28E35700 Alternative Investments

Private Equity Performance Measurement

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Our Agenda Today

- **How risk and return can be measured in private equity context?**
    - How we can define and obtain Alpha and Betas?
- **Do traded (public) risk factors explain private equity returns?**
    - Which traded factors explain VC deal returns?
    - Which traded factors explain Buyout deal returns?
- **How we can measure private equity fund performance?**
    - Internal rate of return (IRR), Multiple, and Public Market Equivalent (PME)
- **Advantages and disadvantages of deal-level and cashflow-level data**

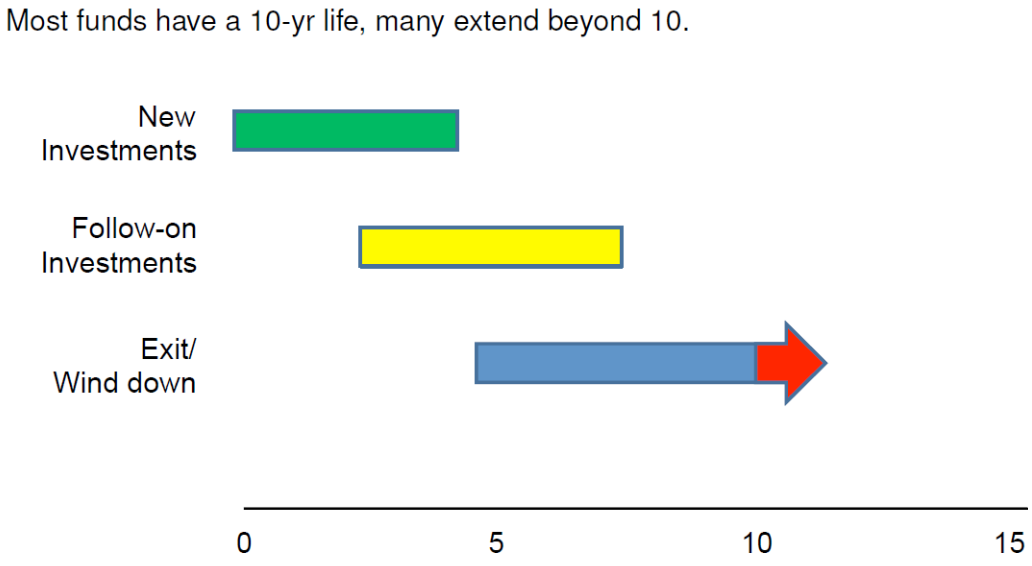
References

1. **Ang and Sorensen (2012), Risks, Returns, and Optimal Holdings of Private Equity: A Survey of Existing Approaches**
2. Korteweg and Sorensen (2011, RFS), Risk and Return Characteristics of Venture Capital Backed Entrepreneurial Companies
3. Franzoni, Nowak, Phalippou (2011, JF): Private Equity Performance and Liquidity Risk

