Model solutions to exam 2019-10-24.

Multiple choice 1e, 2d, 3a, 4a, 5b, 6e, 7b, 8b, 9c, 10d.

I (a) A market has a **natural monopoly** if the cost of setting up a new firm is so high that there is room for only one. Example: the cost of constructing the grid for electricity transmission is so high that duplicating the grid would make no economic sense.

(b) **Complements** are more useful when consumed together, for example paper and pencils. An increase in the price of a good decreases the demand for its complements.

(c) **Income elasticity of demand** describes how demand reacts to changes in income as the ratio of percentage changes. Example: if a 10% increase in consumer income results in a 2% increase in sales for some good then its income elasticity is $2/10 = 0.2$.

(d) **Marginal cost of public funds** (MCF) measures the welfare cost of raising additional tax revenue. It takes into account that taxation, in addition to merely causing a transfer of wealth equal to tax revenue, tends to reduce economic activity in the markets where the tax revenue is raised.

II The impact of a tax on prices depends on the shapes of demand and supply curves. The long-run effect of a tax decrease will partly show up as a decrease in consumer price and partly as an increase in producer price, regardless of how competitive an industry is. The OVD was arguing for a very extreme prediction, where all of the tax cut would show up in consumer prices while producer prices would remain unchanged. While logically possible (if demand is completely inelastic or supply completely elastic), such extreme outcomes are almost never observed in practice. OVD's assumption is neither simpler nor more reasonable than the opposite extreme, whereby the entire tax decrease would show up as an increase in producer prices and not benefit consumers at all.

III (a) Optimal level of production is found at the intersection of marginal cost and marginal revenue. Note that $p^d(q) = 10 - q/4$. In this case the effective marginal cost is a constant 4 (3 plus 1 for the tax) euros for each can. Set up the revenue function to find MR:

$$R(q) = (10 - q/4)q = 10q - q^2/4$$
$$MR(q) = \frac{\partial R(q)}{\partial q} = 10 - q/2$$

Solve optimal p and q from MR = MC:

$$10 - q/2 = 4 \implies$$
$$q^* := 12 \implies$$
$$p^* := p^d(12) = 10 - 3 = 7$$

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Should Acme Inc enter? It should, if total profits are positive at the profit maximizing price and quantity.

$$\pi^* = p^* q^* - 4q^* - \text{FC}$$
$$= 7 \times 12 - 4 \times 12 - 30$$
$$= 84 - 48 - 30 = 6$$

Acme Inc should enter because $\pi(q^*) > 0$.

(b) Now the marginal cost is $3 + t$ where $t \in \{0, 2\}$. We can find the optimal solution following exactly the steps in part (a). Additionally Acme Inc can opt out of production in which case it only loses $1/3$ of the fixed cost of 30. This gives us the maximum profits in all three cases.

$$\pi(t = 2) = -5$$
$$\pi(t = 0) = 19$$
$$\pi(\text{no production}) = -30 \times 1/3 = -10$$

The expected profits for launching in Bulvania without finding out t in advance is:

$$E(\pi) = -5 \times 0.5 + 19 \times 0.5 = 7$$

Notice that Acme Inc should launch irrespective of the realized tax, because if $t = 2$, launching will results in less losses than withdrawing; $-5 > -10$. If Acme Inc finds out that t = 2 beforehand, they can withdraw without the fixed cost of 30 and the expected profits are:

$$E(\pi) = 0 \times 0.5 + 19 \times 0.5 = 9.5$$

The expected profits for finding out t in advance are 2.5 higher than without finding out. This is how much Acme Inc would be willing to pay at most for the services of XPols. Intuitively, these gains come from the timing of when the value of t is realized. In the latter case t is revealed earlier due to the services of XPols, which enables Acme Inc to avoid the fixed costs if $t = 2$, unlike in the first case.

IV (a) All commuters can benefit from the Metro in a way that does not depend on the number of other users. In order to calculate the efficient frequency of train service, we can use the optimality condition for efficient provision of public goods, stating that at the efficient level marginal cost, MC, is equal to marginal benefit, MB.

First calculate marginal benefit as a function of train frequency q (trains/hour):

$$\text{MB}_i(q) = \frac{\partial \text{TB}_i(q)}{\partial q}$$
$$\text{MB}_i(q) = 20 - 0.5q$$

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Marginal cost is obtained as the derivative of variable costs (in millions €):

$$VC(q) = 2q,$$
$$MC = 2$$

Efficient provision is then given by:

$$MC = MB(q)$$

To get the comparable $MB(q)$ function, we need to aggregate all benefits from commuters and measure the benefits in millions. Aggregated total benefit and marginal benefit in millions € are:

$$TB(q) = \sum_i TB_i(q) = 0.2 \cdot (20q - 0.25q^2) = 4q - 0.05q^2$$
$$MB(q) = 4 - 0.1q$$

The efficiency condition is then:

$$MB(q) = MC(q)$$
$$4 - 0.1q = 2$$
$$q^* := 20$$

We need to check that the optimal frequency is feasible and total benefits exceed total costs. Figures are in millions:

$$TB(q) = 4 \cdot 20 - 0.05 \cdot 20^2 = 60$$
$$TC(q) = 2 \cdot 20 + 12 = 52$$
$$TB(q) > TC(q)$$

The efficient provision of metro service is therefore 20 trains per hour.

