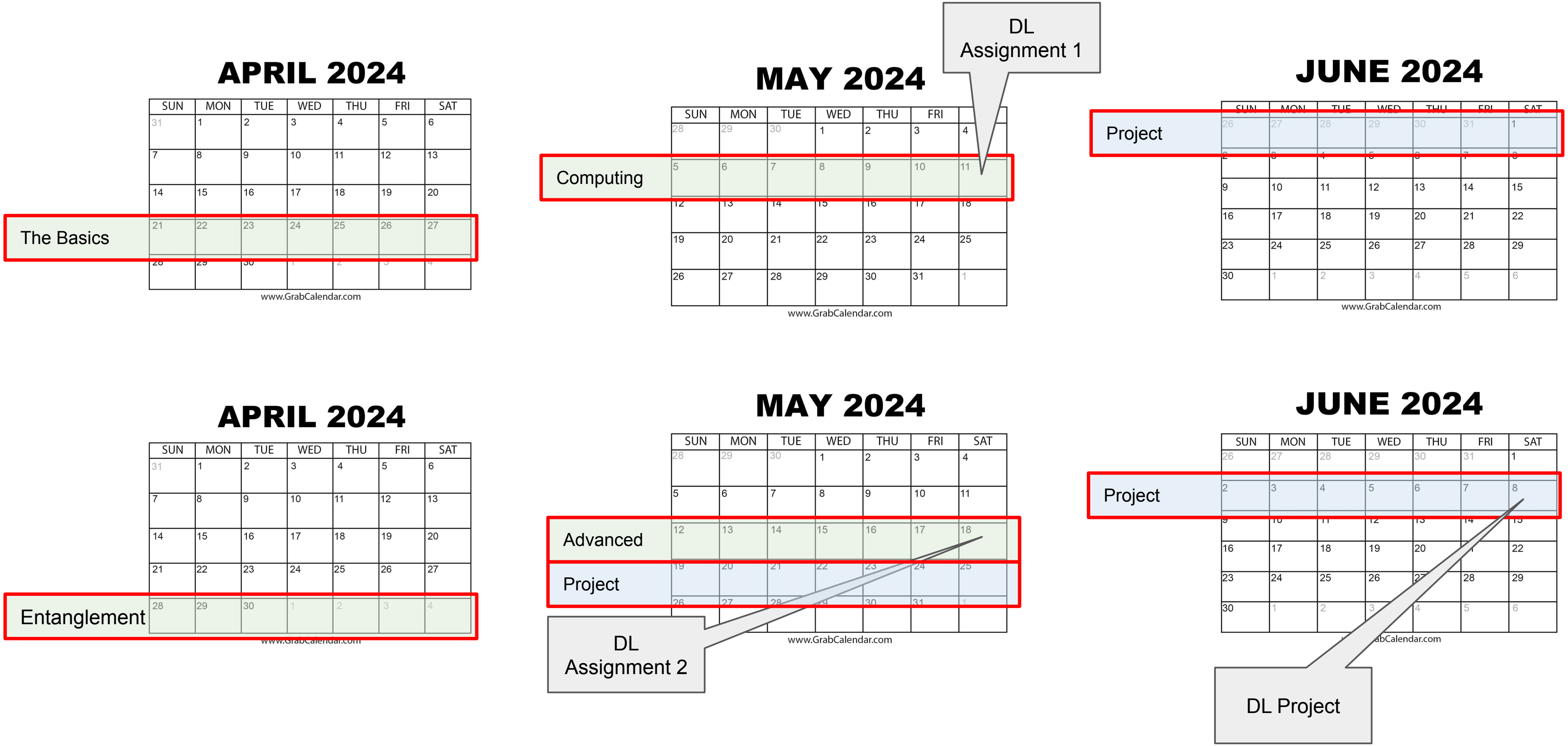
Grade 4: 51-75 points

Grade 5: 76 - 100 points

Examples:

- no assignment and no project but quiz is perfect
  -> 20 points -> grade 1
- only assignment 1 and nothing else
  -> 10 points -> grade 0
- only assignment 1 and half assignment 2 and the quiz
  -> approx. 40 points -> grade 2
- quiz, both assignments and almost than half of the project
  -> 74 points -> grade 5
- same situation like above and feedback
  -> grade 5

Project list will be announced on 2nd May


APRIL 2024
The Basics
MAY 2024
Computing
DL Assignment 1
JUNE 2024
Project
APRIL 2024
Entanglement
MAY 2024
Advanced
Project
DL Assignment 2
JUNE 2024
Project
DL Project
www.GrabCalendar.com

Programming Assignment 1 - Quantum Circuit Simulator

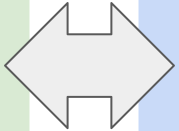
Theory

- The mathematics of quantum circuits (qubit states and quantum gates)
- The exponential dimensions of the states (complex vectors) used to represent a computation with multiple qubits

Practice

- Writing Python scripts to generate and operate on complex vectors and matrices required to simulate the quantum circuit using a classical computer
- Observing in practice the exponential time and space (seconds and bits) needed to simulate the quantum computation

Programming Assignment 2 - Quantum Circuit Optimizer



Theory

- Changing the structure of quantum circuits by applying local transformations (circuit identities) leaves the computation unchanged
- The width and depth of a quantum circuit
- The parallel execution of quantum gates

Practice

- Writing Python code for applying circuit identities for reducing:
  - depth of quantum circuit
  - number of quantum gates
- Benchmarking the execution time of the quantum circuit simulator with the optimized circuit

Practical Quantum Computing

Lecture 01
An Overview of the Course

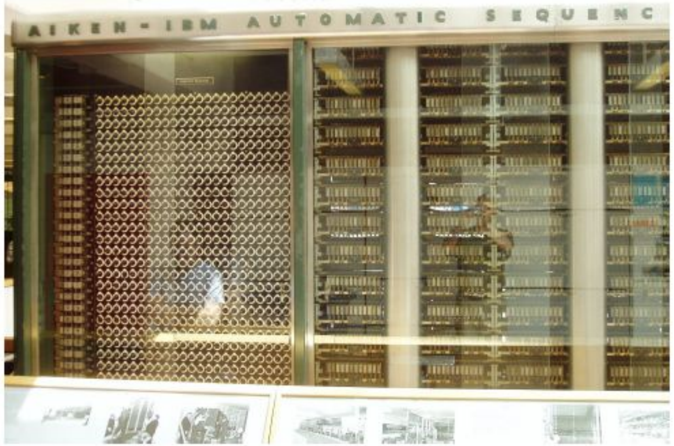
Learning goals - 01 Introduction (The Basics)

1. **Quantum software**
    1. what is it? - the definition
    2. why is it needed? - the motivation
    3. how is it working? - architecture and design
2. **Quantum circuits**
    1. components and structure
    2. faulty vs reliable circuits
    3. the cost of running reliable quantum circuits
3. **Quantum advantage over classical**
    1. what is quantum supremacy?
    2. why are quantum circuits hard to simulate classically?
4. **Roadmap for the rest of the course**