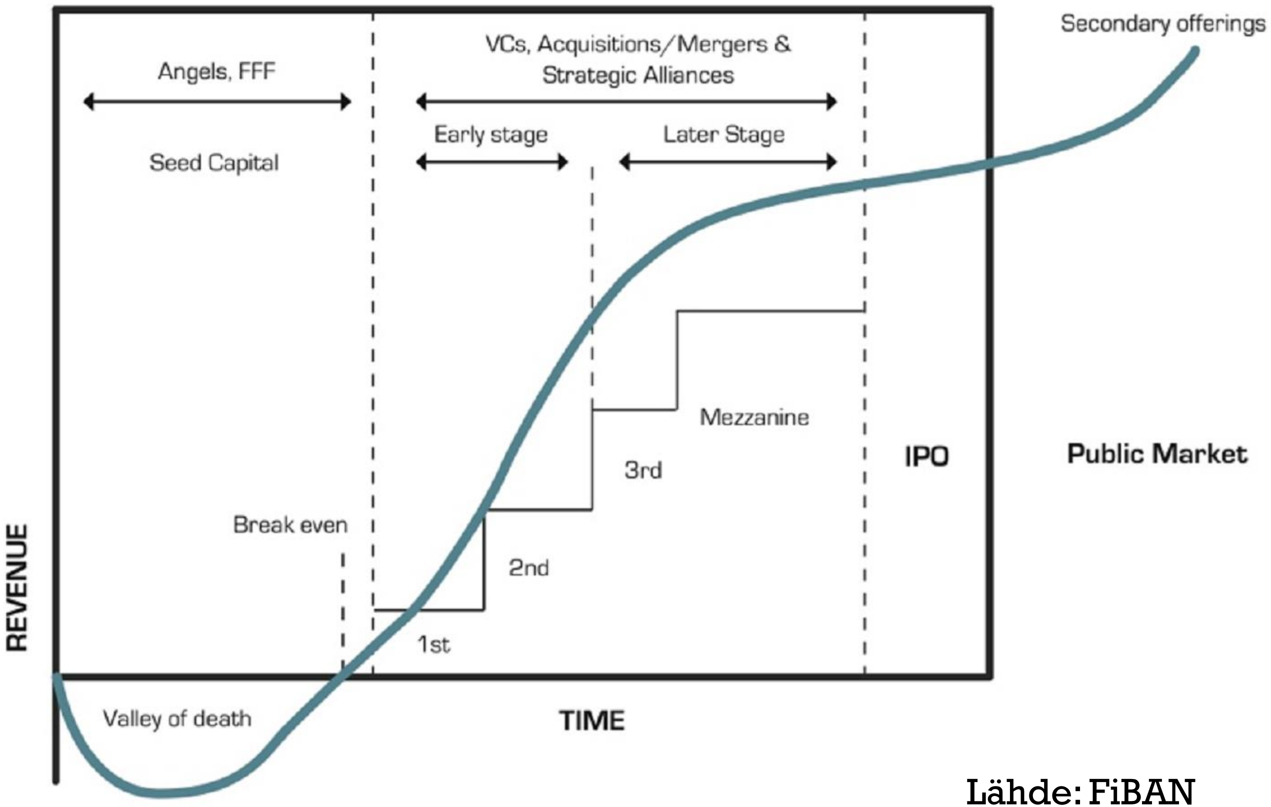
Basics of PE World

Stages in PE Investment Process

1. **Establishing a fund**
2. **Mapping potential investments**
3. **Making an investment**
4. **Developing the target company**
5. **Exit (IPO, trade sale, secondary buyout)**



Investment Rounds and Revenue / Risk



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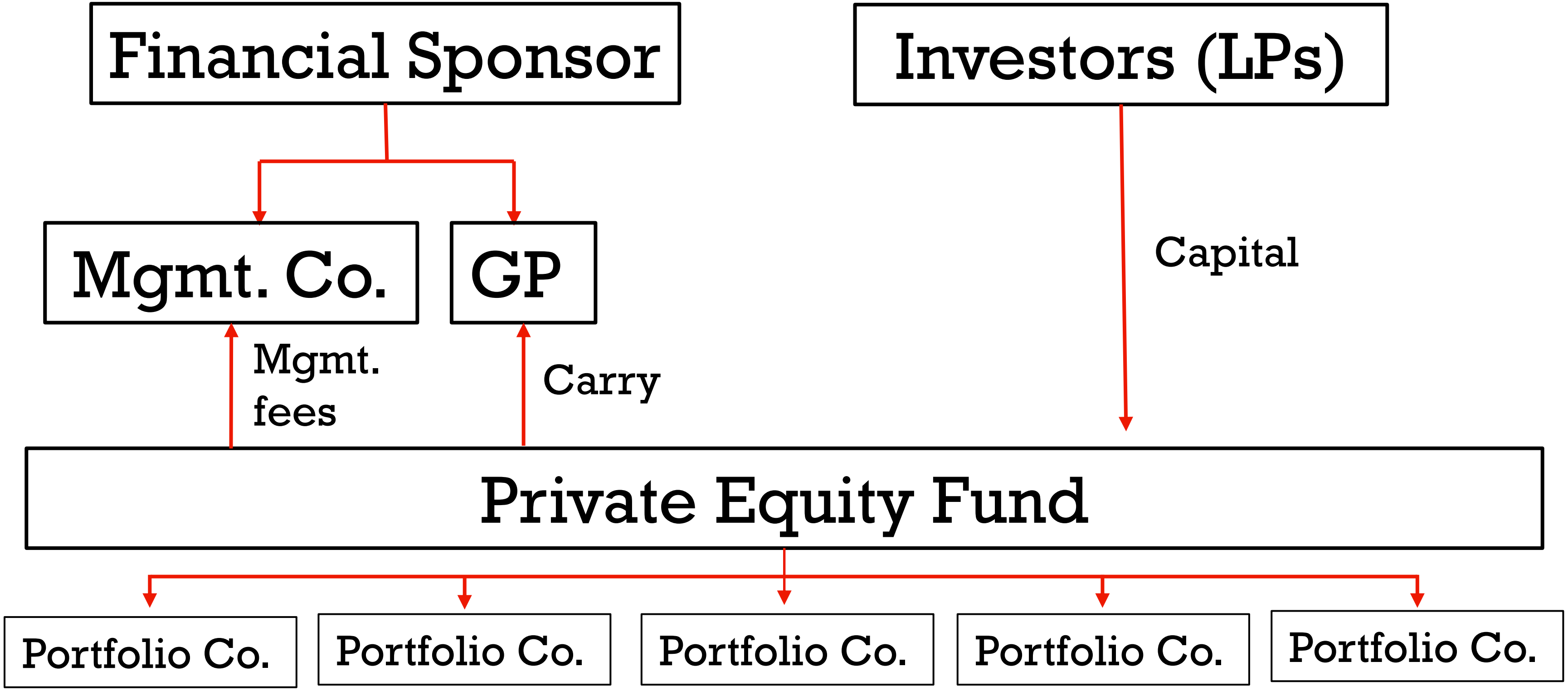
Lähde: FiBAN

