Information Economics Final Exam

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The exam is 2 hours and has a total of 120 points. Please answer as many questions as you can. Answer shortly but justify your answers and explain accurately what you are doing. If you are confused about some question statement, please explain clearly what you assume when answering. Point totals reflect the difficulty of the problem and give a rough estimate for how long the question should take. The exam is closed book, calculators are not permitted

1. Some conceptual questions, answer briefly and relate your answer to a model we studied in class.

   (a) *(10 points)* Suppose that I cannot influence the difficulty of this micro exam. The difficulty randomly determined, and no amount of effort I put in changes the likelihood that it is difficult. Your score is determined by the time you spend studying and the difficulty – the more difficult the test, the more you need to study to do well.

   I would prefer that you spend all your free time studying, no matter the difficulty. Your TA, on the other hand, knows that this class is mostly a waste of time, but still would like you to study a bit more than you'd like to for every level of difficulty. The TA and I both know the difficulty of the exam. Why might I have the TA tell you about how hard the exam is instead of telling you myself?

   Solution. *This seems like an application of cheap talk. Both I and the TA would be engaging in cheap talk when we tell you about the exam. We've learned from our cheap talk model that informative communication between two agents with conflicting interests is possible if the difference is small, but is impossible if the difference is large.*

   (b) *(10 points)* Your car insurance provider would like to induce you to drive safely. Suppose your insurance contract specifies that they'll insure all but $100 worth of damages if you are involved in a collision with a car and all but \$200 worth of damages if you are involved in a collision with a tree. What does this suggest about the probabilities of these two events?[1]

Solution. *we know from our moral hazard model that these payments are determined by the likelihood ratios of the events. This suggests that* $Pr(tree|not\,safe)/Pr(tree|safe) > Pr(car|not\,safe)/Pr(car|safe)$.

2. A government is designing sin taxes, taxes intended to correct distortions in consumption due to self-control problems. Specifically, there is a single consumer with private type θ drawn uniformly from $[0, 1]$.There is a single good, and the consumer's utility for q units of the good and transfer t is

$$u(q; \theta) + \beta v(q; \theta) - t$$

where we interpret u as their utility from consuming q units of the good today and $\beta v(q; \theta)$ as their additional utility (or dis-utility) tomorrow due to that consumption. $\beta < 1$ captures their self-control problem.

The government would like to design the optimal direct mechanism to maximize

$$\max_{q,t} E(u(q(\theta); \theta) + v(q(\theta); \theta) + t(\theta)),$$

subject to incentive constraints, as well as the constraint that the consumer can always choose not to participate and receive utility 0. Assume u, v are twice continuously differentiable and have uniformly bounded derivatives.

(a) *(10 points)* What are the incentive compatibility constraints for this problem?

Solution. *A mechanism is incentive compatible if for all* θ, θ'

$$u(q(\theta), \theta) - \beta v(q(\theta), \theta) - t(\theta) \geq u(q(\theta'), \theta) - \beta v(q(\theta'), \theta) - t(\theta').$$

[1] You've moved to America because I don't want to figure out how to make the Euro symbol.

(b) *(20 points)* Show that the incentive compatibility constraints imply that

$$t(\theta) = u(q(\theta);\theta)+\beta v(q(\theta);\theta)-\int_0^\theta u_\theta(q(s),s)+\beta v_\theta(q(s),s)\,ds-V(0)$$

Solution. *Let $V(\theta)$ be the value funtion. The incentive constraints can be rewritten as*

$$V(\theta) \geq V(\theta') - u(q(\theta'),\theta') + \beta v(q(\theta'),\theta') + u(q(\theta'),\theta) - \beta v(q(\theta'),\theta)$$

So

$$\begin{aligned} &u(q(\theta),\theta') - u(q(\theta),\theta) + \beta v(q(\theta),\theta') - \beta v(q(\theta),\theta) \geq V(\theta) - V(\theta') \\ &\geq u(q(\theta'),\theta) - u(q(\theta'),\theta') + \beta v(q(\theta'),\theta) - \beta v(q(\theta'),\theta'). \end{aligned}$$

Dividing through by $\theta - \theta'$ and taking limits gives us

$$V'(\theta) = u_\theta(q(\theta),\theta) - \beta v_\theta(q(\theta),\theta)$$

which can then be integrated to get

$$V(\theta) - V(0) = \int_0^\theta u_\theta(q(s),s) + \beta v_\theta(q(s),s)\,ds.$$

Finally, since $V(\theta) = u(q(\theta);\theta) + \beta v(q(\theta);\theta) - t(\theta)$, solving for t gives

$$t(\theta) = u(q(\theta);\theta)+\beta v(q(\theta);\theta)-\int_0^\theta u_\theta(q(s),s)+\beta v_\theta(q(s),s)\,ds-V(0).$$

(c) *(10 points)* Under what additional assumptions on u and v is the condition from part b along with the additional condition that $q(\theta)$ is increasing jointly necessary and sufficient for incentive compatibility. (It's fine to reference a result from the notes, but be as precise as possible about how it applies here).