In the exercise session and programming assignment of this week

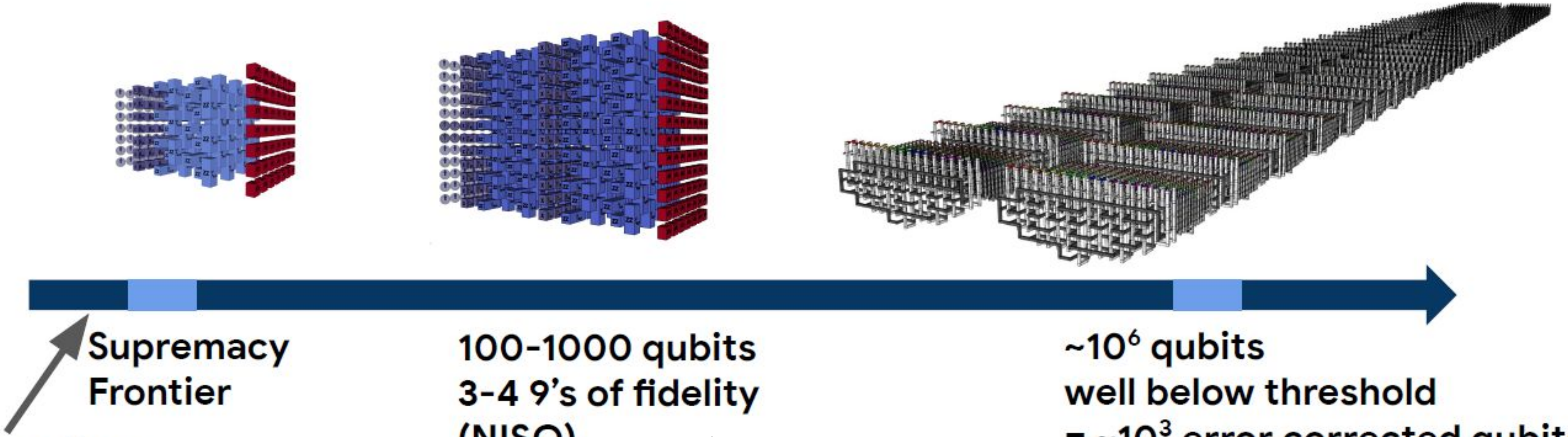
- basics of quantum circuit simulator
- build our own quantum circuit simulator

Looking at history



Harvard Mark 1

The NISQ Age Ended in December 2023



Supremacy
Frontier

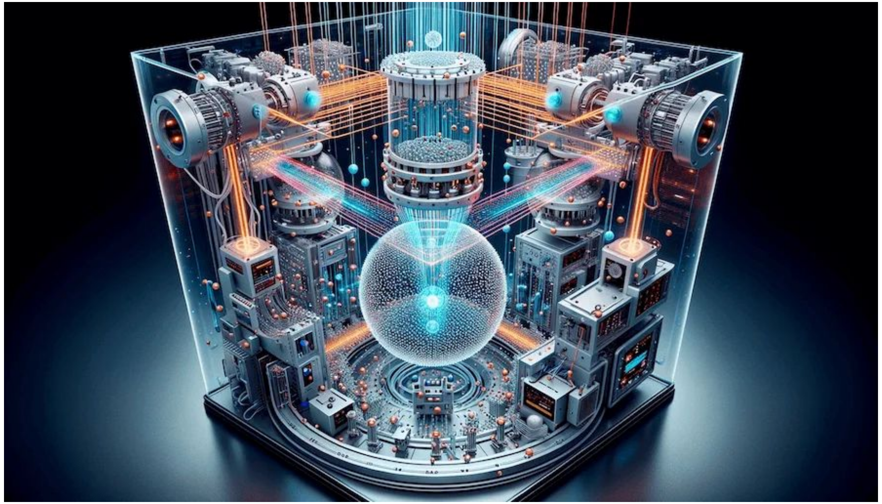
we are here

100-1000 qubits
3-4 9's of fidelity
(NISQ)

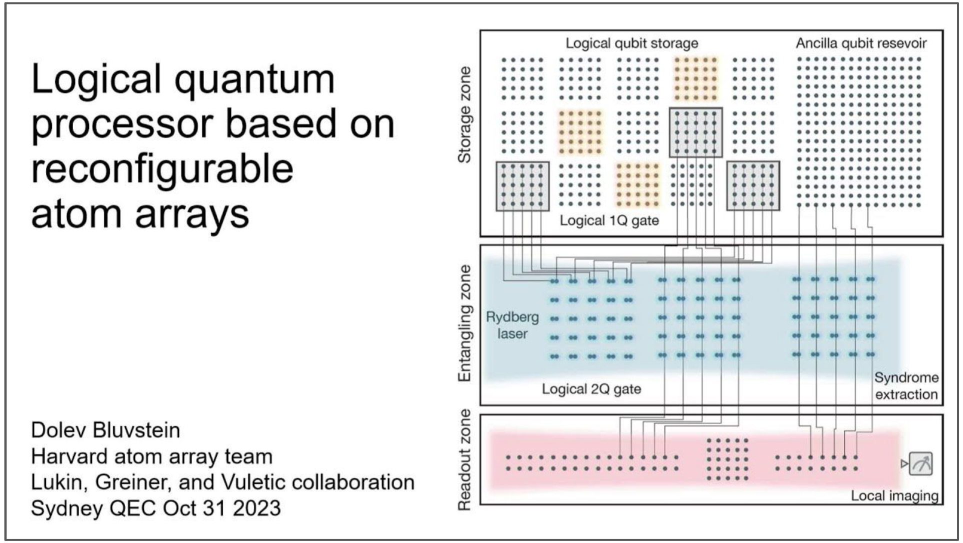
~10^6 qubits
well below threshold
= ~10^3 error corrected qubits

Are there impactful algorithms for noisy intermediate scale quantum (NISQ) processors?

The Early Fault-Tolerant QC Age began in December 2023

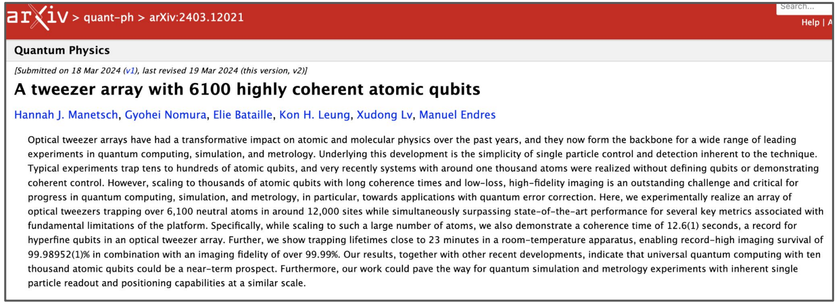


https://www.quera.com/blog-posts/key-advantages-of-neutral-atom-quantum-computer-architectures



Logical quantum processor based on reconfigurable atom arrays

Dolev Bluvstein
Harvard atom array team
Lukin, Greiner, and Vuletic collaboration
Sydney QEC Oct 31 2023



arXiv > quant-ph > arXiv:2403.12021

Quantum Physics

[Submitted on 18 Mar 2024 (v1), last revised 19 Mar 2024 (this version, v2)]

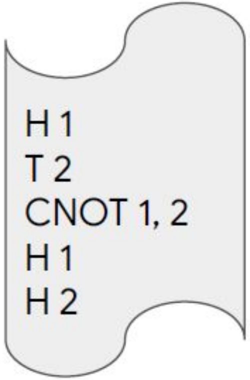
A tweezer array with 6100 highly coherent atomic qubits

Hannah J. Manetsch, Gyohei Nomura, Elie Bataille, Kon H. Leung, Xudong Lv, Manuel Endres

Optical tweezer arrays have had a transformative impact on atomic and molecular physics over the past years, and they now form the backbone for a wide range of leading experiments in quantum computing, simulation, and metrology. Underlying this development is the simplicity of single particle control and detection inherent to the technique. Typical experiments trap tens to hundreds of atomic qubits, and very recently systems with around one thousand atoms were realized without defining qubits or demonstrating coherent control. However, scaling to thousands of atomic qubits with long coherence times and low-loss, high-fidelity imaging is an outstanding challenge and critical for progress in quantum computing, simulation, and metrology, in particular, towards applications with quantum error correction. Here, we experimentally realize an array of optical tweezers trapping over 6,100 neutral atoms in around 12,000 sites while simultaneously surpassing state-of-the-art performance for several key metrics associated with fundamental limitations of the platform. Specifically, while scaling to such a large number of atoms, we also demonstrate a coherence time of 12.6(1) seconds, a record for hyperfine qubits in an optical tweezer array. Further, we show trapping lifetimes close to 23 minutes in a room-temperature apparatus, enabling record-high imaging survival of 99.98952(1)% in combination with an imaging fidelity of over 99.99%. Our results, together with other recent developments, indicate that universal quantum computing with ten thousand atomic qubits could be a near-term prospect. Furthermore, our work could pave the way for quantum simulation and metrology experiments with inherent single particle readout and positioning capabilities at a similar scale.