Private Equity Fund Structure



Risk and Return in PE World

Risk and Return in PE World

- **Standard models work for traded liquid assets**

$$E[R_i(t)] - R_f(t) = \alpha_i + \beta_i[R_m(t) - R_f(t)]$$

- **CAPM assumptions do not hold in the PE world**
    - Frictions are large, rebalancing is infrequent, and idiosyncratic risk is not diversifiable
- **In practice, $\boldsymbol{\alpha}$ is overestimated, and $\boldsymbol{\beta}$ is underestimated**
    - No observable market values
    - Returns are infrequently observed (rounds and exits do not arise “monthly”)
    - Selection bias (only good companies are observed)

Ang and Sorensen (2012)

How to address problems of standard discrete-time CAPM?

1. Cash flow streams between the LPs and GPs

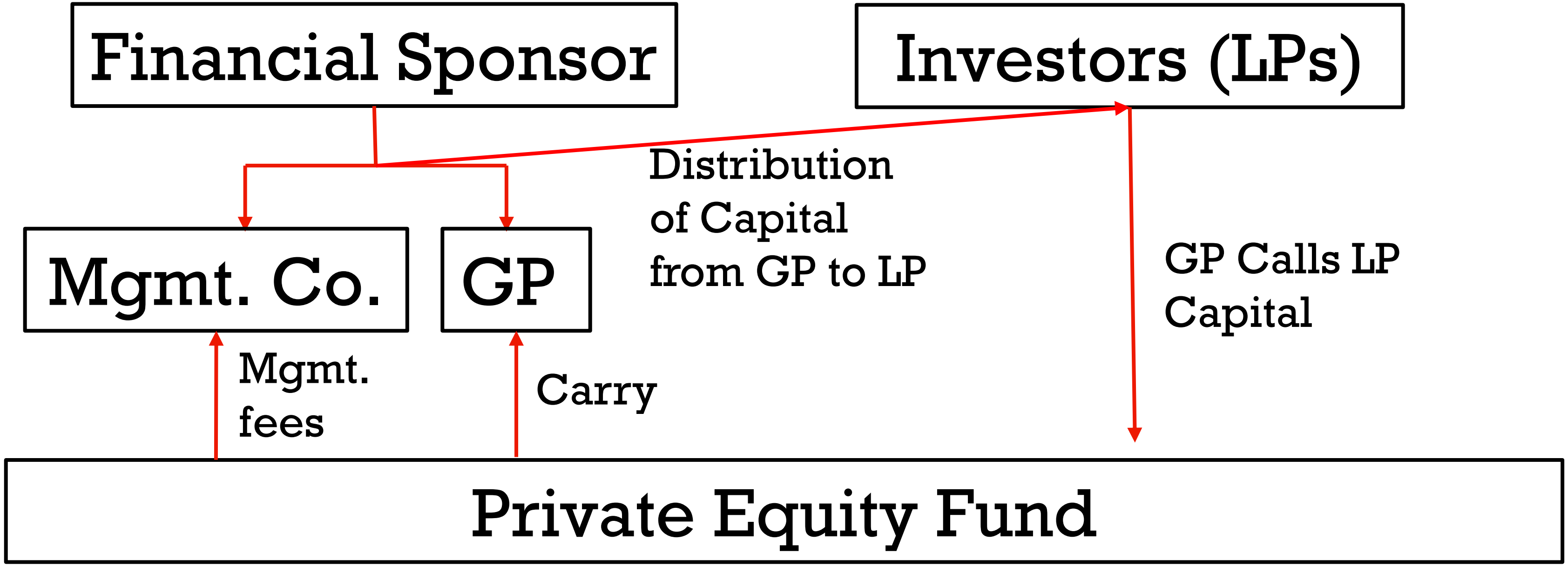
   (Fund-level data)

2. Portfolio company-level performance data

   (Deal-level data)

Ang and Sorensen (2012)

Cash flow streams between the LPs and GPs



Ang and Sorensen (2012)

PE Fund Performance Measures

1. **Internal Rate of Return (IRR)**
2. **Total Value to Paid-in Capital Multiple (TVPI or Multiple)**
3. **Public Market Equivalent (PME)**

- **IRR and Multiple (TVPI) are absolute performance measures.**

- **PME measures performance relative to the market.**
    - Does a PE fund outperform the ETF on S&P500?

