# 28E35700 Alternative Investments

## Evaluating Hedge Fund Risk and Return

## Spring 2024


## Juha Joenväärä

Aalto University
School of Business

# Agenda

- **Performance measument:**
  - Alphas and Betas
  - Risk Factor Selection
  - Operational risk / Fraud indicators
  - Tail Risk and Tail Correlation
  - Trade-off: Positive Convexity vs Alpha

# Indented Learning Outcomes

- You know how *investment fund* skill and risk – alpha and beta – are defined, can be obtained and interpreted.

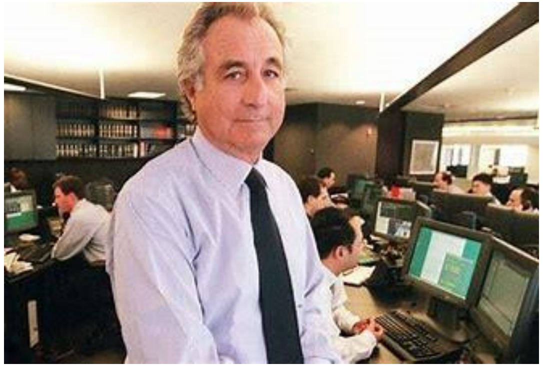
- You understand why the standard performance evaluation framework is difficult to apply to *hedge funds* and what are the limitations of quantitative due diligence based on the regression models.

- You understand how tail risk and tail correlation are measured and linked to diversification benefits. Sophisticated investors prefer hedge funds that hedge but deliver also alpha

# Could You Name Some Famous Hedge Funds?

1. Some HF that has delivered extremely high returns?

2. Some HF has failed/ been fraud?



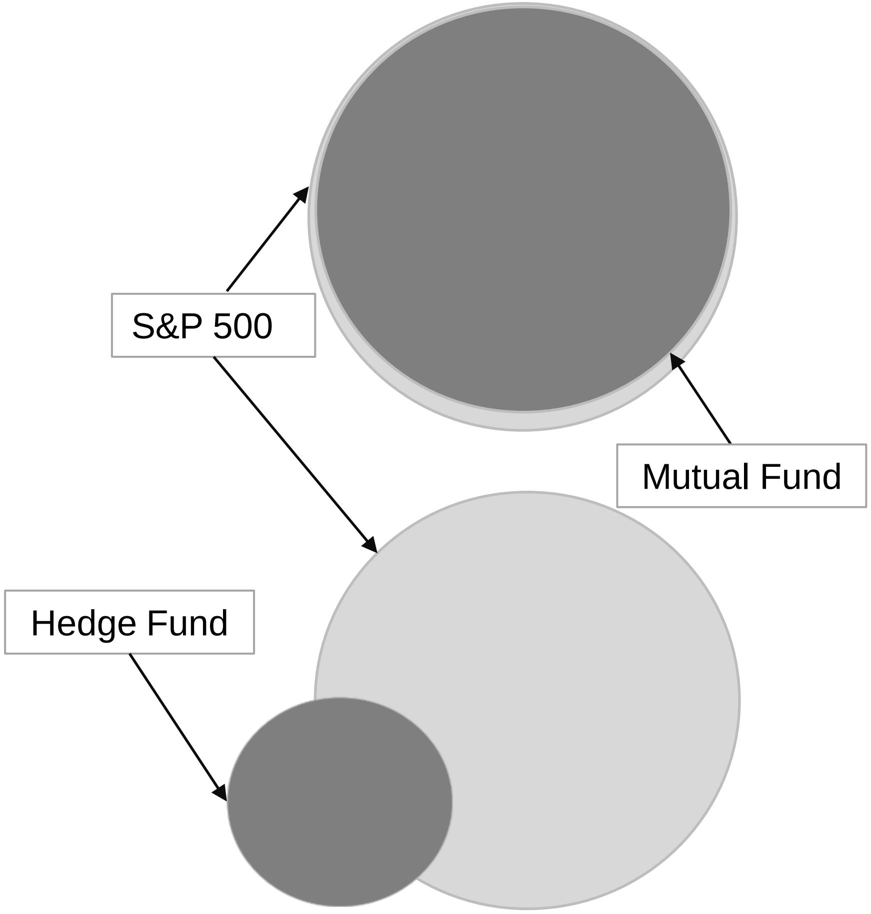
# How a Hedge Fund Differs from a Mutual Fund?

- **What is a mutual fund?**

  A mutual fund is a type of investment vehicle consisting of a portfolio of stocks, bonds, or other securities. Mutual funds give small or individual investors access to diversified, professionally managed portfolios at a low price.

- **What is a hedge fund?**

  Hedge funds are actively managed investment pools whose managers use a wide range of strategies, often including buying with borrowed money and trading esoteric assets, in an effort to beat average investment returns for their clients. They are considered risky alternative investment choices. Hedge funds require a high minimum investment or net worth, excluding all but wealthy clients.




Aalto University
School of Business

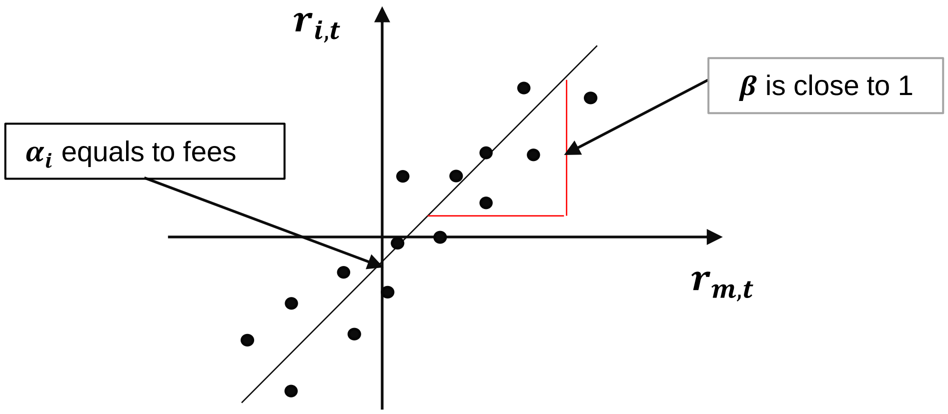
# Mutual Fund Performance Evaluation

**Typical mutual fund manager generates returns based on the systematic risk he takes – the so-called beta risk – and the value his abilities contribute to the investment process – his so-called alpha.**

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon$$

**Investors are unlikely to pay the manager much for returns from beta risk.**

**Investors are willing to pay for alpha.**




**What you can say about typical mutual fund's $\alpha_i$ and $\beta_i$?**

**$\alpha_i$ is often slighly negative equaling to fees to paid for the fund manager**
**$\beta$ is close to 1.**

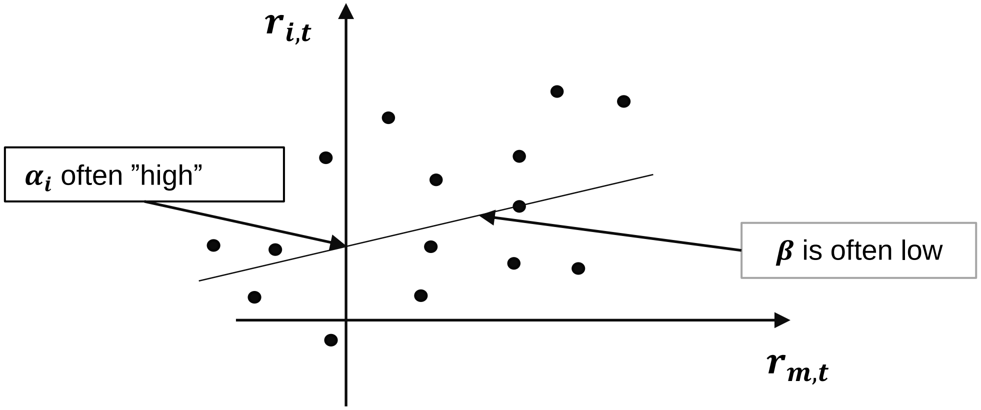
**$R^2$ is typically above 90%.**

# Hedge Fund Performance Evaluation

Hedge fund manager typically engages in sophisticated investment strategies that involve concentrated positions, short sales, illiquid assets, complex derivatives, and leverage.

Hedge fund has a potential to generate alpha, and, therefore, investors are often willing to pay high management-based and performance-based fees.

However, hedge fund investment strategies can be very risky and deliver poor performance when the diversification benefits are mostly needed.




Can you apply the same regression framework for hedge funds?

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon$$

$\alpha_i$ would be often positive and $\beta_i$ lower than for the mutual funds. Average fund's $\beta_i = 0.40$.

$R^2$ is typically 20 - 40%.