Solution. *We need single crossing, which in this case is the requirement that*

$$\frac{\partial^2 u}{\partial q \partial \theta} + \beta \frac{\partial^2 v}{\partial q \partial \theta} > 0$$

.

(d) *(15 points)* Let $u(q;\theta) = \theta q$ and $v(q;\theta) = -q^2$. Solve for the optimal mechanism.

Solution. *These satisfy single crossing, so we can replace the IC constraints with the envelope condition. Plugging this into the objective, we have*

$$\max \int_0^1 [\theta q - q^2 + \theta q - \beta q^2 - \int_0^\theta q(s)\, ds]\, d\theta$$

Changing the order of integration, we can reformulate this as

$$\max \int_0^1 [2\theta q(\theta) - (1+\beta) q(\theta)^2 - (1-\theta) q(\theta)]\, d\theta$$

So maximizing this pointwise

$$2(1+\beta)q(\theta) = 3\theta - 1$$

so $q(\theta) = \max\{0, \frac{3\theta-1}{2(1+\beta)}\}$, $t(\theta)$ *follows directly.*

3. A college is deciding how to do undergraduate admissions. They can condition admissions on the applicant's test score $t \in [0,1]$ drawn uniformly. There is a single applicant, who knows their test score. When applying to the college, an applicant of type t can send a message $m \in \{t, \emptyset\}$, i.e. they either disclose their test score t or report nothing.

   The college chooses an admission probability a to maximize $-(a-t)^2$. So they admit a student with probability equal to their expected test score t, given their application (so if there is no disclosure, $Pr(admission|m) = E(t|\emptyset)$). An applicant receives utility 1 from being admitted and 0 otherwise.

(a) *(15 points)* Show that there is an equilibrium of this game where the applicant reports their test score regardless of its value.

Solution. *Everyone reports a score. If you don't report a score believe you are the lowest type. Then, the payoff from not reporting is* 0*. The payoff from reporting is* $t > 0$*, so every type reports their score.*

(b) *(15 points)* Show that the equilibrium you found in part a is the only equilibrium where the applicant plays a strategy where they report t iff $t \geq \tau$ for some $\tau \in [0, 1]$.

Solution. *Consider any such equilibrium with* $\tau > 0$*. The payoff from not reporting is* $E(t|\emptyset) = \tau/2$*. So all types* $t \in (\tau/2, \tau]$ *have a strict incentive to report if* $\tau > 0$*. So this cannot be an equilibrium.*

Suppose that students can perfectly anticipate their test score and can pay for test prep. Test prep is unobservable to the college and increases their test score by $x \in \mathbb{R}_+$ for cost $c(x)$. So a student who knows they'll score $t = .5$ without prep and chooses $x = .1$ has a test score of .6, which they can report to the college. The college's payoffs are the same – they still want to admit a student with probability equal to their expected test score without prep. Suppose that $c(x)$ is twice continuously differentiable and convex.

(c) *(15 points)* Show that there is an equilibrium where no student gets any text prep (i.e. $x(t) = 0$ for all t) if and only if $c'(0) \geq 1$.

Solution. *Consider an equilibrium where everyone reports their score and no one pays for test prep. In this equilibrium, the college believes whatever score you report (and for* $t > 1$ *off path beliefs that are best at discouraging deviations are that* $t = 0$*, so you never choose those). Then if you report a score of* $t + x$*, the college believes that your type is* $t + x$*, which means that is the probability of admission. So for type* t*,* $x = 0$ *must solve*

$$\max_{x \in [0, 1-t]} t + x - c(x)$$

This has first-order condition

$$c'(x) = 1,$$

and at the boundaries $c'(0) \geq 1$ *or* $c'(1-t) \leq 1$. *Since* c *is convex,* 0 *solves this problem iff* $c'(0) \geq 1$.