Ang and Sorensen (2012)

Internal Rate of Return (IRR)

- **IRR measures lifetime returns between LP and GP**

$$\boldsymbol{Present\ Value} = \sum \frac{CF(t)}{(1+\boldsymbol{IRR})^t} = \sum \frac{Dist(t) - Call(t)}{(1+\boldsymbol{IRR})^t} = \mathbf{0}$$

$$\Rightarrow \frac{\left(\sum \frac{Dist(t)}{(1+\boldsymbol{IRR})^t}\right)}{\left(\sum \frac{Call(t)}{(1+\boldsymbol{IRR})^t}\right)} = 1$$

  - *CF(t) = Cash flow at t*
  - $Dist(t)$ = Distribution of Capital from GP to LP
  - *Call(t)* = Capital Calls paid by LP to GP

- **IRR brings the net present value (NPV) to zero.**

Ang and Sorensen (2012)

IRR is a problematic measure of economic performance

- IRR does **not** calculate performance **relative to a benchmark or market return**

- IRR calculation **implicitly assumes** that invested and returned capital can be **reinvested at the IRR rate.**
  - If a fund makes an early small investment with **a large quick return**, the investment can **largely define the IRR for the entire fund**, regardless of the performance of subsequent investments. See Ludo Phalippou's example in Linkedin.

- Phalippou (2011) suggests that **GPs may actively manage** their investments to **inflate fund IRRs.**

Ang and Sorensen (2012)

Problems in IRR: Sensitive to timing of cash-flows
Example Phalippou (2011)

Time	Fund A	Fund B
0	-100	-100
1	150	0
2	0	0
3	50	0
4	0	0
5	0	100
6	0	0
7	0	0
8	0	100
9	0	0
10	0	0
11	0	0
12	0	0
IRR	68%	11%

Early cash-flows leads to huge IRR

Total Value to Paid-in Capital Multiple (TVPI)

- **The total amount of capital returned to the *LP* investors (net of fees) divided by the total amount invested (including fees):**

$$TVPI = \frac{\sum Dist(t)}{\sum Call(t)}$$

- **No adjustment** for the **time value of money.**
  - **IRR** assumes that capital can be **reinvested at the *IRR* rate**
  - ***TVPI* multiple** assumes that capital can **be reinvested at a zero rate**

→ TVPI multiple is **less susceptible for manipulation**

Ang and Sorensen (2012)

Public Market Equivalent (PME)

- **PME** is the ratio of the discounted value of the **LP's inflows divided by the discounted value of outflows,** with the discounting performed using **realized market returns:**

$$PME = \frac{\left(\sum \frac{Dist(t)}{\prod\left(1 + R_m(t)\right)}\right)}{\left(\sum \frac{Call(t)}{\prod\left(1 + R_m(t)\right)}\right)}$$

- PME **compares the return** that could be earned on the **called capital,** were it **invested in** the **public index, with** the returns earned in **private equity.**

- **PME greater than one is equivalent to a positive economic return** for the LPs when PE investments have the same risk as the general market.
- The public market equivalent measure (PME) is used to **evaluate performance relative to the market** (or S&P500 ETF).

Ang and Sorensen (2012)

Only difference between measures is the way how measures are discounted

$$TVPI = \frac{\sum Dist(t)}{\sum Call(t)}$$

$$PME = \frac{\left(\sum \frac{Dist(t)}{\prod\left(1 + R_m(t)\right)}\right)}{\left(\sum \frac{Call(t)}{\prod\left(1 + R_m(t)\right)}\right)}$$

$$\frac{\left(\sum \frac{Dist(t)}{(1 + \boldsymbol{IRR})^t}\right)}{\left(\sum \frac{Call(t)}{(1 + \boldsymbol{IRR})^t}\right)} = 1$$

Ang and Sorensen (2012)

Examples for estimating performance measures: Gredil , Griffiths and Stucke (2014)

- **The four input variables* are the same in all cases:**
  - A sequence of contributions into the PE portfolio: $C = \{c_0, c_1, \ldots, c_n\}$
  - A sequence of distributions from the PE portfolio: $D = \{d_0, d_1, \ldots, d_n\}$
  - A residual value of the PE portfolio at time n: NAV_{PE}
  - A reference benchmark (*e.g.*, the public market): $M = \{m_0, m_1, \ldots, m_n\}$