Quantum Software just changed in December 2023, too

Fidelity: 90



```
H 1
T 2
CNOT 1, 2
H 1
H 2
```

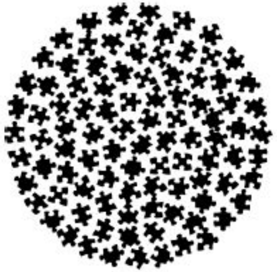
Write it on a piece of paper!

99

Useful to record Instructions.

Can still eyeball circuits.

99.9



At supremacy frontier.

Depth and gate minimization.

Simple modularity.

99.99

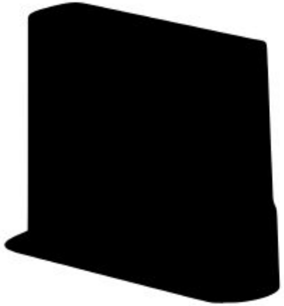
...

Complex modularity.

Automatic compiling.

Beginning of hardware independent abstractions.

99.9999999....

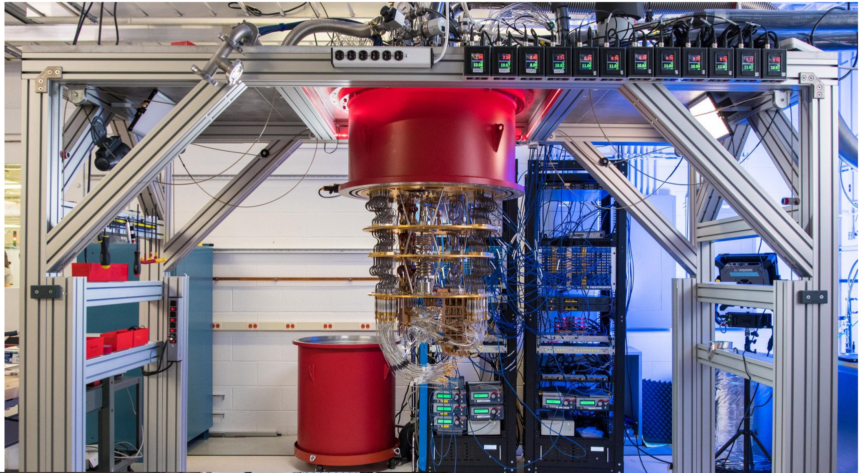


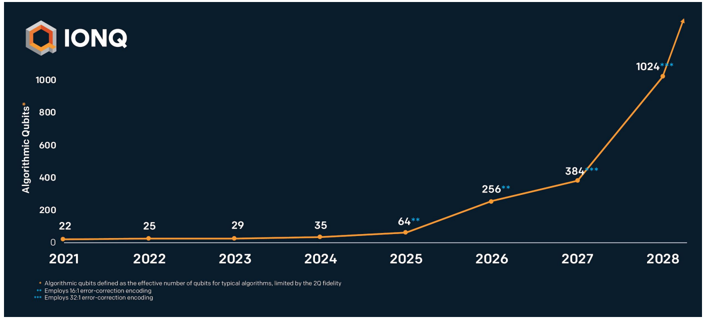
Architecture.

Operating systems.

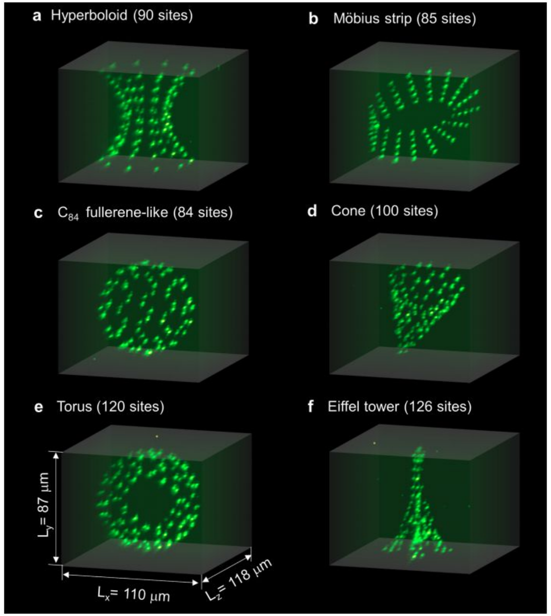
High level languages.

Quantum Computers

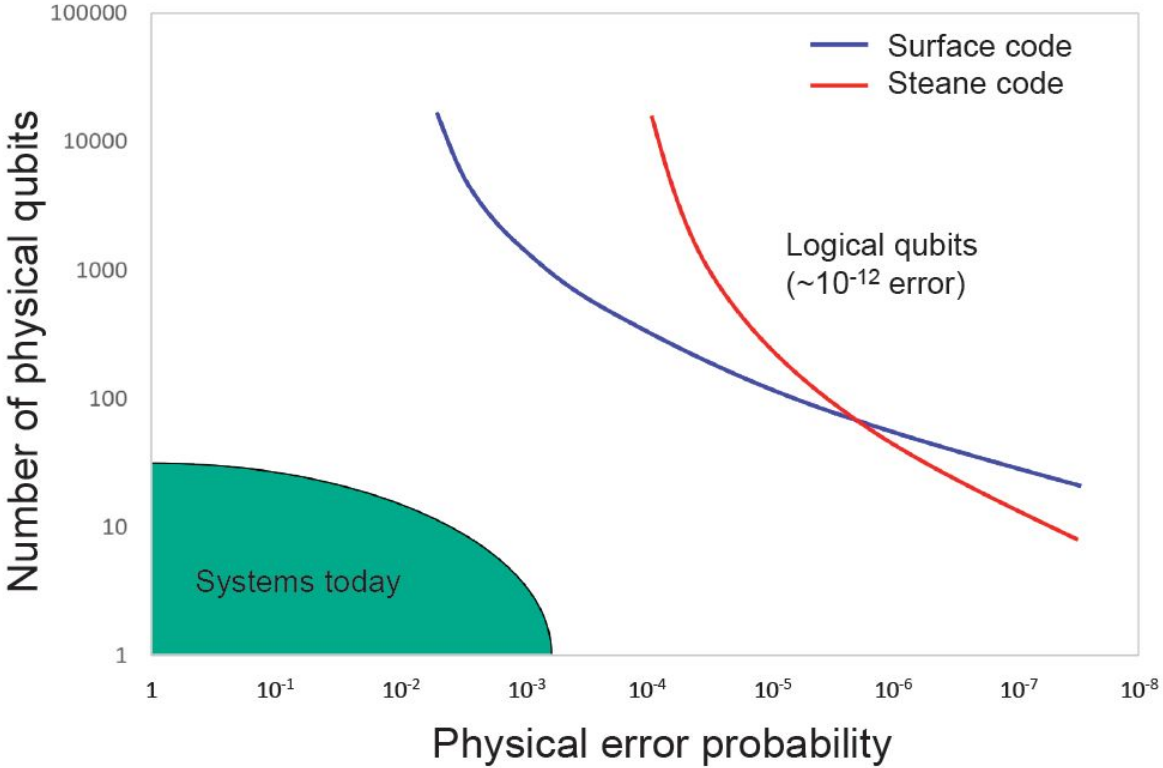




* Algorithmic qubits defined as the effective number of qubits for typical algorithms, limited by the 2Q fidelity
** Employs 16:1 error-correction encoding
*** Employs 32:1 error-correction encoding



Fault-tolerance and the million of qubits



National Academies of Sciences, Engineering, and Medicine 2019. Quantum Computing: Progress and Prospects. Washington, DC: The National Academies Press. https://doi.org/10.17226/25196.

