Model Solutions, Exam 2020-10-22

I (a) In everyday language "being efficient" often refers to an intensive use of some resource. Someone working very hard is described as efficient, whereas to an economist someone who values leisure and is barely putting in any effort can be working just as efficiently—efficiency requires that there is no room for costless improvement.

(b) In economics a lottery is any combination of mutually exclusive outcomes, each associated with a payoff and a probability. Elsewhere lottery is a form of gambling, usually with low stakes and a large main prize.

(c) In economics a public good, once created, is not depleted by consumption and is open for everyone to consume, such as open software. Elsewhere any good provided by the public sector is often called a public good, such as education or health care.

(d) In economics, welfare consists of surpluses generated by economic agents such as consumers, producers and the government, measured in monetary terms. Elsewhere welfare can refer to anything that is deemed desirable for human well-being.

II (a) A higher subsidy will increase supply and so reduce the price, resulting in an unambiguous increase in consumer surplus.

(b) A lower supply quota will reduce output, which also increases the price, resulting in an unambiguous decrease in consumer surplus.

(c) Increased government purchases increase demand, raising both the price and the quantity supplied, so producer surplus is unambiguously increased.

(d) A lower price floor reduces the price the producers get but also increases the quantity sold, so the impact on producer surplus is ambiguous. With sufficiently elastic demand the increase in quantity dominates and producer surplus is increased.

(e) A higher tax increases the tax revenue per unit traded but also reduces the quantity traded, hence the change in tax revenue is ambiguous. If demand and supply are sufficiently elastic then the reduction in quantity dominates and tax revenue goes down.

III "A small price cut lowered your revenue a little, so a further price cut will likely just make it worse. You should instead try raising your price, it will probably raise your revenue."

Additional comment: This cousin seems to be facing inelastic demand (-2/3). To maximize profits he should increase the price at least until demand becomes elastic. Until then, any price increase is surely good for profits: revenue is going up, while quantity (and any variable costs he might have) are going down.

IV All monetary values in millions of euros, quantities in tons.

(a) Each year X Inc maximizes its profits by setting to marginal revenue equal to marginal cost. Inverse demand is $P^D(q) = (140 - q)/40 = 3.5 - 0.025q$, total revenue is $\text{TR}(q) = qP^D(q) = q(140 - q)/40 = 3.5q - 0.025q^2$, so marginal revenue is $\text{MR}(q) = \text{TR}'(q) = 3.5 - 0.05q$.

Setting this MR equal to marginal cost yields $3.5 - 0.05q = 1 \implies q^* = 50$, which is below the capacity constraint of 100. Optimal price is therefore $p^* = P^D(50) = 3.5 - 0.025 \times 50 = 2.25$.

Finally, we need to check whether profits would be negative, taking into account fixed costs: $\pi^* = (p^* - 1)q^* - 10 = 1.25 \times 50 - 10 = 52.5$. So profits are indeed maximized by selling fuzz at $p = 2.25$. Note that the previously incurred cost 750 was sunk and thus irrelevant.

(b) Owners are better off by selling the firm if they are paid at least the present value of its future profits. Opportunity cost of capital gives the discount rate $r = 0.05$. X Inc operates for $T = 20$ more years at the maximized yearly profit of $\pi^* = 52.5$. (Profits beyond $T = 20$ would require a whole new investment, so they are not relevant here).

We can apply the formula for the present value of a perpetuity by expressing this profit stream as a sum of two perpetuities: a positive one that starts this year and a negative one that starts T years later. Hence the present value, and thus the owners' reservation value, is

$$\frac{\pi^*}{r} - \frac{1}{(1+r)^{T+1}}\frac{\pi^*}{r} = \frac{\pi^*}{r}\left(1 - \frac{1}{(1+r)^T}\right) = \frac{52.5}{0.05}\left(1 - \frac{1}{1.05^{20}}\right) \approx 654.$$

(c) Fixed costs do not affect optimal pricing so the outside investors would find it optimal to produce the same quantity, and so the reduced capacity constraint $100/2 = 50 \geq q^*$ would not bind. Yearly profits are improved by the 40% reduction in fixed costs, $\pi^{**} = \pi^* - 0.4 \times 10 = 48.5$. Taking into account the immediate modification cost, and using the present value formula seen in part IVb, the outside investors' reservation price for X should be

$$-30 + \frac{\pi^{**}}{r}\left(1 - \frac{1}{(1+r)^T}\right) = -30 + \frac{48.5}{0.05}\left(1 - \frac{1}{1.05^{20}}\right) \approx 674.$$

V All monetary values in thousands of euros, quantities in daily broadcasting hours.