- **Based on the sequence of contributions and distributions, their future values at time n are defined as follows**
  - The future value of contributions at time n is: $FV(C) = \left\{c_0 \cdot \frac{m_n}{m_0}, c_1 \cdot \frac{m_n}{m_1}, \ldots, c_n\right\}$
  - The future value of distributions at time n is: $FV(D) = \left\{d_0 \cdot \frac{m_n}{m_0}, d_1 \cdot \frac{m_n}{m_1}, \ldots, d_n\right\}$

Notations are slightly different since here we follow Gredil , Griffiths and Stucke (2014) instead of Ang and Sorensen (2012).
Calls = Contributions (how much LP contributes).

Multiple (TVPI) and PME

$$KS\text{-}PME = \frac{\sum FV(D) + NAV_{PE}}{\sum FV(C)}$$

$$KS\text{-}PME = TVPI(FV(C), FV(D), NAV_{PE})$$

C represents contributions into, and D represents distributions from, the PE portfolio. Their corresponding future values FV (C) and FV (D) are as of Dec-31, 2010.

	Actual Values				Future Values		
	C	D	NAV_{PE}	Index	FV (C)	FV (D)	NAV_{PE}
Dec-31, 2001	100	0	...	100	131	0	...
Dec-31, 2002	0	0	...	78	0	0	...
Dec-31, 2003	100	25	...	100	130	33	...
Dec-31, 2004	0	0	...	111	0	0	...
Dec-31, 2005	50	150	...	117	56	168	...
Dec-31, 2006	0	0	...	135	0	0	...
Dec-31, 2007	0	150	...	142	0	138	...
Dec-31, 2008	0	0	...	90	0	0	...
Dec-31, 2009	0	100	...	113	0	115	...
Dec-31, 2010	0	0	75	131	0	0	75
Total	250	425			317	453	
		TVPI	2.00			KS-PME	1.67

(131/117) x 50
= 55,98

Tools for estimating PE performance measures

- **EXCEL examples for PE performance measures**
    - https://www.insead.edu/sites/default/files/assets/dept/centres/gpei/docs/Measuring_PE_Fund-Performance-2019.pdf
- **Direct Alpha Method**
    - https://directalphamethod.info/
- **R codes done by Karl Polen (CIO of pension fund)**
    - http://past.rinfinance.com/agenda/2014/talk/KarlPolen.pdf
    - https://rpubs.com/kpolen/16062

How to address the problems of standard discrete-time CAPM?

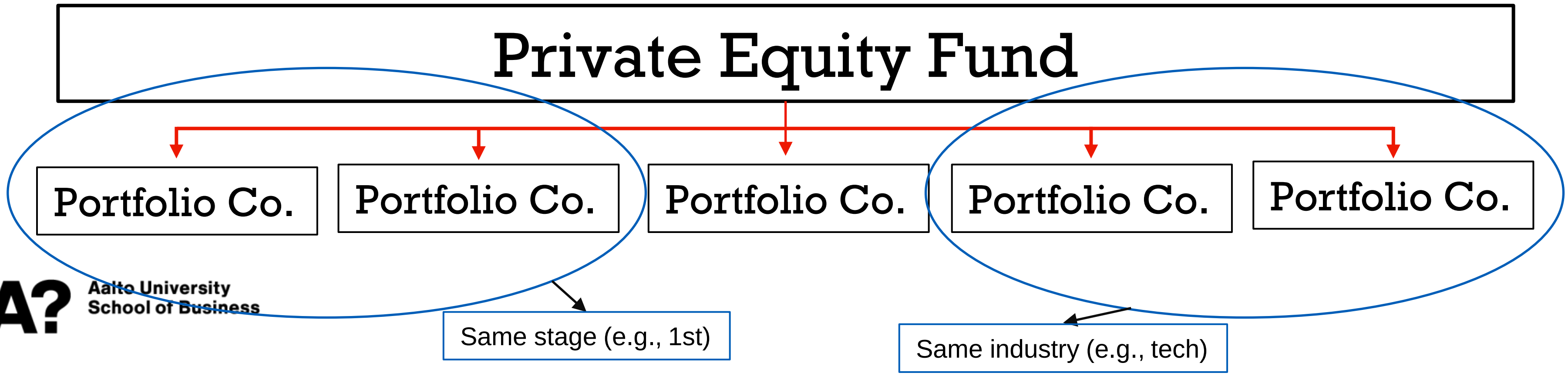
1. **Cash flow streams between the LPs and GPs (Fund-level data)**

2. **Portfolio company-level performance data (Deal-level data)**

Ang and Sorensen (2012)