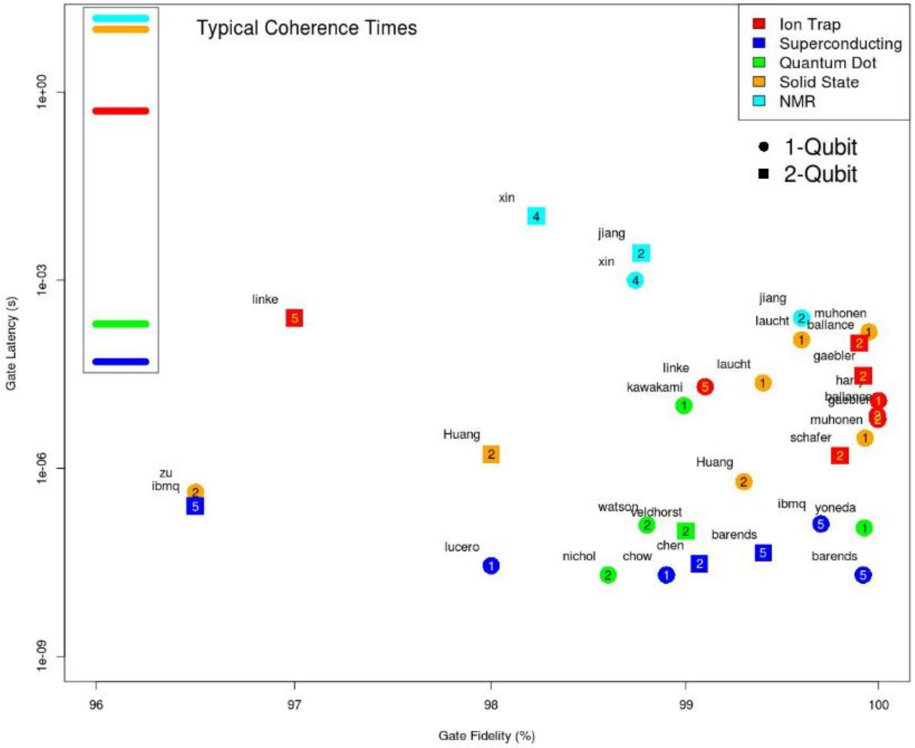
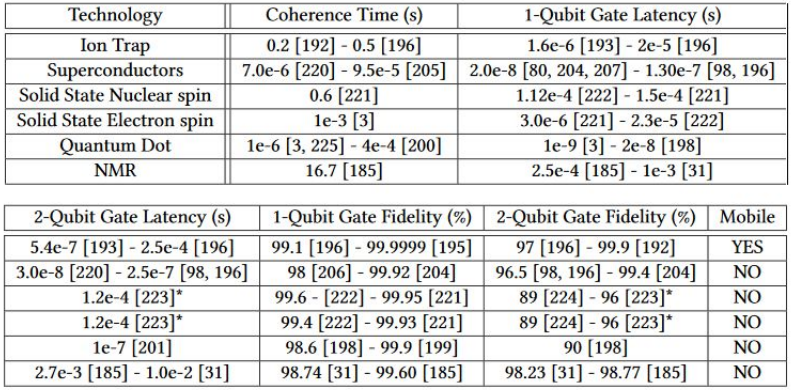


Fig. 1. Experimental latencies and fidelities of 1- and 2-qubit gates for different technologies. Approximate coherence times are shown in the inset for comparison. Higher fidelity and lower latency (relative to coherence time) are desirable. Numbers in points indicates total number of qubits in system. Technologies included are Ion Trap [192–196], Superconducting [80, 98, 204–208, 220, 220], Quantum Dot [198, 199, 201, 258], Solid State [221, 222, 259, 260], and NMR [31, 185]

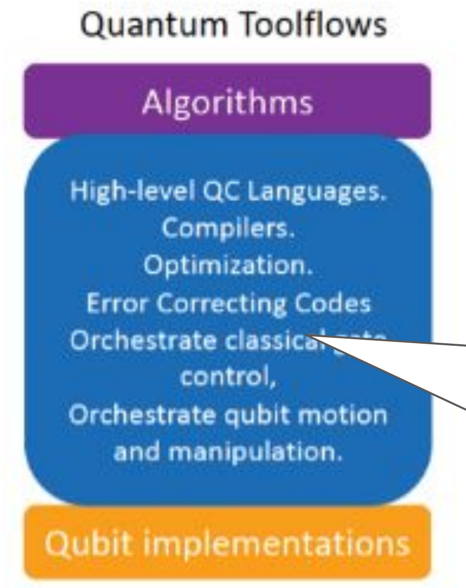


Technology	Coherence Time (s)	1-Qubit Gate Latency (s)	2-Qubit Gate Latency (s)	1-Qubit Gate Fidelity (%)	2-Qubit Gate Fidelity (%)	Mobile
Ion Trap	0.2 [192] - 0.5 [196]	1.6e-6 [193] - 2e-5 [196]	5.4e-7 [193] - 2.5e-4 [196]	99.1 [196] - 99.9999 [195]	97 [196] - 99.9 [192]	YES
Superconductors	7.0e-6 [220] - 9.5e-5 [205]	2.0e-8 [80, 204, 207] - 1.30e-7 [98, 196]	3.0e-8 [220] - 2.5e-7 [98, 196]	98 [206] - 99.92 [204]	96.5 [98, 196] - 99.4 [204]	NO
Solid State Nuclear spin	0.6 [221]	1.12e-4 [222] - 1.5e-4 [221]	1.2e-4 [223]*	99.6 - [222] - 99.95 [221]	89 [224] - 96 [223]*	NO
Solid State Electron spin	1e-3 [3]	3.0e-6 [221] - 2.3e-5 [222]	1.2e-4 [223]*	99.4 [222] - 99.93 [221]	89 [224] - 96 [223]*	NO
Quantum Dot	1e-6 [3, 225] - 4e-4 [200]	1e-9 [3] - 2e-8 [198]	1e-7 [201]	98.6 [198] - 99.9 [199]	90 [198]	NO
NMR	16.7 [185]	2.5e-4 [185] - 1e-3 [31]	2.7e-3 [185] - 1.0e-2 [31]	98.74 [31] - 99.60 [185]	98.23 [31] - 98.77 [185]	NO

Table 1. Metrics for various quantum technologies.
* Nuclear/Electron Hybrid

Resch S, Karpuzcu UR. Quantum computing: An overview across the system stack. arXiv preprint arXiv:1905.07240. 2019 May 16.

Some of the Quantum Software Philosophies



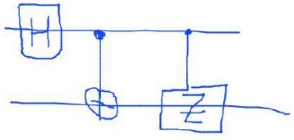
QUALTRAN

Qiskit
Elements for building a quantum future

Cirq

TKET™

PENNYLANE



(ugly quantum circuit)

Open source Python frameworks for

Noisy Intermediate Scale Quantum (NISQ) algorithms

Some of the Quantum Software Philosophies

- Hardware details need to be part of programming abstractions as they greatly impact the viability of algorithms

- Hardware should drive features and diverse hardware will have diverse features

- Data structures and abstractions should match context in which they are used (**optimization, simulation, execution**)

- Optimize for workflows that validate heuristics algorithms and for rapid iteration in exploring minimally sized circuits.