(a) Total marginal benefit for a public good is obtained by summing the inverse demands. The inverse demands are: $P_I^D(q) = P_L^D(q) = 24 - q/2$, $P_K^D(q) = 31 - q$, and their sum is $P^D(q) = 79 - 2q$. For the aggregation to be this simple all MBs have to be positive, which we will soon see graphically.

Marginal cost jumps at 12 hours so it is defined piecewise:
$\mathrm{MC}(q) = 15$ if $0 \leq q < 12$, $\mathrm{MC}(q) = 35$ if $12 \leq q < 24$.

At the efficient quantity marginal cost either equals marginal benefit, or jumps above it at a point of discontinuity, or the capacity constraint $q = 24$ is reached before they meet. We could check all three possibilities, or use a graph of MC and MB to show that the optimum is somewhere between 12 and 24 hours, see Figure 1. Solving the equality gives $\mathrm{MC}(q) = \mathrm{MB}(q) \iff 35 = 79 - 2q \implies q^* = 22$.

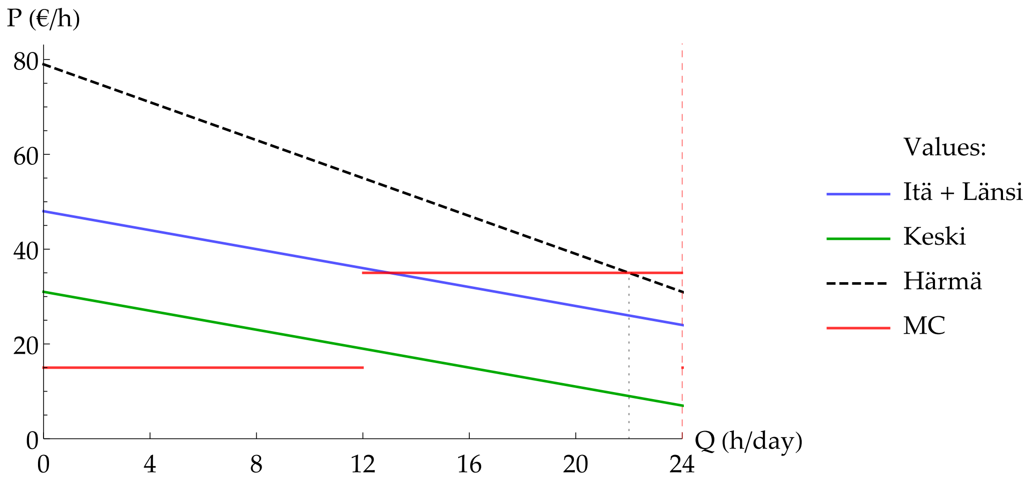

Figure 1: Marginal benefit and marginal cost curves. Itä and Länsi have been aggregated into one.

Finally, we need to compare the total benefit with the total cost of providing the service. Total benefit is the area under the demand curve, $(P^D(0) + P^D(q^*))q^*/2 = (79{+}35){\times}22/2 = 1254$, which exceeds the total cost $15{\times}12{+}(22{-}10){\times}35{+}190 = 720$, so $q^* = 22$ is indeed the efficient quantity of daily broadcasting.

(b) Total benefits are $(24{+}13){\times}22/2 = 407$ in Itä and in Länsi, and $(31{+}9){\times}22/2 = 440$ in Keski. Total cost is shared evenly so every region pays $720/3 = 240$, leaving a total surplus of 167 in Itä and in Länsi and 200 in Keski.

(c) Introducing MCPF $= 2$ doubles the welfare cost of all public spending. Now the MC-curve jumps up above the MB-curve at exactly $q = 12$, where MC now jumps from 30 to 75 while $P^D(12) = 55$. Total benefit is now $(79 + 55) \times 12/2 = 804$. This still exceeds total cost, $12 \times 30 + 380 = 740$, so 12 hours is indeed efficient.

(d) The possibility to use broadcast time for zero-MC reruns at a lower MB can be interpreted as increasing the marginal opportunity cost of live broadcasting by the lost MB of reruns. Compared to (a), MC jumps up by 3×4 everywhere. Now MC does not jump above MB at the discontinuity, as $P^D(12) > 35 + 12$. Solving MC $=$ MB beyond the jump gives $47 = 79 - 2q \implies q = 16$.

Here benefits are higher but costs lower than in (c), so broadcasting $16h$ live (and $8h$ of reruns) is indeed efficient.