Portfolio company-level entry and exit data

- **High statistical power** allows for **a more nuanced differentiation of the risks and returns** across industries and types and over time
- **Return** can be calculated **directly from the initial investment and the distribution of the proceeds at exit (infrequent data)**
- **For buyouts, fewer intermediate cash flows than for VC**



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Challenge: Returns are measured over different periods

- **A technical disadvantage** of portfolio company-level data is that **the returns are measured over different periods.**
  - Returns are measured from **the time of the initial investment to the time of exit**, and the **duration varies substantially across investments.**

- **The standard (discrete-time) CAPM is a one-period model,** where the period may be a day, a month, or a quarter.
  - CAPM does not compound, however, and the **returns must all be calculated over periods of the same length**.
  - Hence, the **standard** discrete-time **CAPM cannot be applied to PE** easily

Ang and Sorensen (2012)

A continuous-time version of the CAPM can be applied to company-level data

- The expected return regression is restated in **log-returns** (continuously-compounded)**:**

$$ln[1 + R_i(t)] - ln\left[1 + R_f(t)\right] = \delta + \beta\left(ln[1 + R_m(t)] - ln\left[1 + R_f(t)\right]\right) + \epsilon_i$$

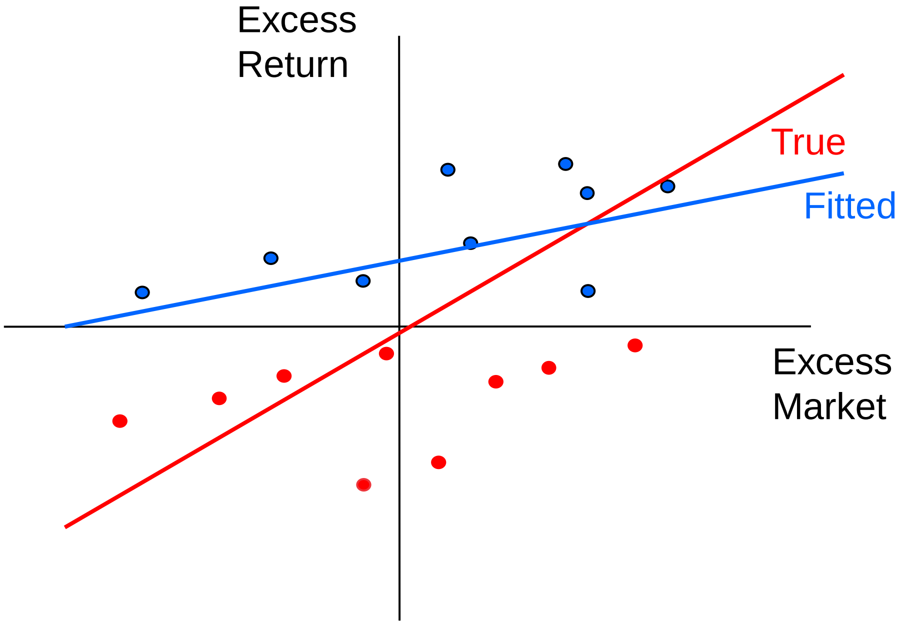
- One complication with the continuous-time CAPM is that **the estimated intercept** of the expected return equation **cannot be interpreted as an abnormal return**, as in the standard discrete-time CAPM.
- Under specific **distributional assumptions** about the way **volatility increases with the duration of an investment**, the **abnormal returns** can be calculated using the following adjustment

$$\alpha = \delta + \frac{1}{2}\sigma^2$$

Ang and Sorensen (2012)

Impact of Selection Bias on Alpha and Beta

Once zombi companies are taken into account alpha and beta estimates are not biased



Source: Korteweg and Sorensen (2011)

Betas for start-up portfolio companies

Betas without selection bias adjustment

	CAPM		FF3	
Intercept (δ)	-0.028	***	-0.022	***
Market (β)	2.077	***	1.810	***
SMB			-0.326	*
HML			-1.043	***
Sigma (σ)	1.339		1.354	