# Can we improve the regression model's explanory power?

**In the 1970 view, there is <u>one source of systematic risk</u>, the market index.**

- **Researchers have documented dozens of dimensions of systematic risks.**
- **Maybe we add those additional sources of systematic risk to our performance evaluation model?**

$$r = \alpha + \beta_1 f_1 + \beta_2 f_2 + \beta_3 f_3 + \cdots + \beta_k f_k + \varepsilon$$

- **After adding <u>the right factors</u>, the $\alpha$ would be equal to zero/fees and $R^2$ close to 100%.**

**<u>John Cochrane's view (AFA Keynote Address):</u>**

- **I tried telling a hedge fund manager, "you don't have alpha. I can replicate your returns with a value-growth, momentum, carry, and short-vol strategy."**
- **The hedge fund manager said, "'Exotic beta' is my alpha. I understand those systematic factors and know how to trade them. You don't. So, I have deserved my fees"**

# If we have ”right” systematic risk factors in our model then the alpha would be zero (equal to fees)

- Here is our extended model with additional systematic risk factors

$$r = \alpha + \beta_1 f_1 + \beta_2 f_2 + \beta_3 f_3 + \cdots + \beta_k f_k + \varepsilon$$

- When we have right factors, then the alpha is close to zero (or equal to fees)

$$r = -0.01 + \beta_1 f_1 + \beta_2 f_2 + \beta_3 f_3 + \cdots + \beta_k f_k + \varepsilon$$

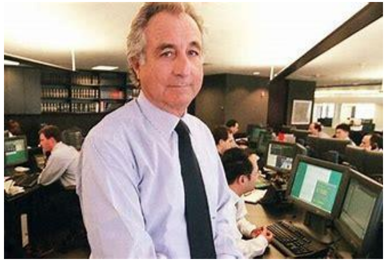
- In practice, it is however very difficult know what are the right systematic factors, and therefore we often have positive alpha

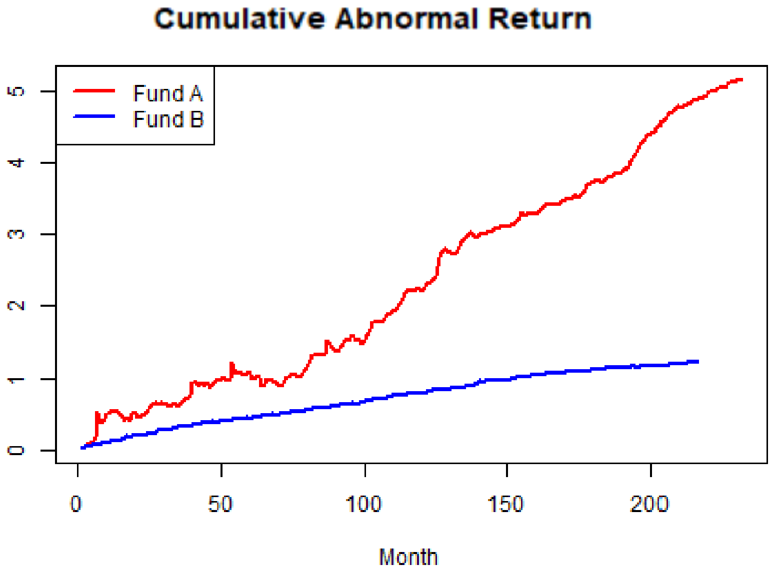
$$r = 0.02 + \beta_1 f_1 + \beta_2 f_2 + \beta_3 f_3 + \cdots + \beta_k f_k + \varepsilon$$

# Alphas and Betas for Famous Hedge Funds

1. **Renaissance Technologies Medallion Fund**

2. **Bernie Madoff's Fund**






| | Alpha | | Betas | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Fund | Alpha | *t*-stat | Mkt_RF | SMB | HML | RMW | CMA | UMD | STR |
| Fund A | 26.7** | 5.5 | -0.062 | -0.018 | 0.080 | 0.042 | -0.256 | 0.071 | -0.130 |
| Fund B | 6.9** | 11.6 | 0.044** | -0.038* | -0.019 | 0.015 | 0.006 | -0.008 | 0.001 |

The fund's returns are regressed against the market, size (SMB), value (HML), operating profitability (RMW), CMA (investments), momentum (UMD), and equity option straddle (STR) factors)

# Back to Cochrane’s point

- **Hedge fund manager has a point.**
  - How many investors have through their exposures to carry trade or short volatility “systematic risks,” and are ready to consider those as “passive,” mechanical investments?
  - To an investor who hasn’t heard of it and holds the market index, a new factor is alpha. And has nothing to do with informational inefficiency.

- **Most active management and performance evaluation, just isn’t well described by the alpha-beta, information-systematic, selection-style split anymore.**
  - There is no more alpha.
  - There is just beta you understand and beta you don’t understand.

# Three examples of how to select factors

# Renaissance Institutional Equities Fund

## Models & signals

- **“There are an underlying set of models that constitute all our funds”**
- **Different equity risk models**
  1. 40% of signals are based on fundamentals
  2. 40% on technical
  3. 20% others
     - E.g., social networking, analysts’ opinions, insider trading

## “Everything we do is data driven”

- **“The big black boxes need input”**
- **Inputs are divided into:**
  - Risk models
  - Cost models
  - Predictive signals

| | Alpha | Market | SMB | HML | BAB | QMJ | CS_MOM (Equities) |
|---|---|---|---|---|---|---|---|
| **Renaissance Institutional Equities Fund** | 0.0005 (0.334) | 0.572 (12.631***) | -0.182 (-2.587*) | 0.293 (4.601***) | 0.317 (5.498***) | 0.670 (8.155***) | 0.176 (3.302**) |



The fund’s returns are regressed against the market, size (SMB), value (HML), Betting Against Beta (BAB), Quality Minus Junk (QMJ) and Cross-Sectional Momentum

# Two Sigma Spectrum Fund

**“Factors are not generic”**

- “You can’t compare us to anyone else”
- E.g., value ratio
  - The accounting value of a firm/market value of a firm
  - The inputs in just this one ratio could be construed and built in 20 different ways
- E.g., earnings yield
  - The range in some instances may be -4% to +12%
- Different firms have different inputs for these models, despite being called the same name
- There are a broad range of inputs and outputs
- They try to look at multiple definitions of a factor
- Including stability and correlation models

**Portfolio**

- Portfolio optimization occurs at the stock level
- They have risk premia models covering:
  - Value
  - Carry
  - Quality
  - Low volatility
  - Size
  - Momentum
  - Weighted residual forecasts
  - Consolidated forecasts
- There are more than 50 models
- With different weights and portfolio properties

| | Alpha | Market | RMW | MOM | BAB | QMJ | TS_MOM (Currencies) |
|---|---|---|---|---|---|---|---|
| **Two Sigma Spectrum Fund** | 0.005 (5.391***) | 0.052 (1.703) | 0.237 (3.101**) | 0.108 (3.962***) | 0.071 (1.916) | -0.131 (-1.989*) | 0.034 (1.682) |

The fund’s returns are regressed against the market, operating profitability (RMW), momentum (MOM), Betting Against Beta (BAB), Quality Minus Junk (QMJ) and Time-Series Momentum for Currencies factors

# Warren Buffett's Alpha

- Warren Buffett's Berkshire Hathaway has realized a **Sharpe ratio of 0.79 with significant alpha to traditional risk factors.**
    - The alpha became insignificant, however, when we controlled for exposure to the factors "betting against beta" and "quality minus junk."
    - Buffett's leverage is about 1.7 to 1, on average.
    - **Buffett's returns** appear to be neither luck nor magic but, rather, **a reward for leveraging cheap, safe, high-quality stocks.**

- Decomposing Berkshire's **portfolio into publicly traded stocks and private companies**
    - public stocks have performed the best

    → Buffett's returns are more the result of stock selection than of his effect on management.

# How to Identify Frauds?

# Quantitative and Qualitative Due Diligence

**Use of statistical tools**

- Red flags based on fund returns
- If a fund is suspicious → additional investigation

**Investment strategy**

- Is risk and return profile plausible?
- Exposure to typical risk factors of the investment strategy
- How related to other funds with similar style/strategy

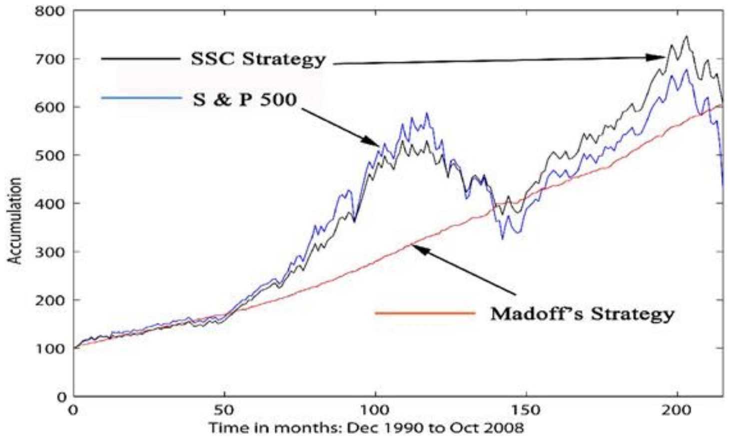
**If the passes quantitative DD → perform qualitative DD**