Practical Quantum Computing - Quantum Software

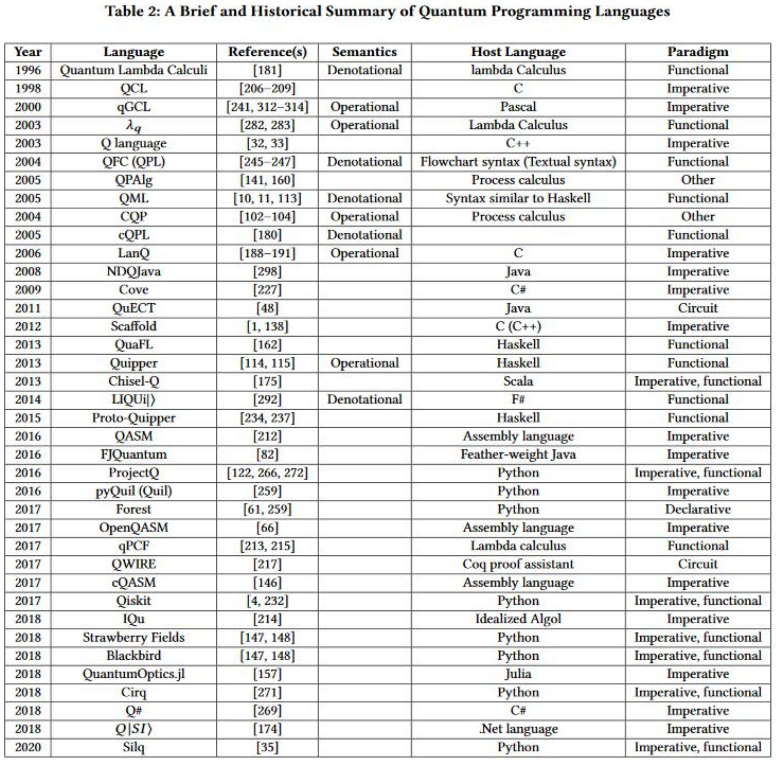


Table 2: A Brief and Historical Summary of Quantum Programming Languages

Year	Language	Reference(s)	Semantics	Host Language	Paradigm
1996	Quantum Lambda Calculi	[181]	Denotational	lambda Calculus	Functional
1998	QCL	[206–209]		C	Imperative
2000	qGCL	[241, 312–314]	Operational	Pascal	Imperative
2003	λ_q	[282, 283]	Operational	Lambda Calculus	Functional
2003	Q language	[32, 33]		C++	Imperative
2004	QFC (QPL)	[245–247]	Denotational	Flowchart syntax (Textual syntax)	Functional
2005	QPAlg	[141, 160]		Process calculus	Other
2005	QML	[10, 11, 113]	Denotational	Syntax similar to Haskell	Functional
2004	CQP	[102–104]	Operational	Process calculus	Other
2005	cQPL	[180]	Denotational		Functional
2006	LanQ	[188–191]	Operational	C	Imperative
2008	NDQJava	[298]		Java	Imperative
2009	Cove	[227]		C#	Imperative
2011	QuECT	[48]		Java	Circuit
2012	Scaffold	[1, 138]		C (C++)	Imperative
2013	QuaFL	[162]		Haskell	Functional
2013	Quipper	[114, 115]	Operational	Haskell	Functional
2013	Chisel-Q	[175]		Scala	Imperative, functional
2014	LIQUi|⟩	[292]	Denotational	F#	Functional
2015	Proto-Quipper	[234, 237]		Haskell	Functional
2016	QASM	[212]		Assembly language	Imperative
2016	FJQuantum	[82]		Feather-weight Java	Imperative
2016	ProjectQ	[122, 266, 272]		Python	Imperative, functional
2016	pyQuil (Quil)	[259]		Python	Imperative
2017	Forest	[61, 259]		Python	Declarative
2017	OpenQASM	[66]		Assembly language	Imperative
2017	qPCF	[213, 215]		Lambda calculus	Functional
2017	QWIRE	[217]		Coq proof assistant	Circuit
2017	cQASM	[146]		Assembly language	Imperative
2017	Qiskit	[4, 232]		Python	Imperative, functional
2018	IQu	[214]		Idealized Algol	Imperative
2018	Strawberry Fields	[147, 148]		Python	Imperative, functional
2018	Blackbird	[147, 148]		Python	Imperative, functional
2018	QuantumOptics.jl	[157]		Julia	Imperative
2018	Cirq	[271]		Python	Imperative, functional
2018	Q#	[269]		C#	Imperative
2018	$Q\|SI\rangle$	[174]		.Net language	Imperative
2020	Silq	[35]		Python	Imperative, functional

Zhao J. Quantum Software Engineering: Landscapes and Horizons. arXiv preprint arXiv:2007.07047. 2020 Jul 14.

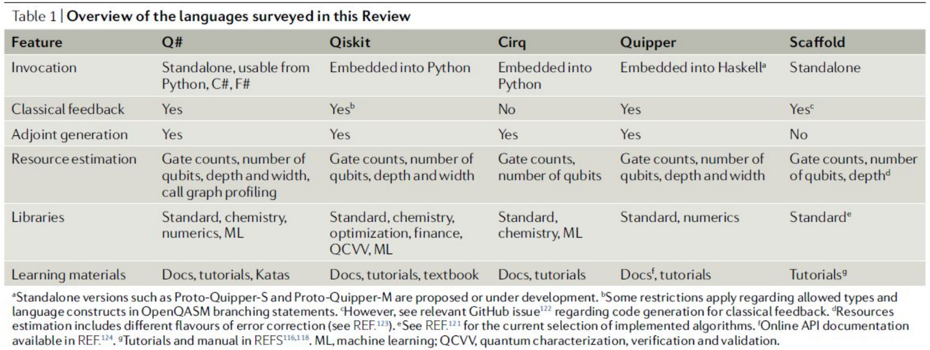
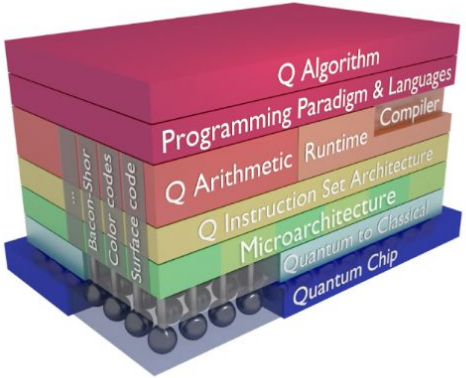


Table 1 | **Overview of the languages surveyed in this Review**

Feature	Q#	Qiskit	Cirq	Quipper	Scaffold
Invocation	Standalone, usable from Python, C#, F#	Embedded into Python	Embedded into Python	Embedded into Haskell[a]	Standalone
Classical feedback	Yes	Yes[b]	No	Yes	Yes[c]
Adjoint generation	Yes	Yes	Yes	Yes	No
Resource estimation	Gate counts, number of qubits, depth and width, call graph profiling	Gate counts, number of qubits, depth and width	Gate counts, number of qubits	Gate counts, number of qubits, depth and width	Gate counts, number of qubits, depth[d]
Libraries	Standard, chemistry, numerics, ML	Standard, chemistry, optimization, finance, QCVV, ML	Standard, chemistry, ML	Standard, numerics	Standard[e]
Learning materials	Docs, tutorials, Katas	Docs, tutorials, textbook	Docs, tutorials	Docs[f], tutorials	Tutorials[g]

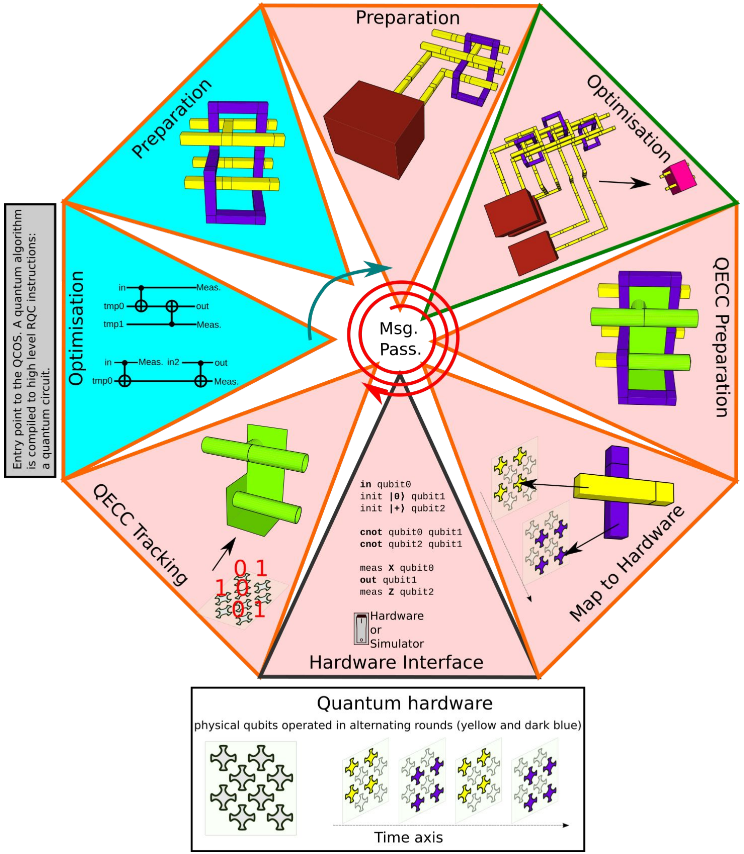
[a] Standalone versions such as Proto-Quipper-S and Proto-Quipper-M are proposed or under development. [b] Some restrictions apply regarding allowed types and language constructs in OpenQASM branching statements. [c] However, see relevant GitHub issue[122] regarding code generation for classical feedback. [d] Resources estimation includes different flavours of error correction (see REF.[123]). [e] See REF.[121] for the current selection of implemented algorithms. [f] Online API documentation available in REF.[124]. [g] Tutorials and manual in REFS[116,118]. ML, machine learning; QCVV, quantum characterization, verification and validation.