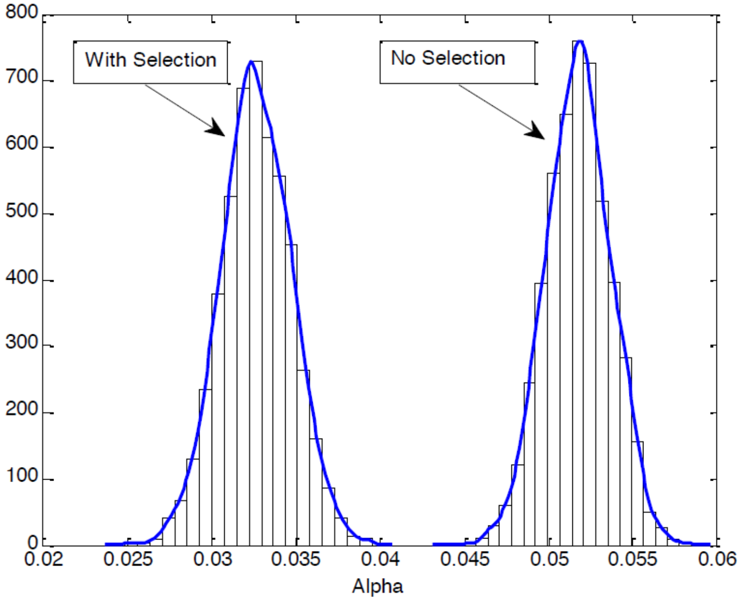
Betas with selection bias adjustment

	CAPM		FF3	
Intercept (δ)	-0.056	***	-0.054	***
Market (β)	2.751	***	2.343	***
SMB			1.065	***
HML			-1.639	***
Sigma (σ)	0.411	***	0.403	***

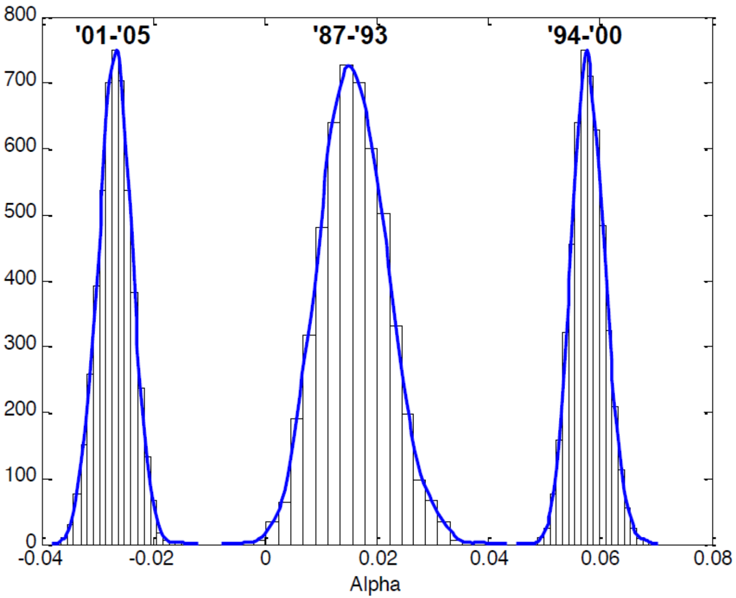
Source: Korteweg and Sorensen (2011)

Monthly alphas $(\alpha = \delta + \frac{1}{2}\sigma^2)$ for start-up portfolio companies

Alphas with and with selection adjustment



Alphas are time-varying



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Source: Korteweg and Sorensen (2011)

Betas with VC-specific factor

VC Factor is the log-change in the total dollar volume of VC investments in the month of the observation.

Betas when VC-specific Factor

Intercept	-0.053***	-0.052***
Market	0.934***	0.979***
SMB		0.520***
HML		-1.009***
VC Factor	**0.582*****	**0.477*****
Sigma	0.044***	0.403***

Source: Korteweg and Sorensen (2011)

Portfolio Sorts Instead of Continuous-Time CAPM

- **The non-linear adjustment alpha in the continuous-time version of the CAPM leads to high alphas when the volatility is high**
  - Alphas appear to be unreasonably high compared to studies using fund-level data

  → Are the assumptions really correct?
- **Franzoni, Nowak, and Phalippou (2012) sidestep this problem by estimating the CAPM after forming portfolios of deals rather than individual deals.**
  - This substantially lowers volatility and reduces the magnitude of the adjustment.
  - It does, however, reduce the other advantages of using individual deals.
  - It reduces statistical power, and the analysis must use a modified IRR approximation of returns.