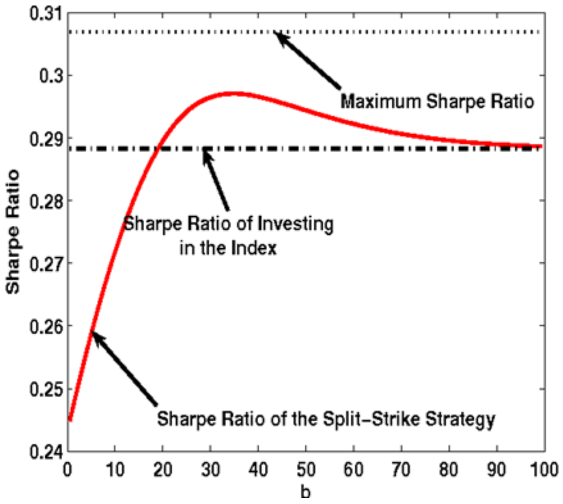
- Ask the right questions

# Fairfield Sentry’s Investment Strategy

The Fund seeks to obtain capital appreciation of its assets principally through the utilization of a non-traditional options trading strategy described as `split strike conversion’, to which the Fund allocates the predominant portion of its assets. The investment strategy has defined risk and reward parameters. The establishment of a typical position entails (i) the purchase of a group or basket of equity securities that are intended to highly correlate to the S&P 100 Index, (ii) the purchase of out-of-the-money S&P 100 Index put options with a notional value that approximately equals the market value of the basket of equity securities and (iii) the sale of out-of-the-money S&P 100 Index call options with a notional value that approximately equals the market value of the basket of equity securities. The basket typically consists of between 40 to 50 stocks in the S&P 100 Index. The primary purpose of the long put options is to limit the market risk of the stock basket at the strike price of the long puts. The primary purpose of the short call options is to largely finance the cost of the put hedge and to increase the stand-still rate of return. The `split strike conversion' strategy is implemented by Bernard L. Madoff Investment Securities LLC (BLM), a broker-dealer registered with the Securities and Exchange Commission, through accounts maintained by the Fund at that firm. The services of BLM and its personnel are essential to the continued operation of the Fund, and its profitability, if any. The Investment Manager, in its sole and exclusive discretion, may allocate a portion of the Fund's assets (never to exceed, in the aggregate, 5% of the Fund's Net Asset Value, measured at the time of investment) to alternative investment opportunities other than its `split strike conversion' investments.

# Split Strike Conversion (SSC) Strategy vs Madoff's Fairfield Sentry




| | Excess returns | | | | Betas | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Fund | Mean | SD | Sharpe | Alpha | Mkt_RF | SMB | HML | RMW | CMA | UMD | STR | AdjR2 |
| Madoff | 6.949 | 2.393 | 2.904 | 6.903** | 0.044** | -0.038* | -0.019 | 0.015 | 0.006 | -0.008 | 0.001 | 0.078 |
| | | | | 11.623 | 3.129 | -2.358 | -0.820 | 0.680 | 0.187 | -0.726 | 0.067 | |

# Performance Flags

**Fund managers have an incentive to report attractive returns because investors are performance sensitive**

- Goetzmann et al. (2003 *JF*)

**Fund managers can manipulate performance measures like the Sharpe ratio to make return series more attractive**

- Getmansky, Lo, Makarov (2004 *JFE*), Goetzmann et al. (2007 *RFS*)

**Fund managers can misreport returns**

- Bollen and Pool (2009 *JF*), Agarwal, Daniel, Naik (2009 *RFS*)

# Operational Flags

## Brown, Goetzmann, Liang, and Schwarz

- Use Form ADVs to study "problem funds" (regulatory violations)
- In 2008 *JF*, show that problem funds much more likely to trigger operational flags such as conflicts of interest
- In 2009 FAJ, construct the $\omega$-score to proxy for operational risk and show it is related to premature fund closure

## Dimmock and Gerken (2010 JFE)

- Also study Form ADVs and "problem funds"
- Prior regulatory violations help predict 38% of subsequent violations

# Bollen and Pool (2011, RFS): Red Flags

**Prob (Fraud) = a + b x Flag 1 + c x Flag 2 +…**

- Flag 1: Kink in Return Distribution at Zero
- Flag 2: Low Correlation with Other Assets
- Flag 3: Unconditional Serial Correlation
- Flag 4: Conditional Serial Correlation
- Flag 5: Data Quality

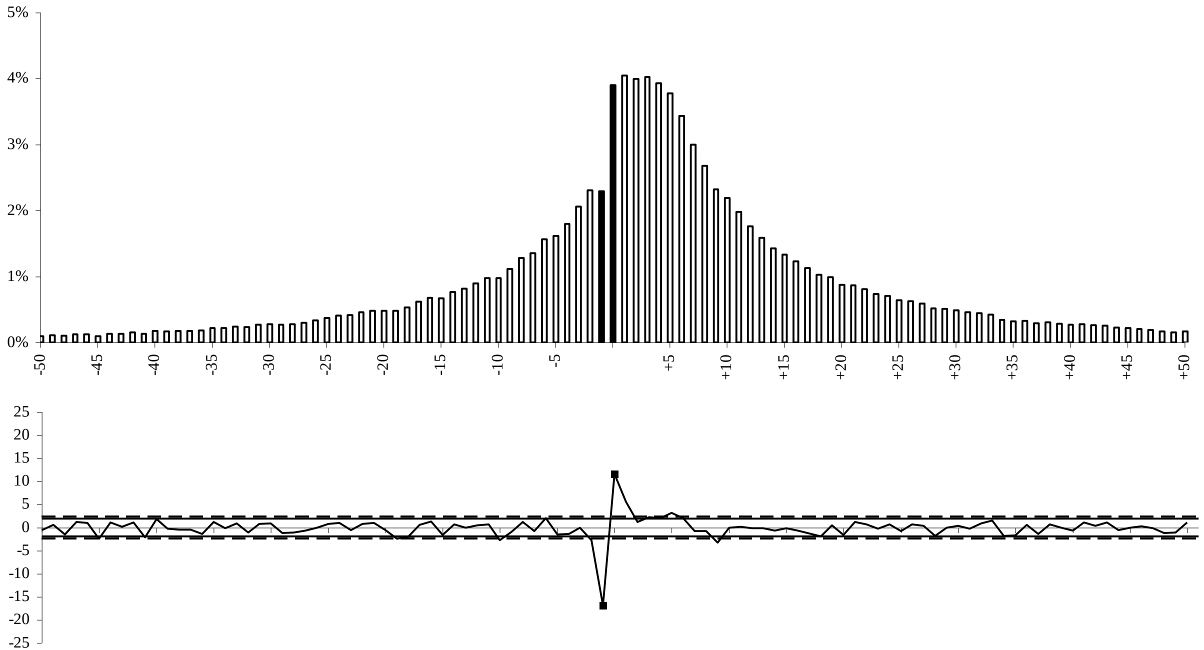
**If the Flag gets value of 1 if the fund is suspicious, and otherwise 0**

# Predicting Hedge Fund Frauds: 1994-2017

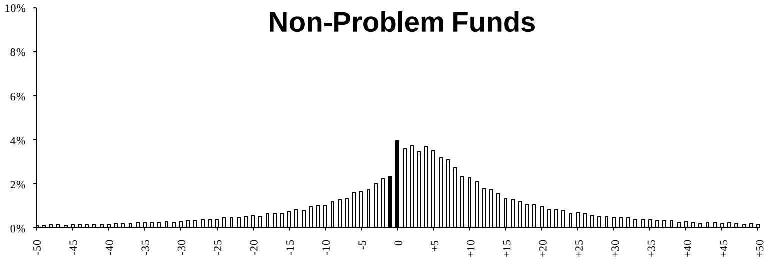
| | | Fraud indicator variable | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Kink flag | Estimate | 0.131 | | | | | | | | | |
| | Statistic | (1.77) | | | | | | | | | |
| | AME | [0.003] | | | | | | | | | |
| Max Adj-R2 flag | Estimate | | 0.064 | | | | | | | | |
| | Statistic | | (0.73) | | | | | | | | |
| | AME | | [0.002] | | | | | | | | |
| Index flag | Estimate | | | 0.141 | | | | | | | |
| | Statistic | | | (2.04) | | | | | | | |
| | AME | | | [0.003] | | | | | | | |
| AR flag | Estimate | | | | -0.116 | | | | | | |
| | Statistic | | | | (-1.35) | | | | | | |
| | AME | | | | [-0.003] | | | | | | |
| CAR flag | Estimate | | | | | 0.143 | | | | | |
| | Statistic | | | | | (2.30) | | | | | |
| | AME | | | | | [0.003] | | | | | |
| Repeat flag | Estimate | | | | | | 0.159 | | | | |
| | Statistic | | | | | | (1.12) | | | | |
| | AME | | | | | | [0.004] | | | | |
| Uniform flag | Estimate | | | | | | | 0.160 | | | |
| | Statistic | | | | | | | (1.16) | | | |
| | AME | | | | | | | [0.004] | | | |
| # Zero flag | Estimate | | | | | | | | -0.078 | | |
| | Statistic | | | | | | | | (-0.50) | | |
| | AME | | | | | | | | [-0.002] | | |
| % Negative flag | Estimate | | | | | | | | | 0.041 | |
| | Statistic | | | | | | | | | (0.29) | |
| | AME | | | | | | | | | [0.001] | |
| Number of flags | Estimate | | | | | | | | | | 0.057 |
| | Statistic | | | | | | | | | | (1.93) |
| | AME | | | | | | | | | | [0.001] |
| Controls? | | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Number of observations | | 20083 | 20083 | 20083 | 20083 | 20083 | 20083 | 20083 | 20083 | 20083 | 20083 |
| Number of funds | | 4580 | 4580 | 4580 | 4580 | 4580 | 4580 | 4580 | 4580 | 4580 | 4580 |
| Number of firms | | 1965 | 1965 | 1965 | 1965 | 1965 | 1965 | 1965 | 1965 | 1965 | 1965 |