Heim B, Soeken M, Marshall S, Granade C, Roetteler M, Geller A, Troyer M, Svore K. Quantum programming languages. Nature Reviews Physics. 2020 Nov 16:1-4.



Varsamopoulos S, Bertels K, Almudever CG. Comparing neural network based decoders for the surface code. IEEE Transactions on Computers. 2019 Oct 23;69(2):300-11.

Aggregated architecture of a large quantum computer


Entry point to the QCOS. A quantum algorithm is compiled to high level RQC instructions: a quantum circuit.

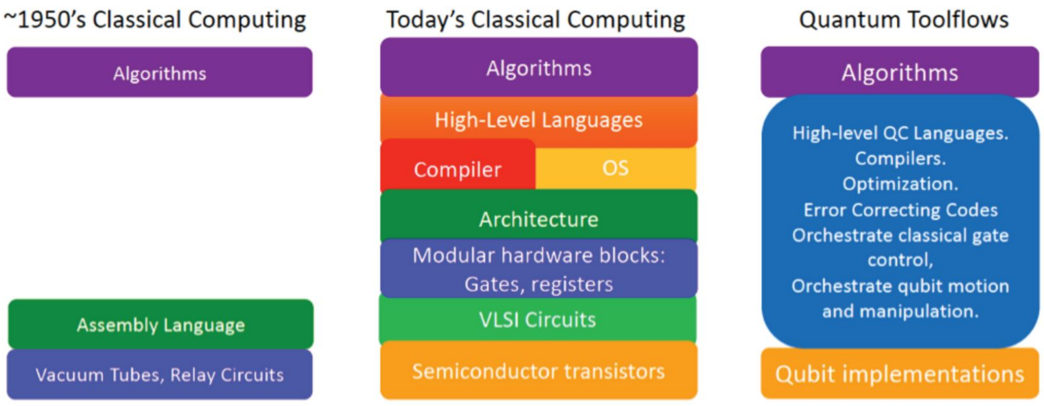
```
in qubit0
init |0⟩ qubit1
init |+⟩ qubit2

cnot qubit0 qubit1
cnot qubit2 qubit1

meas X qubit0
out qubit1
meas Z qubit2
```

Quantum hardware

physical qubits operated in alternating rounds (yellow and dark blue)



From: https://cra.org/ccc/events/quantum-computing/

A brief introduction into the topics

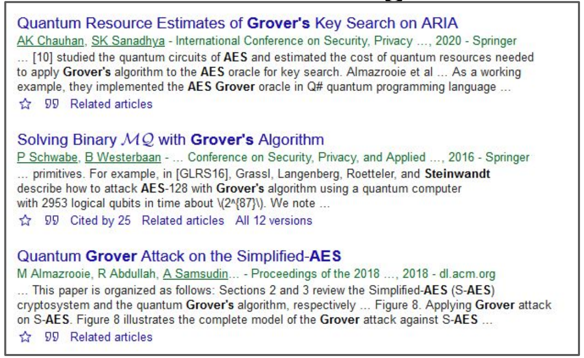
Grover’s Algorithm

For N = 1000 entries

- classical exhaustive search method needs 1000 steps
- Grover’s algorithm needs approx. 32 steps

Grover’s algorithm is a framework

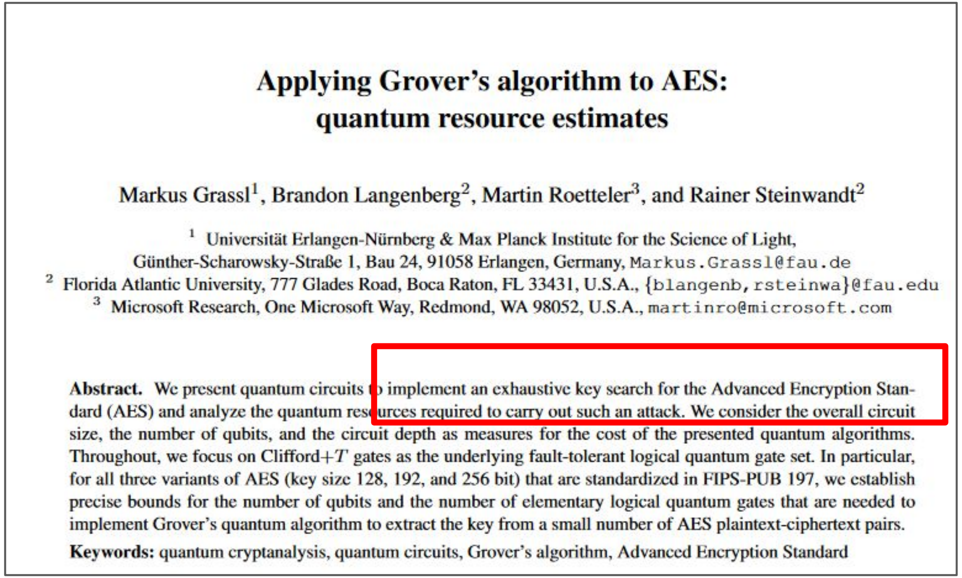
- No exponential speedup like Shor’s alg.
- Extended for different problems
    - cryptanalysis AES
    - combinatorial optimisation
    - travelling salesman



Quantum Resource Estimates of **Grover's** Key Search on ARIA
AK Chauhan, SK Sanadhya - International Conference on Security, Privacy ..., 2020 - Springer
... [10] studied the quantum circuits of **AES** and estimated the cost of quantum resources needed to apply **Grover's** algorithm to the **AES** oracle for key search. Almazrooie et al ... As a working example, they implemented the **AES Grover** oracle in Q# quantum programming language ...
☆ 99 Related articles

Solving Binary $\mathcal{MQ}$ with **Grover's** Algorithm
P Schwabe, B Westerbaan - ... Conference on Security, Privacy, and Applied ..., 2016 - Springer
... primitives. For example, in [GLRS16], Grassl, Langenberg, Roetteler, and **Steinwandt** describe how to attack **AES**-128 with **Grover's** algorithm using a quantum computer with 2953 logical qubits in time about \(2^{87}\). We note ...
☆ 99 Cited by 25 Related articles All 12 versions

Quantum **Grover** Attack on the Simplified-**AES**
M Almazrooie, R Abdullah, A Samsudin... - Proceedings of the 2018 ..., 2018 - dl.acm.org
... This paper is organized as follows: Sections 2 and 3 review the Simplified-**AES** (S-**AES**) cryptosystem and the quantum **Grover's** algorithm, respectively ... Figure 8. Applying **Grover** attack on S-**AES**. Figure 8 illustrates the complete model of the **Grover** attack against S-**AES** ...
☆ 99 Related articles



Applying Grover’s algorithm to AES:
quantum resource estimates

Markus Grassl[1], Brandon Langenberg[2], Martin Roetteler[3], and Rainer Steinwandt[2]

[1] Universität Erlangen-Nürnberg & Max Planck Institute for the Science of Light,
Günther-Scharowsky-Straße 1, Bau 24, 91058 Erlangen, Germany, `Markus.Grassl@fau.de`
[2] Florida Atlantic University, 777 Glades Road, Boca Raton, FL 33431, U.S.A., `{blangenb,rsteinwa}@fau.edu`
[3] Microsoft Research, One Microsoft Way, Redmond, WA 98052, U.S.A., `martinro@microsoft.com`

Abstract. We present quantum circuits to implement an exhaustive key search for the Advanced Encryption Standard (AES) and analyze the quantum resources required to carry out such an attack. We consider the overall circuit size, the number of qubits, and the circuit depth as measures for the cost of the presented quantum algorithms. Throughout, we focus on Clifford+T gates as the underlying fault-tolerant logical quantum gate set. In particular, for all three variants of AES (key size 128, 192, and 256 bit) that are standardized in FIPS-PUB 197, we establish precise bounds for the number of qubits and the number of elementary logical quantum gates that are needed to implement Grover’s quantum algorithm to extract the key from a small number of AES plaintext-ciphertext pairs.