Ang and Sorensen (2012)

Franzoni, Nowak, Phalippou (2012): Private Equity Performance and Liquidity Risk

- **PE (Buyout) is likely to be subject to aggregate liquidity shocks:**
  - Large transactions in entire companies.
  - Likely to generate high transaction costs and need for a liquid market
  - High leverage that needs to be refinanced (buyout funds).
  - If liquidity in credit markets shuts down, private equity (buyout) funds are obliged to liquidate their investments

- **Question: Do PE (buyout) returns load on traded (public) liquidity factor?**
  - To what extent a liquidity risk premium can explain the high returns on PE

Steps in Portfolio Sorts

1. **Estimate MIRR for each portfolio company**
2. **Form portfolios based on the month when the investment is started**
3. **Treat the portfolio returns as a time-series of returns on the whole asset class private equity**
4. **Estimate alphas and betas using one regression of this series of (portfolio) returns on the series of average factor realizations over the portfolio life**

Source: Franzoni, Nowak, and Phalippou (2012)

Table I

Cash Flows of a Typical Investment

The table shows the cash flows of a representative investment. It lasts for four years, pays a final dividend equal to 1.5 times the original investment, and pays an intermediate dividend in year 2.5 equal to half of the initial investment. We show the computation of the modified IRR (MIRR) with a re-investment rate of 5% per semester. At the bottom of the table we report the present value of the dividends using two different discount rates.

Date (in Years)	Cash Flows	Re-invested Dividend (at 5% per Semester)
0	−100	0
0.5	0	0
1	0	0
1.5	0	0
2	0	0
2.5	50	50
3	0	53
3.5	0	55
4	150	208
MIRR = $(208/100)^{1/4} - 1 = 20\%$		
IRR = 21%		
Present value of dividends at 15% discount rate		119
Present value of dividends at 18% discount rate		108

Source: Franzoni, Nowak, and Phalippou (2012)

$$(1+MIRR)^T = \frac{D_1 \prod_{t=1}^{T-1}(1+x_t) + D_2 \prod_{t=2}^{T-1}(1+x_t) + \cdots + D_{T-1}(1+x_{T-1}) + D_T}{I_0 + \frac{I_1}{(1+x_0)} + \frac{I_2}{\prod_{t=0}^{1}(1+x_t)} + \cdots + \frac{I_T}{\prod_{t=0}^{T-1}(1+x_t)}}$$

$$= \frac{FV(Div, x_t)}{PV(Inv, x_t)}, \qquad (1)$$

where $FV(\cdot, x_t)$ and $PV(\cdot, x_t)$, respectively, denote the forward and present value of a stream of cash flows computed using the discount rate x_t. Note that, when no cash is returned to investors (i.e., the dividends are all zero), the MIRR equals -100%.

We now give a numerical example of the construction of MIRR and its sensitivity to the reinvestment assumption. To do so we use the typical cash flow pattern shown in the Introduction (Table I) and assume a constant reinvestment rate of 5% per semester. The final value of the dividends is

$$FV\,(\text{Inv}, 5\%) = 50(1.05^3) + 150 = 208.$$

The annualized MIRR is thus

$$MIRR = \left(\frac{208}{100}\right)^{1/4} - 1 = 20\%.$$

Source: Franzoni, Nowak, and Phalippou (2012)

Alphas and Betas for the CAPM, FF3 and Pastor-Stambaugh (2003) models

Model:	Market	FF	PS
	Panel A: Risk Models		
IML_PS			0.638
			(3.539)
Rm-Rf	0.948	1.395	1.294
	(6.688)	(5.443)	(5.227)
HML		0.719	1.020
		(2.450)	(3.466)
SMB		−0.124	−0.040
		(−0.497)	(−0.167)
Constant	0.006	0.000	−0.002
	(6.003)	(0.035)	(−0.712)
Sigma	0.049	0.048	0.046
Adj. R^2	0.849	0.853	0.865
N	139	139	139

Buyout deals loads on the MKT, HML, and P&S traded liquidity factor

Alpha (Constant) of PE goes to zero when the liquidity factor is included!

Market Beta is 1 and significant Alpha

Source: Franzoni, Nowak, and Phalippou (2012)

Annual Performance by Deciles of Liquidity Conditions

The difference between bad versus good liquidity conditions is a striking 46% per year.

Figure 3. Annual performance by deciles of liquidity conditions. The figure plots the average investment MIRR in each decile of the Pástor and Stambaugh (2003) liquidity condition variable.

Source: Franzoni, Nowak, and Phalippou (2012)

Takeaway

- **PE (buyout investments) loads on a liquidity risk factor**
    - Important in terms of performance of PE in extreme events
    - Compensation for liquidity risk is about 3%

- **Compensation for liquidity risk can account for abnormal performance of PE (alpha=0)**
    - Based on this paper, it is not clear how would VC investments load to liquidity risks
    - Newer papers show that traded liquidity risk factor does not explain all private equity alpha away

Source: Franzoni, Nowak, and Phalippou (2012)

Conclusions

- **We learned to interpret alphas and betas to VC and buyout deals**
  - Traded public factors are intuitively related to public risk factors
  - Maybe some diversification benefits

- **We learned how we can measure private equity fund performance**
  - IRR, Multiple, and PME

- **We understand how both cashflow and deal level data can be used in measuring risk and return (performance)**