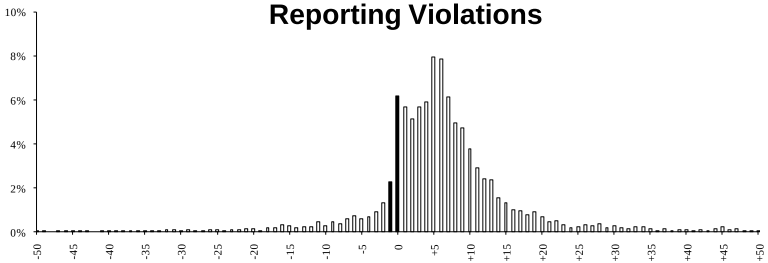
# Flag 1: Kink in Return Distribution at Zero

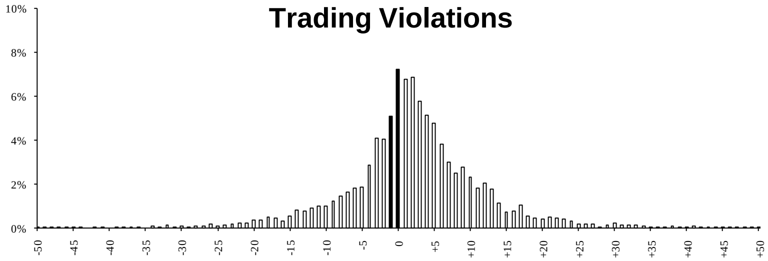
CISDM data 1994 – 2005
Bollen and Pool (2009 *JF*)




Aalto University
School of Business

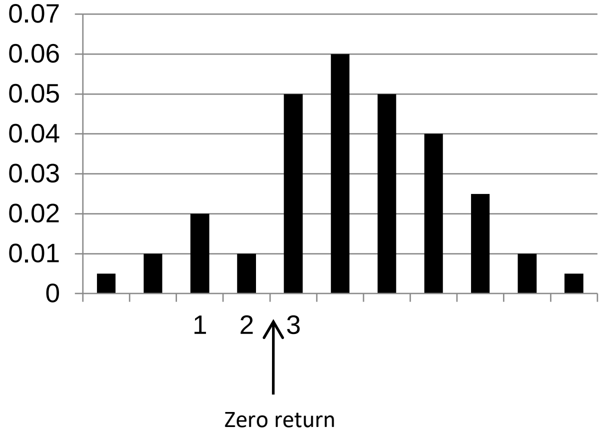

Non-Problem Funds
10%
8%
6%
4%
2%
0%
-50 -45 -40 -35 -30 -25 -20 -15 -10 -5 0 +5 +10 +15 +20 +25 +30 +35 +40 +45 +50


Reporting Violations
10%
8%
6%
4%
2%
0%
-50 -45 -40 -35 -30 -25 -20 -15 -10 -5 0 +5 +10 +15 +20 +25 +30 +35 +40 +45 +50

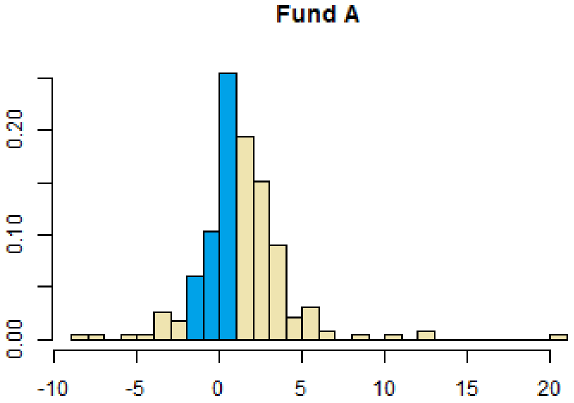

Trading Violations
10%
8%
6%
4%
2%
0%
-50 -45 -40 -35 -30 -25 -20 -15 -10 -5 0 +5 +10 +15 +20 +25 +30 +35 +40 +45 +50

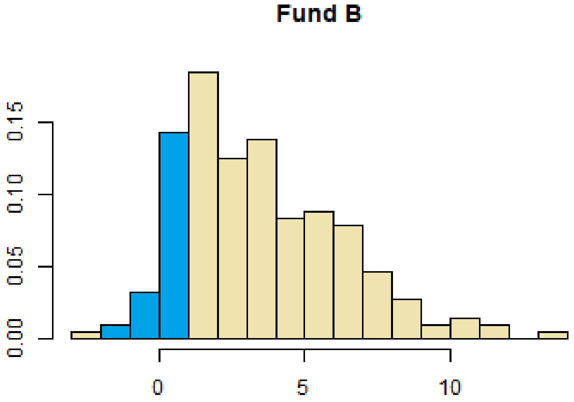
# Testing for a Kink for Individual Fund

- Choose optimal bin size equal to $1.058464 \times \sigma \times N^{-1/5}$ for sample size $N$
- Use the approach of Burgstahler and Dichev (1997 *JAE*)
- Let $p_1$, $p_2$, and $p_3$ be the percentage of observations in each of 3 bins
- Reject hypothesis of a smooth distribution if height of bin 2 is significantly different than the average height of surrounding bins




$$z \equiv \frac{p_2 - \frac{1}{2}(p_1 + p_3)}{\frac{1}{N}\sqrt{N(p_2 - p_2^2) + \frac{1}{4}N(p_1 - p_1^2 + p_3 - p_3^2) + Np_2(p_1 + p_3) - \frac{1}{2}Np_1p_3}} \sim N(0,1)$$




- Bin width = 0.0195
- $p_1 = 0.06$
- $p_2 = 0.10$
- $p_3 = 0.26$
- $p_2 - (p_1 + p_3) / 2 = -0.05$
- *z-stat* = −1.94
- => Borderline kinky returns

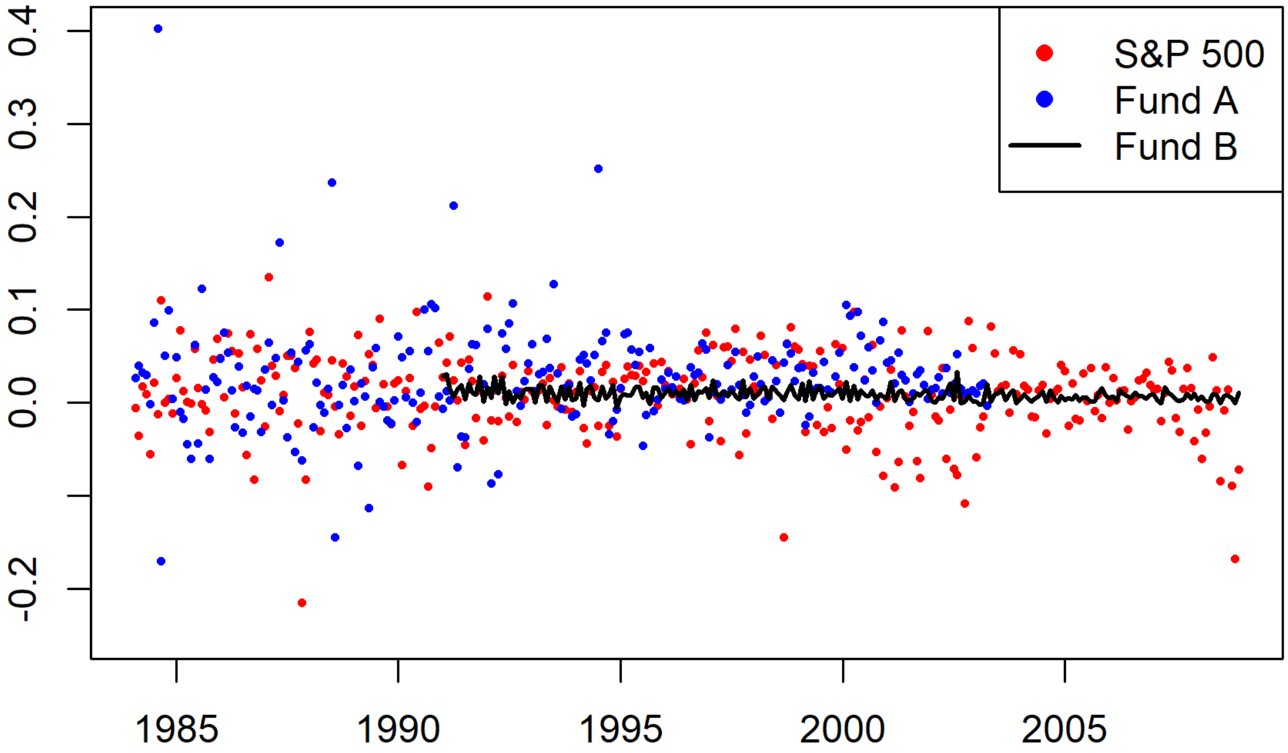
- Bin width = 0.0026
- $p_1 = 0.01$
- $p_2 = 0.03$
- $p_3 = 0.14$
- $p_2 - (p_1 + p_3) / 2 = -0.04$
- *z-stat* = −2.47
- => Kinkier returns

# Flag 2: Low Correlation with Other Assets

**Harry Markopolos computed a correlation of 0.06 between Madoff and S&P 500, whereas split-strike conversion strategy should have a correlation around 0.50**