Keywords: quantum cryptanalysis, quantum circuits, Grover’s algorithm, Advanced Encryption Standard

Fault-Tolerance and its Cost

For N = 1000 entries

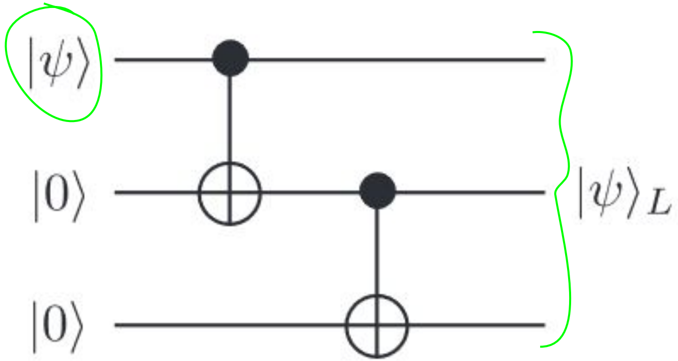
- Grover's algorithm needs approx. 32 steps
- How long does a step take?
  - Depends on speed of quantum computer gates
  - Fault-tolerance, reliability of the computer
- Qubit can be affected by noise (e.g. depolarising noise)

$$\rho \to (1-p)\rho + \frac{p}{3}(X\rho X + Y\rho Y + Z\rho Z)$$

- Threshold theorem: *a quantum computer with noise can efficiently and accurately simulate an ideal quantum computer, if the level of noise is below a certain threshold*
  - Assuming threshold is not reached
  - Use methods to mitigate, detect, correct errors

Repetition and more complex codes

$$|0\rangle_L = |000\rangle\,, \qquad |1\rangle_L = |111\rangle \xrightarrow{\text{Introduce redundancy}} |\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}.$$

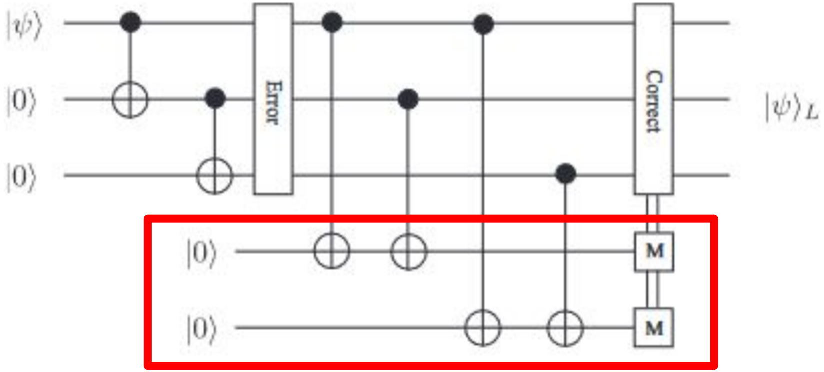


Circuit: Encoding a state in a logical state

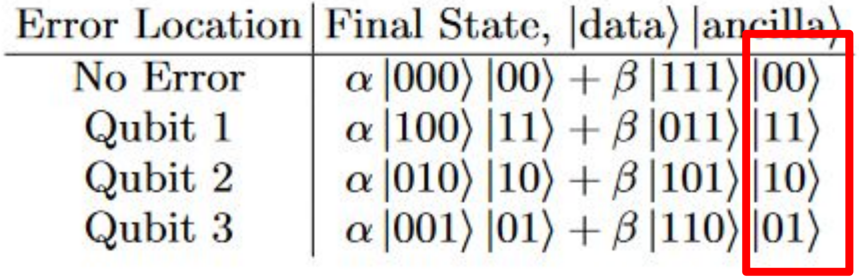
$$\begin{aligned}\alpha\,|0\rangle + \beta\,|1\rangle &\rightarrow \alpha\,|0\rangle_L + \beta\,|1\rangle_L \\ &= \alpha\,|000\rangle + \beta\,|111\rangle \\ &= |\psi\rangle_L\,.\end{aligned}$$

[*] Devitt et. al, *Quantum Error Correction for Beginners*, available at https://arxiv.org/pdf/0905.2794.pdf

Syndromes, Correction, Flags



Ancillae used for syndrome measurement



Error Location	Final State, $|\text{data}\rangle\ |\text{ancilla}\rangle$
No Error	$\alpha\,|000\rangle\,|00\rangle + \beta\,|111\rangle\,|00\rangle$
Qubit 1	$\alpha\,|100\rangle\,|11\rangle + \beta\,|011\rangle\,|11\rangle$
Qubit 2	$\alpha\,|010\rangle\,|10\rangle + \beta\,|101\rangle\,|10\rangle$
Qubit 3	$\alpha\,|001\rangle\,|01\rangle + \beta\,|110\rangle\,|01\rangle$

- Syndrome measurements *have to be repeated*
- Repetition code protects only against a single type of error: detects two errors, corrects one

Digitization of noise is based on the observation that any interaction between a set of qubits and environment can be expressed in the form

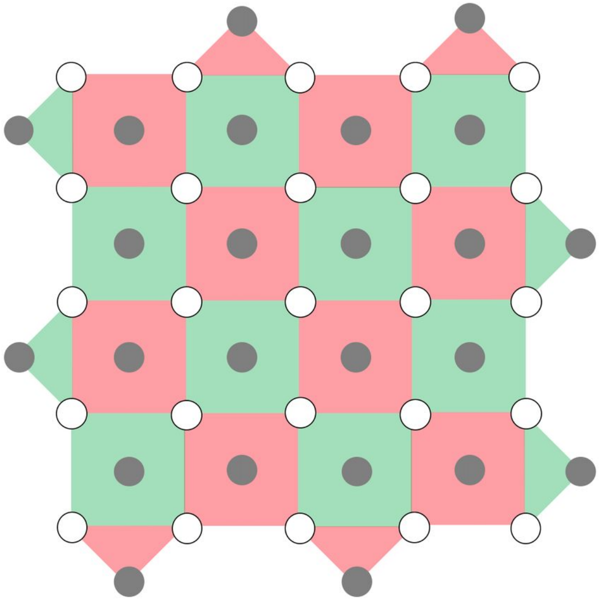
$$G = c_I\sigma_I + c_x\sigma_x + c_y\sigma_y + c_z\sigma_z$$

where,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Need to protect against *phase errors*, too