(b) Efficient provision is by definition the quantity at whcich marginal benefit equal marginal cost. Therefore, setting a uniform price that is equal to the marginal cost does not distort efficiency. As a result the optimal frequency is still 20 trains per hour. Setting a uniform price that is equal to marginal cost per user covers all variable costs.

(c) For the Metro system to break even the fixed costs of maintenance must raised in the form of a uniform consumer price, hence price must equal average cost.

Average cost per hourly train is, in millions €

$$AC(q) = \frac{VC(q)}{q} + \frac{FC}{q} = \frac{2q}{q} + \frac{12}{q} = 2 + \frac{12}{q}$$

If users are charged the average cost then this is average cost must be divided by the number of commuters, 0.2 million. (Notice that since commuters have the same preferences, either all or none of them will find the price acceptable). If service is provided at q trains per hour then the average cost price paid by each commuter must be

$$\bar{p}(q) = \frac{\text{AC}(q)}{0.2} = 10 + \frac{60}{q}$$

For a commuter to be willing to pay this price their marginal benefit from q must be at least as high. The only thing that remains is to find the level of service q where $\bar{p}(q) = \text{MB}_i(q)$. If there were many such points where the Metro system breaks even and commuters are willing to pay the price we would choose the one with highest total benefit, however here it turns out that, at any level of service q, the average cost of production is strictly higher than the marginal benefit $\text{MB}_i(q) = 20 - 0.5q$ for individual commuters. Hence consumers are not willing to pay the average cost price at any level of service q. No service can be provided.

This can be shown in several ways. One alternative is to use a graph to show that there is no level of production that would be feasible under average cost pricing, see Figure 1.
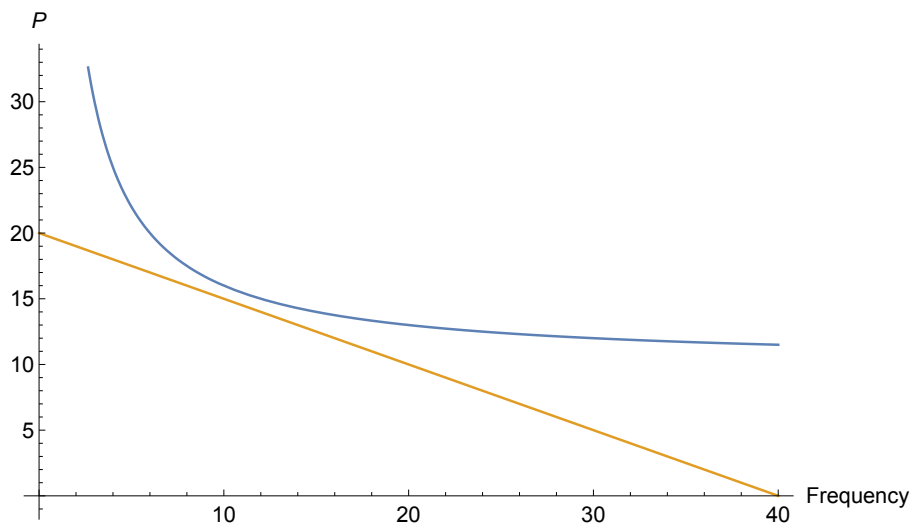

Figure 1: Average cost $10 + 60/q$ (blue) and marginal benefit $20 - 0.5q$ (orange) as functions of train frequency q. The lack of an intersection proves that average cost pricing is not feasible for any q.

Another alternative is to minimize the difference between average cost and marginal benefit $\bar{p}(q) - \text{MB}_i(q)$. As this difference remains positive even at its minimum it shows that no level of service q exists where average cost pricing would be feasible.

Define a function, $f(q)$, for the difference between price and marginal benefit:

$$f(q) = \bar{p}(q) - \mathrm{MB}_i(q) = 10 + \frac{60}{q} - (20 - 0.5q)$$

$$f(q) = \frac{60}{q} + 0.5q - 10$$

Find the frequency that minimizes this difference using a first-order-condition.

$$\frac{\partial f(q)}{\partial q} = \frac{-60}{q^2} + 0.5 = 0$$

$$\frac{60}{q^2} = 0.5 \iff q^2 = 120$$

$$q_m := \sqrt{120} \qquad \text{(the negative root can be discarded as meaningless)}$$

Check that this is the minimum point by taking the second derivative:

$$f''(q_m) = \frac{120}{q_m^3} > 0$$

The positive sign proves that q_m is indeed the minimum. At the minimum

$$\bar{p}(q_m) = 10 + \frac{60}{\sqrt{120}} \approx 15.48$$

$$\mathrm{MB}_i(q_m) = 20 - 0.5 \cdot \sqrt{120} \approx 14.52$$

So the average cost price would always exceed the marginal benefit, hence it is not feasible to provide service under average cost pricing.