**If fund returns are uncorrelated with other assets, this suggests they are random (but likely positive) and evidence of fraud**

**Alternatively, low correlation could be generated by pure idiosyncratic bets, so if flag is triggered it could be a false positive (but my intuition is that most funds have *some* exposure to *some* style factors)**



S&P 500
Fund A
Fund B
0.4
0.3
0.2
0.1
0.0
-0.2
1985
1990
1995
2000
2005

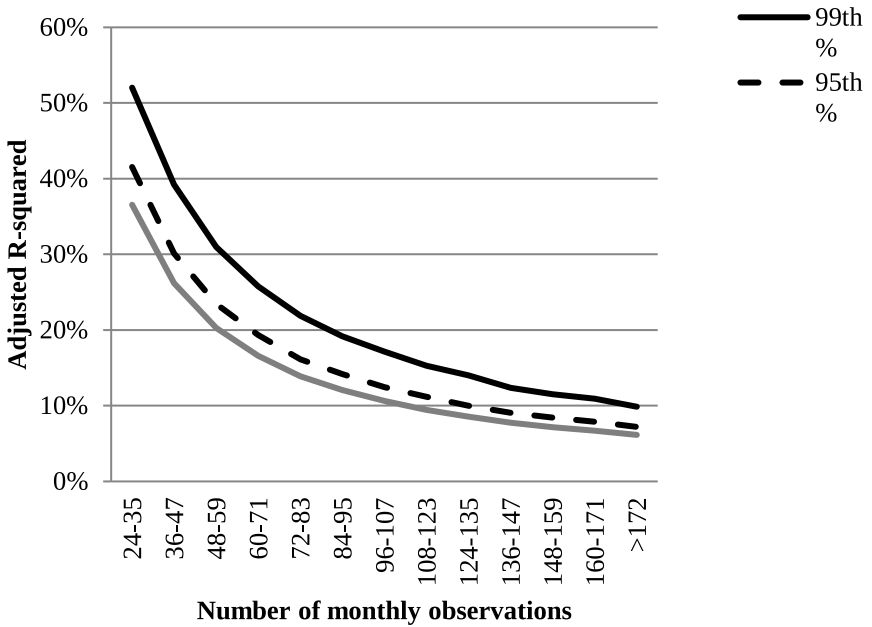
# Testing for Low Correlation

**For each fund, find optimal subset of 14 factors that maximize adjusted R-squared**

**Test whether adjusted R-squared is significantly different from zero through simulation as in Foster, Smith, and Whaley (1997 *JF*)**

- For each fund generate 100 random series of standard normal variates of length equal to fund's history
- Find optimal subset for each and record adjusted *R*-squared
- Use 90th percentile as cutoff levels for significance

# Average Critical Values for $R^2$




Fund A and B both has more than 200 return observations

| Fund | Alpha | Mkt_RF | SMB | HML | RMW | CMA | UMD | STR | Adj. R2 |
|---|---|---|---|---|---|---|---|---|---|
| Medallion | 26.779 | -0.062 | -0.018 | 0.080 | 0.042 | -0.256 | 0.071 | -0.130 | -1.202 % |
| Madoff | 6.903 | 0.044 | -0.038 | -0.019 | 0.015 | 0.006 | -0.008 | 0.001 | 7.843 % |

# Fraud vs Skilled Fund

**Low $R^2$ may also indicate that the fund is skilled**

- Titman and Tiu (2011, RFS) provide a simple argument that suggests that better-informed hedge funds choose to have less exposure to factor risk.
- They show that hedge funds that exhibit lower $R^2$ with respect to systematic factors deliver superior performance
- In contrast, Bollen (2013, JFQA) shows that low –$R^2$ funds fail more often and are exposed to systematic risk

# Calculating HF Returns Involves Problems of Valuation and Misreporting

- A **mutual fund** invested in stocks can compute the **daily value** of its portfolio by using the closing prices of the stocks.
- **Hedge funds** often hold securities that are **not traded on exchanges**.
  - Many derivatives are **traded over-the-counter.**
  - For securities not traded on an exchange, **no closing price exists**.
  - Need to **rely on theoretical models** to estimate the value of some securities
  - Need to **rely on quoted prices** rather than actual transaction prices.

- **In an efficient market**, one would not expect the return of a fund for one month to have information for the return of the fund over the next month.
  - **No serial correlation**

# Flag 3: Unconditional Serial Correlation

**Getmansky et al. (2004 *JFE*) show that if a manager reports a simple moving average of current and past month returns, reported return series will have artificially high Sharpe ratio and unconditional serial correlation**

**We regress fund returns on first lags and use a positive, significant coefficient as an indicator of misreporting**

$$R_t^O = a + bR_{t-1}^O + \varepsilon_t$$

# Flag 4: Conditional Serial Correlation

**Bollen and Pool (2008 *JFQA*) argue that managers have an incentive to smooth losses but not gains, hence the level of serial correlation might be conditional on the magnitude of lagged returns**

**We use their approach to test for serial correlation:**

$$R_t^O = a + b^+ R_{t-1}^O + b^- \left(1 - I_{t-1}\right) R_{t-1}^O + \varepsilon_t$$

**where *I* is an indicator that equals one if the lagged actual return (proxied for by the fitted value of an optimal factor regression) is above its mean and zero otherwise**

# Flag 5: Data Quality

- **Straumann (2008) describes 5 suspicious patterns:**
    - High number of returns exactly equal to zero
    - Low percentage of unique returns *
    - Non-uniform distribution of last digit of returns
    - High number of repeat pairs of returns *
    - Long sequence of constant returns *

- **Bollen and Pool (2011) Additional Data Quality Flag**
    - Too Few Negatives

- **For *, bootstrap critical values using a range of means, volatilities, rounding conventions, and history lengths**

# Flag 5: Too Few Negatives

<u>Harry Markopolos</u> testified to U.S. House of Reps that Madoff reported only 3 months of losses in an 87-month span, compared to 28 down months for S&P 500

Gregoriou and Lhabitant (2009 *JWM*) show 10 months of losses out of 215 for Fairfield Sentry, a Madoff feeder fund

| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | Year |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1990 | | | | | | | | | | | | 2.8% | 2.8% |
| 1991 | 3.1% | 1.5% | 0.6% | 1.4% | 1.9% | 0.4% | 2.0% | 1.1% | 0.8% | 2.8% | 0.1% | 1.6% | 18.6% |
| 1992 | 0.5% | 2.8% | 1.0% | 2.9% | -0.2% | 1.3% | 0.0% | 0.9% | 0.4% | 1.4% | 1.4% | 1.4% | 14.7% |
| 1993 | 0.0% | 1.9% | 1.9% | 0.1% | 1.7% | 0.9% | 0.1% | 1.8% | 0.4% | 1.8% | 0.3% | 0.5% | 11.7% |
| 1994 | 2.2% | -0.4% | 1.5% | 1.8% | 0.5% | 0.3% | 1.8% | 0.4% | 0.8% | 1.9% | -0.6% | 0.7% | 11.5% |
| 1995 | 0.9% | 0.8% | 0.8% | 1.7% | 1.7% | 0.5% | 1.1% | -0.2% | 1.7% | 1.6% | 0.5% | 1.1% | 13.0% |
| 1996 | 1.5% | 0.7% | 1.2% | 0.6% | 1.4% | 0.2% | 1.9% | 0.3% | 1.2% | 1.1% | 1.6% | 0.5% | 13.0% |
| 1997 | 2.5% | 0.7% | 0.9% | 1.2% | 0.6% | 1.3% | 0.8% | 0.4% | 2.4% | 0.6% | 1.6% | 0.4% | 14.0% |
| 1998 | 0.9% | 1.3% | 1.8% | 0.4% | 1.8% | 1.3% | 0.8% | 0.3% | 1.0% | 1.9% | 0.8% | 0.3% | 13.4% |
| 1999 | 2.1% | 0.2% | 2.3% | 0.4% | 1.5% | 1.8% | 0.4% | 0.9% | 0.7% | 1.1% | 1.6% | 0.4% | 14.2% |
| 2000 | 2.2% | 0.2% | 1.8% | 0.3% | 1.4% | 0.8% | 0.7% | 1.3% | 0.3% | 0.9% | 0.7% | 0.4% | 11.6% |
| 2001 | 2.2% | 0.1% | 1.1% | 1.3% | 0.3% | 0.2% | 0.4% | 1.0% | 0.7% | 1.3% | 1.2% | 0.2% | 10.7% |
| 2002 | 0.0% | 0.6% | 0.5% | 1.2% | 2.1% | 0.3% | 3.4% | -0.1% | 0.1% | 0.7% | 0.2% | 0.1% | 9.3% |
| 2003 | -0.3% | 0.0% | 2.0% | 0.1% | 1.0% | 1.0% | 1.4% | 0.2% | 0.9% | 1.3% | -0.1% | 0.3% | 8.2% |
| 2004 | 0.9% | 0.5% | 0.1% | 0.4% | 0.7% | 1.3% | 0.1% | 1.3% | 0.5% | 0.0% | 0.8% | 0.2% | 7.1% |
| 2005 | 0.5% | 0.4% | 0.9% | 0.1% | 0.6% | 0.5% | 0.1% | 0.2% | 0.9% | 1.6% | 0.8% | 0.5% | 7.3% |
| 2006 | 0.7% | 0.2% | 1.3% | 0.9% | 0.7% | 0.5% | 1.1% | 0.8% | 0.7% | 0.4% | 0.9% | 0.9% | 9.4% |
| 2007 | 0.3% | -0.1% | 1.6% | 1.0% | 0.8% | 0.2% | 0.2% | 0.3% | 0.2% | 0.5% | 1.0% | 0.2% | 6.4% |
| 2008 | 0.6% | 0.1% | 0.2% | 0.9% | 0.8% | -0.1% | 0.7% | 0.7% | 0.5% | -0.1% | | | 4.5% |

# Time-varying Performance and Tail Risk

# CalSTRS hunting new risk mitigating strats

## The $312bn system could look to tail-risk hedging plays

**Connor Owen**
9 SEP 2022

The California State Teachers Retirement System (CalSTRS) is seeking additional hedge fund strategies to diversify its risk mitigating strategies (RMS) portfolio.