Surface code



detect X (red), detect Z (green)

measure Z stabilizers, measure X stabilizers

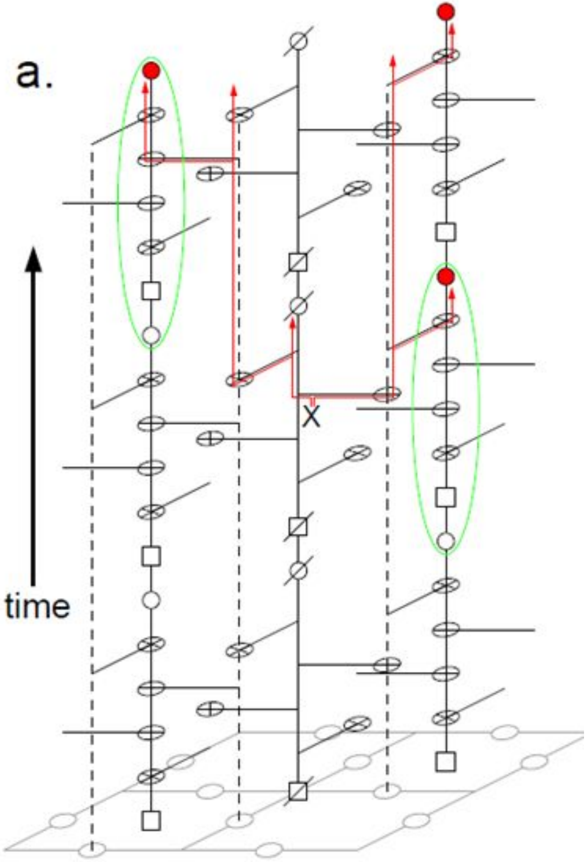
$$|\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}.$$

stabilizers

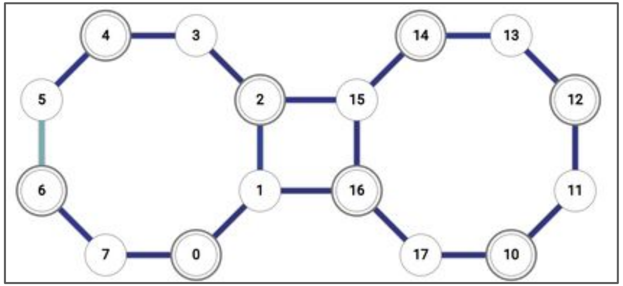
-1 XXX

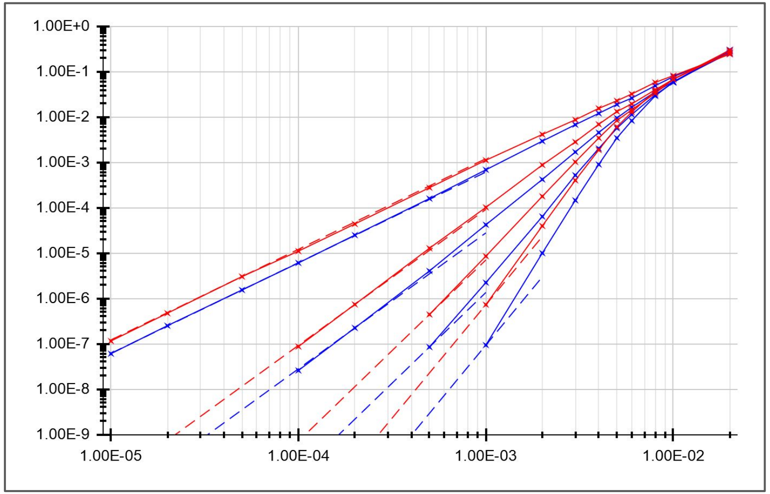
+1 ZZI

+1 ZIZ



Cost of Error Correction





Computer

- Gate duration
- Qubit connectivity
- Qubit and gate quality, realistic noise models

Code distance

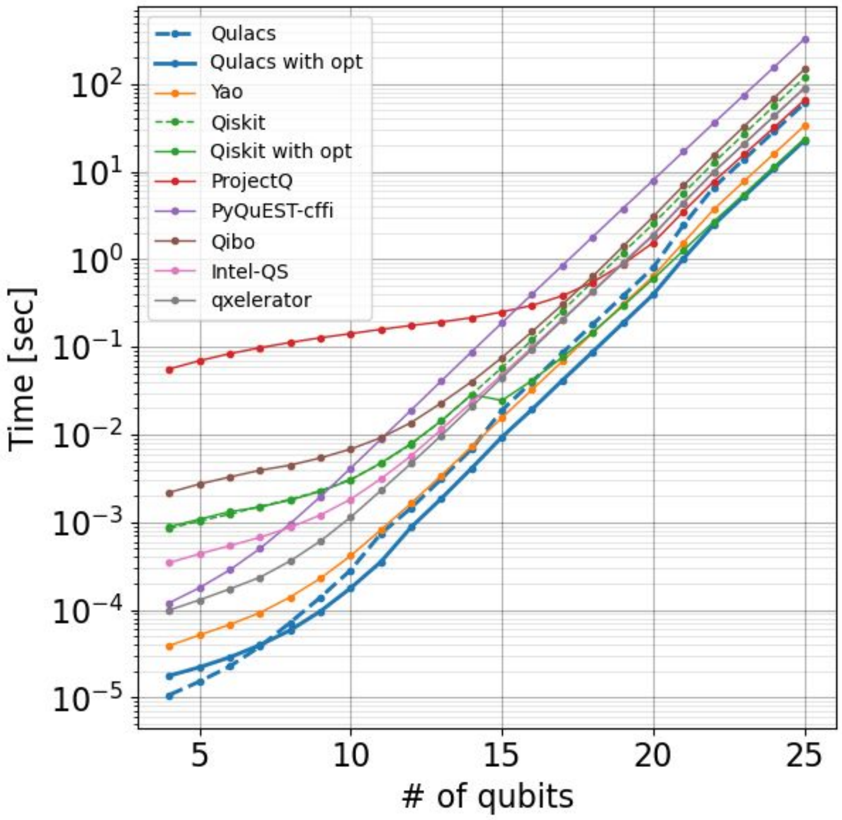
- number of physical qubits
- number of syndrome measurements in time

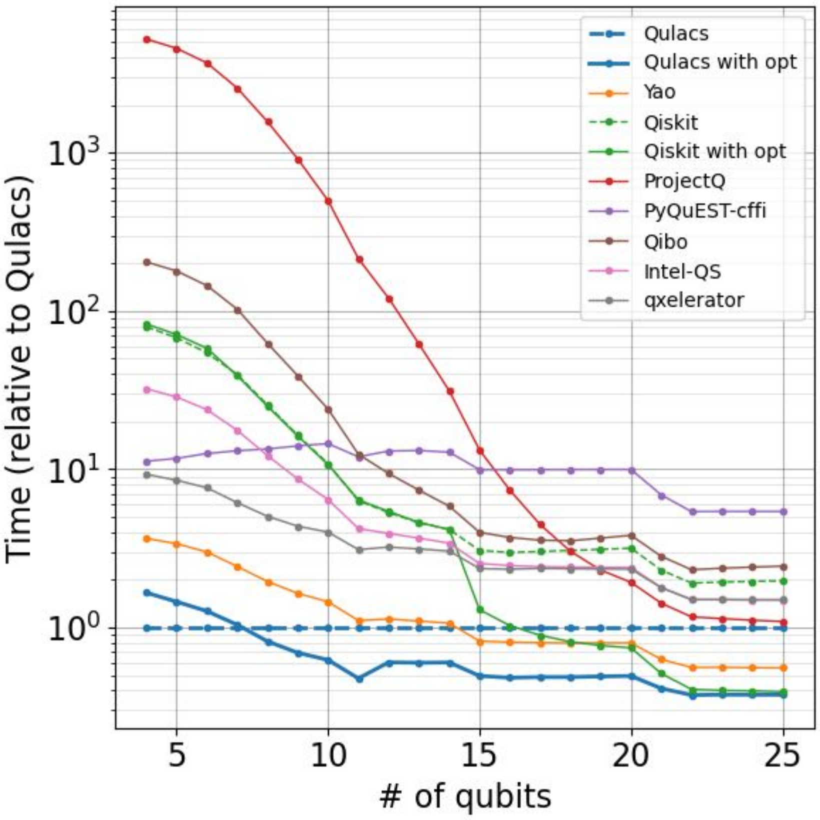
Decoder performance

- what is the error suppression rate? (code dist.)
- how fast does it operate? (infl. code distance)

TOTAL: time overhead -> could negate Grover speed-up if not done right

Quantum circuit software simulators





Space-time volume of a quantum computation