Information Economics Final Exam

June 2nd, 2023

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The exam is 2 hours and has a total of 120 points. Please answer as many questions as you can. Answer shortly but justify your answers and explain accurately what you are doing. If you are confused about some question statement, please explain clearly what you assume when answering. Point totals reflect the difficulty of the problem and give a rough estimate for how long the question should take.

1. Warm up

   (a) *(10 points)* In our canonical principal-agent model, describe the optimal contract when a risk neutral agent is contracting with a risk neutral principal. How does this contract change under limited liability.

   (b) *(10 points)* What is the revelation principle. Briefly describe its role in mechanism design.

2. Consider the following signaling game. There are three equally likely sender types $\theta \in \{1, 2, 3\}$. The sender chooses message $e \in \mathbb{R}_+$ and a receiver chooses action $x \in \mathbb{R}_+$. The receiver receives utility from choosing action x when facing a type θ receives of $u(x, \theta) = -(x - \theta)^2$, and the sender receives utility $u(x, \theta) = x - \frac{1}{\theta}e^5$. Let $e(\theta)$ be the sender's strategy.

   Describe a perfect Bayesian equilibrium (both strategies and beliefs) consistent with each of the following sender strategies, and verify that it is a PBE.

   (a) *(5 points)* $e(1) = e(2) = e(3) = 0$.

   (b) *(10 points)* $e(1) = 0$, $e(2) = 1$, $e(3) = 3^{1/5}$.

   (c) *(10 points)* $e(1) = e(2) = 0$, $e(3) = 3^{1/5}$.

   (d) *(25 points)* Which of the above satisfy the intuitive criterion?

3. There are two siblings who each inherit 50% of an apartment. Sibling i values the apartment at $\theta_i \in [0, 1]$. Valuations are independent, and are uniformly drawn from $[0, 1]$. The siblings hire a mediator to design a mechanism to redistribute the shares in the most efficient way possible. Sibling i's payoff from receiving a share $x_i \in [0, 1]$, $x_i + x_{-i} \leq 1$, of the apartment and transfer $t_i \in \mathbb{R}$ is $x\theta_i - t$. If sibling does not participate in the mechanism, they keep their share and receive a payoff of $\theta_i/2$.

   (a) *(10 points)* In this setting, a direct mechanism is described by a feasible allocation of shares $x_i(\theta_i, \theta_{-i})$ and transfers $t_i(\theta_i, \theta_{-i})$ for each sibling. Write down each of the following constraints:

      - Bayesian incentive compatibility
      - Dominant strategy incentive compatibility
      - Interim individual rationality
      - Ex-ante individual rationality

   (b) *(10 points)* What is the ex-post efficient allocation rule. Construct a Pivot (VCG) mechanism in this setting.

   (c) *(10 points)* Show that in any ex-post efficient BIC mechanism, the expected transfer, $T_i(\theta_i) = E(t_i(\theta_i, \theta_{-i})|\theta_i)$, for an agent of type θ_i must be $\frac{1}{2}\theta_i^2 + V$ for some constant V.

   (d) *(15 points)* The mediator decides to run a second price auction, where the winner pays the loser their bid.[1] This is clearly budget balanced. Show that it is a Bayes Nash equilibrium for bidders to bid $\theta_i/3 + 1/6$.

   (e) *(5 points)* Verify that this auction is interim individually rational. How does this result compare with the Myerson-Satterthwaite Theorem?

[1] So if sibling one bids 1/2 and sibling two bids 1/4, sibling one receives the apartment and pays sibling two 1/4.