The $312bn system has built out its RMS portfolio significantly since increasing the target allocation to 10% back in 2019. Despite setbacks, including Covid-19 and home-working directives, RMS now stands at 9.7% (over $30bn) as of July 31.

Within the portfolio, CalSTRS allocates a total of $17.5bn to trend following managers, $3.5bn to global macro and nearly $1bn to systematic risk premia. The other component of RMS is long US Treasuries, which were allocated close to $9bn.

Scaling up the portfolio in certain areas – namely global macro and risk premia – remains a challenge. However, CalSTRS continues to push for diversification in both existing strategies and potential new ones.

**Potential for tail-risk hedging strategies**

Some parts of the RMS portfolio have shone bright over the past 12 months – notably the trend following strategy, which was up over 30% – but the Treasuries component has struggled, losing close to 20% for the year ending June 30.

"The effectiveness of this hedge will be challenged by continued central bank interest rate hiking and sustained inflation which can reduce bond prices," CalSTRS acknowledged.

A number of Meketa's other clients have turned to tail-risk hedging strategies as a substitute for long Treasury exposure within their RMS portfolios, which can be a more cost-effective option in a rising rate environment.

CalSTRS could also turn to tail hedge strategies when evaluating potential RMS additions. However, within this strategy and others, putting capital to work at a multi-billion-dollar scale remains an obstacle for the investment team.

The system also recently approved its first allocation to a women-owned investment manager within the RMS portfolio, and expects to fund the new hire during this fiscal year.

# Alphas and Betas can be time-varying

- **Benchmark model's alpha and beta can be time-varying**

$$\boldsymbol{r_{i,t} = \alpha_{i,t} + \beta_{i,t} r_{m,t} + \varepsilon}$$

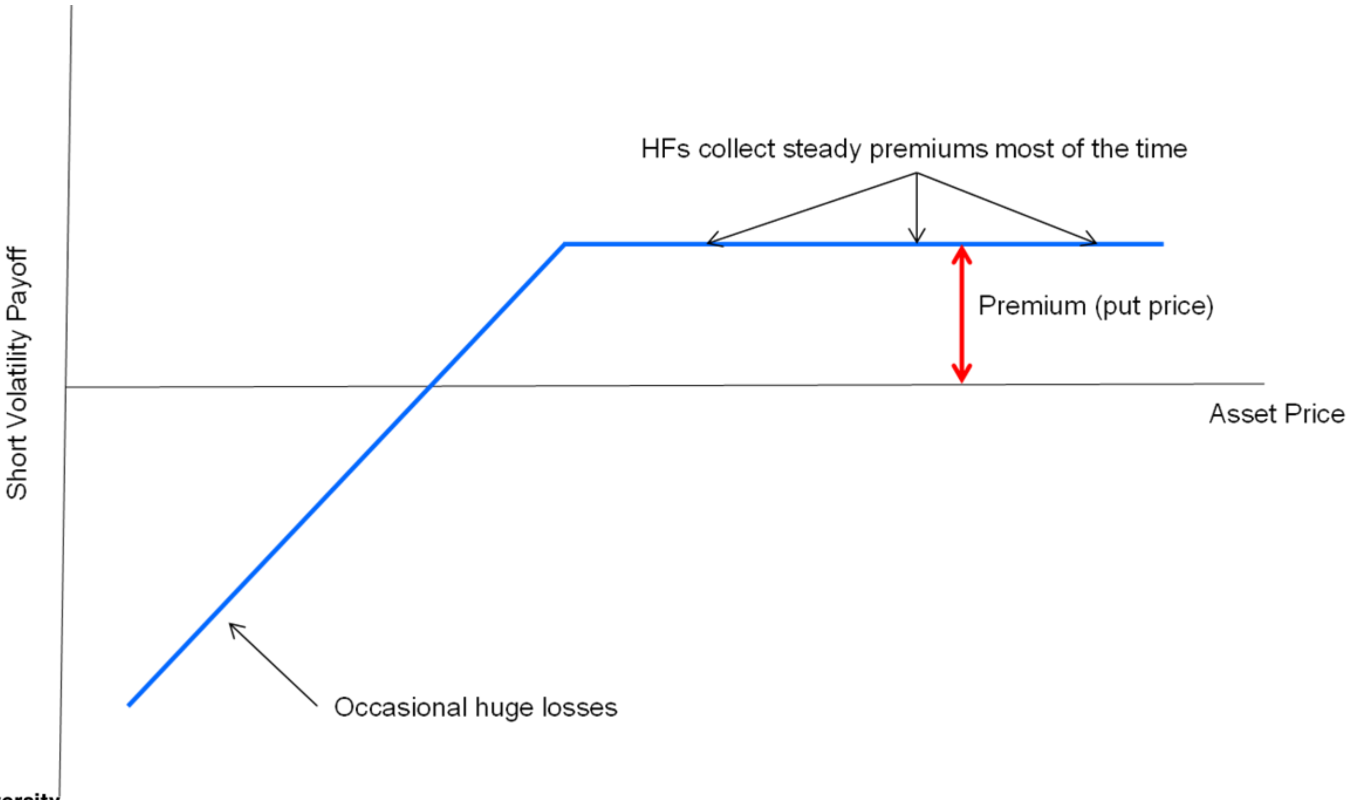
  - $\boldsymbol{\alpha_{i,t}}$ depends on time *t*
  - $\boldsymbol{\beta_{i,t}}$ depends on time *t*

- **Estimate separate alpha and beta for up (normal) and down (crisis) markets**

$$\boldsymbol{r_{i,t} = \alpha_{i,Up} + \beta_{i,Up} r_{m,t} + \varepsilon} \quad \textbf{and} \quad \boldsymbol{r_{i,t} = \alpha_{i,Down} + \beta_{i,Down} r_{m,t} + \varepsilon}$$

- **Ability to diversify depends on their capability to deliver alpha during the crisis**

# Some Hedge Fund Strategy Returns = Short Put



# Merger Arbitrage Strategy

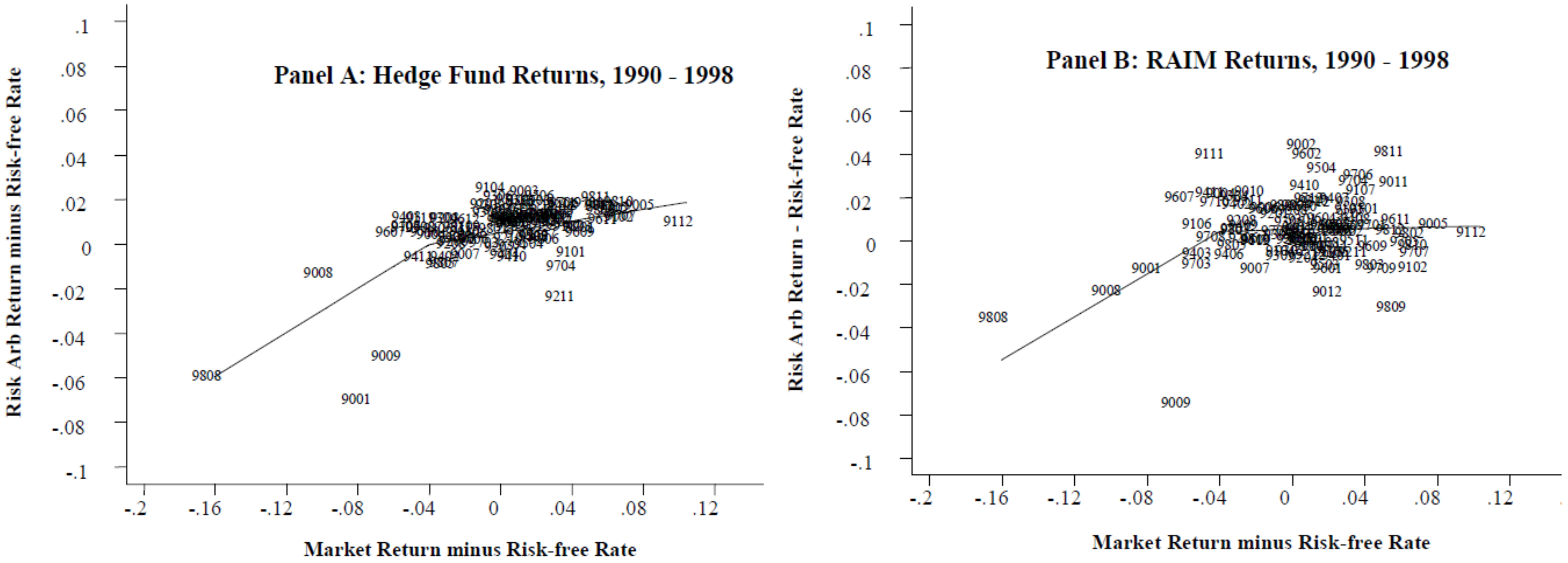



**Figure 5:** This figure compares RAIM returns and hedge fund returns during 1990 - 1998. Panel A presents hedge fund returns obtained from Hedge Fund Research's merger arbitrage index and Panel B presents RAIM returns. Data labels correspond to months (i.e. 9808 is August, 1998). Fitted lines from a piecewise linear regression are also shown.

