Information Economics Final Exam

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The exam is 2 hours and has a total of 120 points. Please answer as many questions as you can. Answer shortly but justify your answers and explain accurately what you are doing. If you are confused about some question statement, please explain clearly what you assume when answering. Point totals reflect the difficulty of the problem and give a rough estimate for how long the question should take.

1. Warm up

   (a) *(10 points)* In our canonical principal-agent model, we saw that when a risk neutral manager is contracting with a risk averse agent, in the absence of moral hazard the manager takes on all risk and pays a deterministic wage to the agent. Explain, informally, how and why this result changes in the presence of moral hazard in the model we studied in class.

   (b) *(10 points)* Can you sketch an example of a signaling game where an equilibrium that fails the intuitive criterion Pareto dominates an equilibrium that satisfies it?

2. A sender has type $\theta \in \{1, 4\}$. The prior probability is $Pr(\theta = 4) = 1/4$. The sender knows their type and they can choose to disclose their type or not to a receiver who then chooses action x to maximize utility $u(x, \theta) = -(x - \theta)^2$. The sender cannot report a type other than their own true type to the receiver. The sender receives utility x from the receiver's action.

   (a) *(5 points)* Argue that all information is revealed in every PBE of this game.

   Now assume the sender can secretly pay a cost of 1 to fabricate evidence that they are the other type, e.g. if the low type pays this cost, then they can also disclose that their type is $\theta = 4$.

   (b) *(10 points)* Is there a separating equilibrium in this game?

(c) *(10 points)* Is there an equilibrium where both types report $\theta = 4$ with probability 1?

(d) *(10 points)* Is there an equilibrium where both types don't disclose with probability 1?

(e) *(15 points)* Can you construct a mixed strategy equilibrium where $\theta = 1$ randomizes between fabricating a signal and not fabricating one.

3. A government wants a firm to decarbonize its production process. The firm has type θ drawn uniformly from $[0, 1]$ which describes the probability that the firm is able to develop a cleaner process.

   The government wants to take into account the difficulty of decarbonization. So, they design a menu of possible *deadlines* $x \in \mathbb{R}_+$ and a *penalties* $p \in \mathbb{R}_+$. the firm's payoff for a given penalty and deadline is determined by their type, if a firm faces deadline x and penalty p receive $-pe^{-\theta x}$. At time 0, the firm can relocate in another country where they do not have to decarbonize, but this costs K for some $K > 0$. The firm is risk neutral, patient (so they don't discount), and is unable to relocate after time 0.

   (a) *(5 points)* The government wants to design a deadline and penalty scheme that satisfy the following constraints:

$$
\begin{aligned}
&-p(\theta)e^{-\theta x(\theta)} \geq -p(\theta')e^{-\theta x(\theta')} \ \forall \theta, \theta' \in [0, 1] \\
&-p(\theta)e^{-\theta x(\theta)} \geq -K \ \forall \theta \in [0, 1].
\end{aligned}
$$

   Explain in words what each of these constraints captures.

   (b) *(15 points)* Note that we can rewrite the first constraint as

$$
\theta x(\theta) - \ln p(\theta) \geq \theta x(\theta') - \ln p(\theta')
$$

   Prove that any deadline that satisfies the constraints must be non-decreasing in θ.

(c) *(15 points)* Show (don't just cite a theorem!) that the constraints imply that

$$\ln p(\theta) = \underline{V} + \theta x(\theta) - \int_0^\theta x(s)\, ds$$

for some $\underline{V} \leq \ln K$. Is the optimal penalty increasing or decreasing?

(d) *(15 points)* Show that any non-decreasing $x(\theta)$ and the corresponding $p(\theta)$ that satisfies the equation in (c) satisfy all the constraints in (a).