Mitchell and Pulvino (2001, JF)

# Expected shortfall conditional on market distress

- **VaR is given by the inverse of the probability function:**

$$\mathrm{VaR}^{\boldsymbol{i}} = -F_{\boldsymbol{r}}^{-1}(\boldsymbol{\gamma})$$

Confidence level $\gamma$ (1% or 5%)

- **Expected shortfall:** $\mathrm{ES} = -E\left[\boldsymbol{r_i} \,\middle|\, \boldsymbol{r_i} \leq -\mathrm{VaR}^{\boldsymbol{i}}\right]$

- **Expected shortfall conditional on market distress:**

$$\mathrm{TCTR}^{i} = -E[\boldsymbol{r_i}|\boldsymbol{r_m} \leq -\mathrm{VaR}^{m}]$$

Aalto University
School of Business

# Example: Empirical VaR and ES

- Let's assume that Fund A has 100 return observations
- To compute **VaR and ES**

1. Sort **Fund A** returns in a **descending order**

**2. VaR at Confidence level 1% = -**6.2% and 5% = -2.3%

**3. ES at Confidence level 5% =**

-(2.3+2.4+3.1+4.4+6.2) / 5 = -3.68%

**Fund A**

| | | | |
|---|---|---|---|
| **Dec 2021** | **1.2%** | **Nov 2013** | **5.2%** |
| **Nov 2021** | **- 4.4%** | **.** | **.** |
| **.** | **.** | **Sep 2013** | **4.2%** |
| **Jan 2019** | **-6.2%** | **.** | **.** |
| **.** | **.** | **.** | **.** |
| **Dec 2018** | **-2.4%** | **Dec 2021** | **1.2%** |
| **.** | **.** | **.** | **.** |
| **Sep 2014** | **-2.3%** | **Sep 2014** | **-2.3%** |
| **.** | **.** | **Dec 2018** | **-2.4%** |
| **Nov 2013** | **5.2%** | **Oct 2013** | **-3.1%** |
| **Oct 2013** | **-3.1%** | **Nov 2021** | **-4.4%** |
| **Sep 2013** | **4.2%** | **Jan 2019** | **-6.2%** |

# Example: ES conditional on market distress

- To compute **ES conditional on market distress**

1. Sort **market returns** in a **descending order**

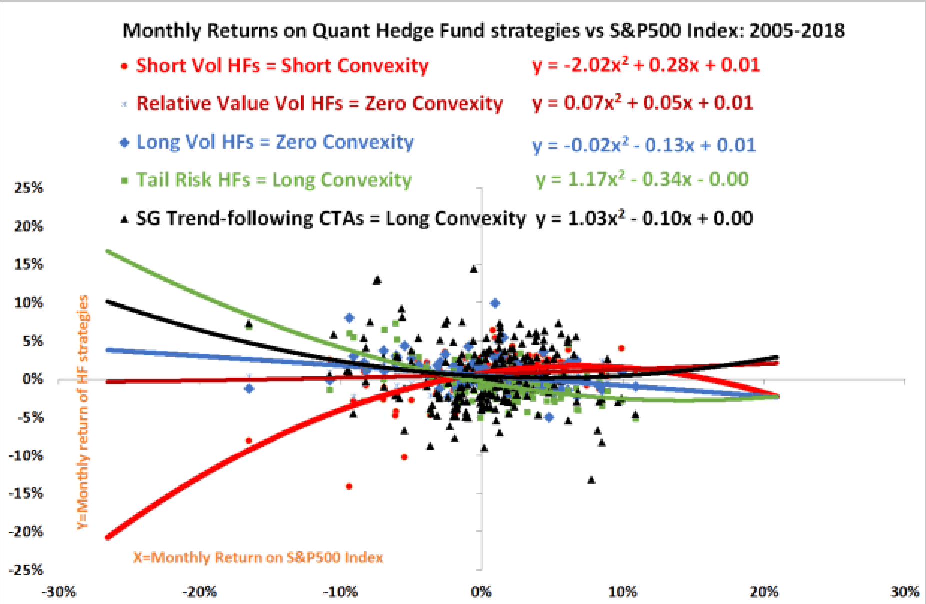
2. **Match Fund A returns by DATE**

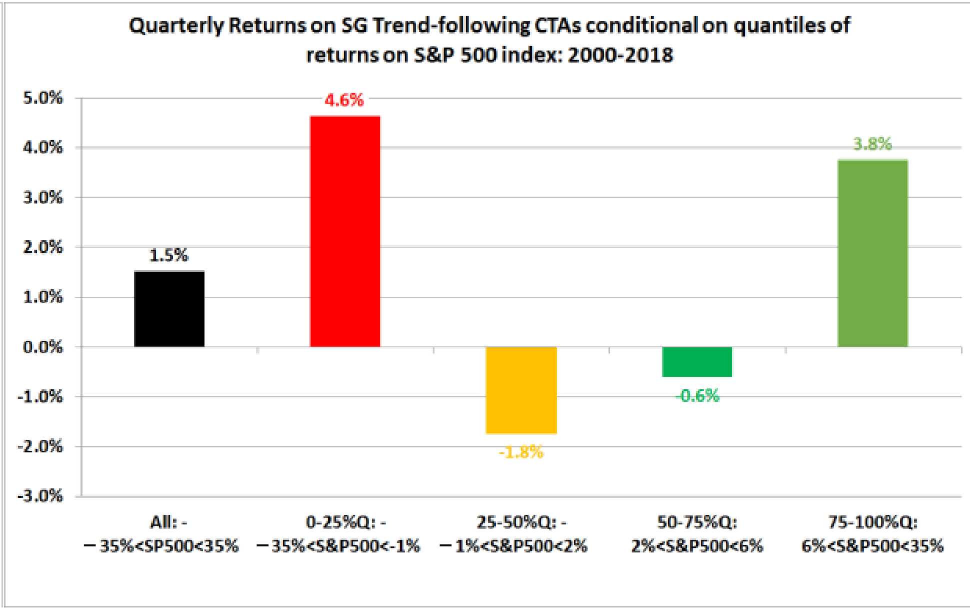
3. **ES conditional on market distress at Confidence level 5% =**

**(-2.3-10.2+5.2-3.1-4.2) / 5 = -1.24%**

| Fund A | | Market return | |
|---|---|---|---|
| Nov 2013 | 1.2% | Nov 2013 | 5.5% |
| . | . | . | . |
| Sep 2013 | 2.3% | Sep 2013 | 3.2% |
| . | . | . | . |
| . | . | . | . |
| Dec 2021 | -2.4% | Dec 2021 | 1.4% |
| . | . | . | . |
| Sep 2014 | -2.3% | Sep 2014 | -2.1% |
| Dec 2018 | -10.2% | Dec 2018 | -2.4% |
| Oct 2013 | 5.2% | Oct 2013 | -3.9% |
| Noc 2021 | -3.1% | Nov 2021 | -4.8% |
| Jan 2019 | 4.2% | Jan 2019 | -8.2% |

# Trade-off: Positive Convexity vs Average Return



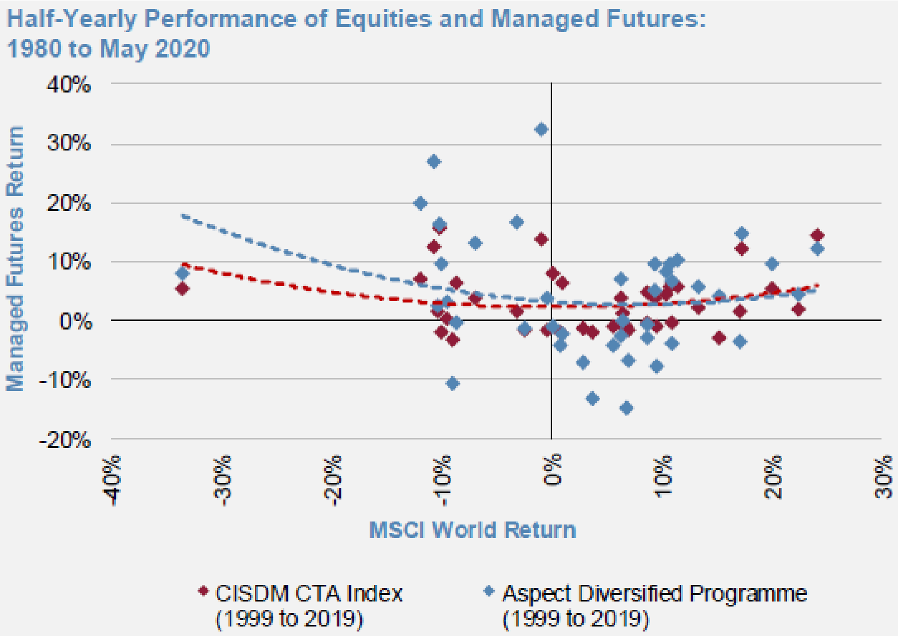



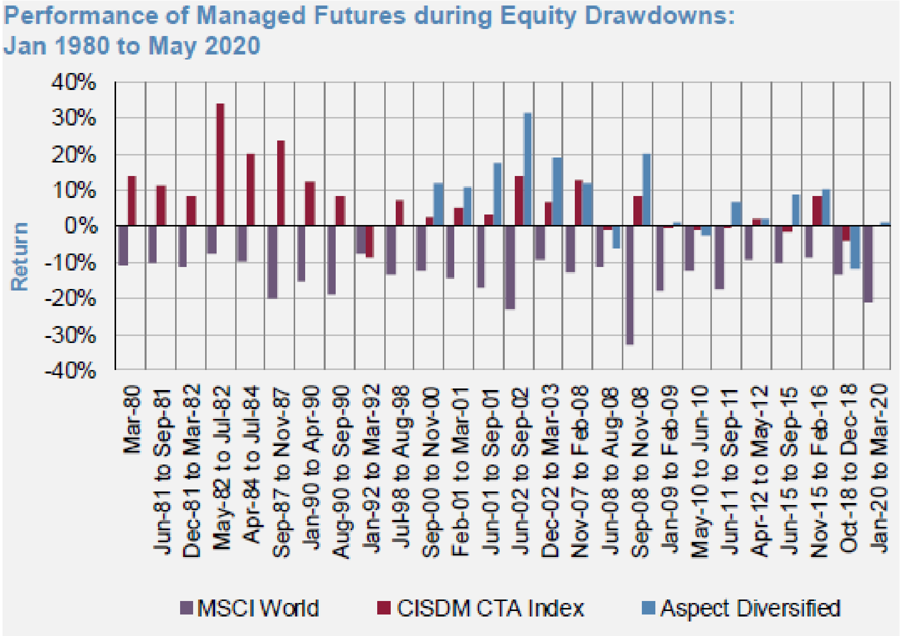

The SG CTA Index is equal weighted for a pool of Commodity-Trading Advisors (CTAs) selected from the larger managers that are open to new investment.

Source of Figures

Aalto University
School of Business

# Aspect Diversified Programme




Aalto University
School of Business

Aspect Capital Limited was established in 1997 by Anthony Todd (chief executive officer), Martin Lueck (research director), Michael Adam, and Eugene Lambert. The firm launched the Aspect Diversified Programme in December 1998 as a systematic trend-following program that invests in financial and commodity futures, currency forwards, and other derivatives. The momentum-based strategy follows medium-term trends in over 140 futures and forward markets. To enhance trend-capturing, the program employs modulating models including momentum, non-price, and cross-market models. The team attempts to maintain a diversified portfolio, with a focus on heavy research and risk management to control volatility, correlation and liquidity effects.

# Appendix

# Definitions of Tail Risk Measures

# 1 Measures of Tail Risk

## 1.1 Value-at-Risk

Let the random variable $R$ denote return on an investment;[1] its probability function is written as

$$F_R(x) = \Pr[R \leq x].$$

Given a fixed confidence level $\alpha$ (usually 1% or 5%), the $(1-\alpha)$-level value-at-risk (VaR) of an investment is defined by the $\alpha$ quantile of the return:

$$\text{VaR} = -\inf\{x \in \mathbb{R} : \alpha \leq F_R(x)\}.$$

If the return distribution is continuous, then VaR is given simply by the inverse of the probability function:

$$\text{VaR} = -F_R^{-1}(\alpha).$$

It is straightforward to estimate VaR using nonparametric estimators based on the empirical distribution function.

## 1.2 Expected Shortfall

A theoretical problem with the value-at-risk measure is its lack of subadditivity; that is, the VaR of a group of investments may exceed the sum of their individual VaRs (Artzner et al. 1999). This characteristic runs counter to the intuitive notion of a diversified portfolio being no more risky than its constituents. The expected shortfall (ES) measure does not suffer from this problem (Acerbi and Tasche 2001); it is defined as follows:

$$\text{ES} = -E[R \mid R \leq -\text{VaR}].$$

## 1.3 Conditional Tail Risk

Let $R^i$ denote the return of an investment fund and $R^m$ the market return. We define Total Conditional Tail Risk (TCTR) as the fund's expected shortfall conditional on market distress:

$$\text{TCTR}^i = -E\left[R^i \,\middle|\, R^m \leq -\text{VaR}^m\right].$$

$\text{TCTR}^i$ can be estimated using the nonparametric estimators based on the empirical distribution function. We can set $\alpha = 5\%$ and use the S&P 500 return as a proxy for market return.

# Addressing return illiquidity

# Fixing Benchmark Model for Addressing Illiquidity

- The OLS estimator is **consistent** when the **errors are *homoscedastic* and *serially uncorrelated***

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon$$

- This is **not** often **true** since hedge funds **invest in illiquid assets**
  - Too high $\alpha_i$ and too low $\beta_i$

- **Potential fix for the serial correlation (stale prices or illiquidity)**
  - Fit MA(2) –model to residuals (Getmansky, Lo and Makarov 2004)
  - Use lagged Factors i.e., so called Dimson betas (Asness et al 2001)
  - Credit Bond Factor is already autocorrelated and therefore address ”factor-based-liquidity” (During the next lecture we talk about Fung-Hsieh (2004) model)

## Adjusted performance measures

Let $r_{i,t}$ denote the true excess return of a hedge fund $i$ in period $t$. Then, true equilibrium returns can be described as follows:

$$r_{i,t} = \alpha_i + \beta_i' X_t + \varepsilon_{i,t}, \qquad t = t_0, \ldots, T_i \tag{6}$$

where $X_t$ is $T_i \times K$ the vector of returns on $K$ zero-cost portfolios, and $\varepsilon_{i,t}$ denotes regression disturbance, which is assumed to be, in each period $t$, an independent realization from normal distribution with zero mean and variance $\sigma^2_{\varepsilon_i}$.

We follow Getmansky, Lo, and Makarov (2004) by assuming that hedge funds' true economic return is not observed. Specifically, $r^o_{i,t}$, observed return for hedge fund $i$, is a weighted average of the fund's true return over the most recent $s+1$ periods, which can be expressed as follows:

$$\begin{aligned} r^o_{i,t} &= \theta_0 r_{i,t} + \theta_1 r_{i,t-1} + \ldots + \theta_s r_{i,t-s}, && (7)\\ 1 &= \theta_0 + \theta_1 + \ldots + \theta_s, && (8)\\ \theta_j &\in [0,1], j = 0, 1, \ldots, s. && (9) \end{aligned}$$

According to Getmansky, Lo, and Makarov (2004), $(1-\theta_0)$ measures the degree to which the fund's returns are smoothed. When value is close to one then the hedge fund is interpreted to invest in liquid assets.

Combining equations (6) and (7) through (9), we can rewrite observed returns as follows

$$r^o_{i,t} = \alpha_i + \theta_0 \beta_i' X_t + \theta_1 \beta_i' X_{t-1} + \ldots + \theta_s \beta_i' X_{t-s} + u_{i,t}, \tag{10}$$

where $u_{i,t}$ follows constrained $MA(s)$-process as

$$\begin{aligned} u_{i,t} &= \theta_0 \varepsilon_{i,t} + \theta_1 \varepsilon_{i,t-1} + \ldots + \theta_s \varepsilon_{i,t-s}, && (11)\\ 1 &= \theta_0 + \theta_1 + \ldots + \theta_s, && (12) \end{aligned}$$

where $\widehat{\alpha}_i$ is the hedge fund $i$ alpha. We have estimated a regression model (10) through (12) by using Maximum Likelihood estimation by applying the procedure provided by Getmansky, Lo, and Makarov (2004).

As Getmansky, Lo, and Makarov (2004) show hedge funds' consistent estimates for alphas, Sharpe ratios, appraisal ratios and variances can be obtained as follows

$$\begin{aligned} SR &= \widehat{SR}^o \sqrt{\widehat{\theta}_0^2 + \widehat{\theta}_1^2 + \cdots + \widehat{\theta}_s^2}, && (13)\\ AR &= \widehat{AR}^o \sqrt{\widehat{\theta}_0^2 + \widehat{\theta}_1^2 + \cdots + \widehat{\theta}_s^2}, && (14)\\ Var &= \frac{\widehat{Var}^o}{\sqrt{\widehat{\theta}_0^2 + \widehat{\theta}_1^2 + \cdots + \widehat{\theta}_s^2}}, && (15) \end{aligned}$$

where $\widehat{SR}^o$, $\widehat{AR}^o$ and $\widehat{Var}^o$ are estimates of Sharpe ratio, appraisal ratio and variance of the observed returns, and $\widehat{\theta}_0, \widehat{\theta}_1, \ldots, \widehat{\theta}_s$ are Maximum Likelihood estimates of $\theta_0, \theta_1, \ldots, \theta_s$ from equation 11. We follow Jagannathan, Malakhov, and Nokikov (2010) and Titman and Tiu (2008) by using $s = 2$.