Model Solutions: Exam 2021-12-17

Multiple choice 1b, 2e, 3e, 4e,[1] 5b, 6e, 7c, 8e

I (a) A **dutch auction** is a descending price live auction. Bidders choose at what price to jump in, the first to bid gets the object and pays the amount of their own bid.

(b) There is a **network externality** if consumers' valuations for a good depend on how many other consumers are consuming the same good.

(c) If workers fear that high performance would lead to decreases in future bonuses for high performance, the incentive pay system is suffering from a **ratchet effect**. This may cause workers to hold back on "too much" effort in order to protect the generosity of the pay system.

II The lower quality of food in Economy class can be part of a **versioning** strategy between Economy and Business class tickets. If the quality of food in Economy class were increased, the high types' willingness to pay for a Business ticket would be reduced, which could lead to lower profits for the airline.

III This is a duopoly problem, where two companies decide their levels of output. Alpha's optimal choice depends on Beta's choice and vice versa. The setup is:

1. Fixed cost: €6 billion
2. Marginal cost: €2 billion per ton
3. Demand: $Q^d(P) = 40 - 8P \Leftrightarrow P^d(Q) = 5 - Q/8$

(a) This is a simultaneous choice situation, because firms make their quantity choice before finding out the competitor's choice. Let's set up Alpha's profit-maximization problem. Its profits (in €bn) are:

$$\begin{aligned}
\Pi_A(Q_A, Q_B) &= P^d(Q_A + Q_B)Q_A - \text{MC} \times Q_A - \text{FC} \\
&= (5 - \frac{Q_A + Q_B}{8}) \times Q_A - 2Q_A - 6 \\
&= 5Q_A - \frac{1}{8}(Q_A^2 + Q_AQ_B) - 2Q_A - 6 \\
&= 3Q_A - \frac{1}{8}(Q_A^2 + Q_AQ_B) - 6
\end{aligned}$$

Let's maximize the profit-function with respect to Alpha's decision variable:

$$\frac{\partial \Pi_A(Q_A, Q_B)}{\partial Q_A} = 3 - \frac{Q_A}{4} - \frac{Q_B}{8} = 0 \Rightarrow$$

$$Q_A = 12 - \frac{Q_B}{2}$$

[1] For this question the most common choice was incorrect, *b*. For moral hazard to be a problem the workers would currently have to be putting in inefficiently little effort, taking into account the value and cost of additional effort. However, we only know that they could be putting in more effort (as is almost always the case).

Since the firms are symmetric, this gives us the best response function for both firms $i \in \{A, B\}$: $\mathrm{BR}(Q_i) = 12 - Q_i/2$. In Nash equilibrium $q = \mathrm{BR}(q)$, so

$$q = 12 - \frac{q}{2} \Rightarrow q = 8,$$

hence equilibrium choices are $Q_A^* = Q_B^* = 8$.
Equilibrium price is $P^* = P^d(Q_A^* + Q_B^*) = 5 - 16/8 = 3$.
Profits of both firms are

$$\Pi_i^* = (P^* - \mathrm{MC})Q_i^* - \mathrm{FC} = (3 - 2) \times 8 - 6 = 2.$$

Both firms choose a capacity of 8 tons and earn a profit of €2 billion.

(b) If Alpha pre-empts Beta then it effectively commits to an output level before Beta can make its choice, but this also increases Alpha's fixed cost by some x.

As the first mover Alpha would know that Beta will take Alpha's decision as a given. The quickest way to solve this is to notice that, ideally from Alpha's point of view, Beta would stay out of the market while Alpha produces the monopoly quantity $Q^m = \mathrm{BR}(0) = 12$. Beta's best quantity response would be $\mathrm{BR}(12) = 12 - 12/6 = 6$, but this would result in negative profits for Beta:

$$\begin{aligned}\Pi^b =& P^d(Q^m + \mathrm{BR}(Q^m)) \times Q^m - 2 \times Q^m - 6\\ =& P^d(12 + 6) \times 6 - 2 \times 6 - 6\\ =& (5 - 18/8) \times 6 - 18 = -1.5 < 0.\end{aligned}$$

Hence, by producing $Q^m = 12$, Alpha deters entry and earns monopoly profits

$$\begin{aligned}\Pi^m &= P^d(Q^m) \times Q^m - 2 \times Q^m - 6 - x\\ &= (5 - 12/8) \times 12 - 24 - 6 - x\\ &= 42 - 30 - x = 12 - x.\end{aligned}$$

This exceeds Alpha's €2bn profit without pre-empting (part IIIa) if $x < 10$ €bn.[2]

(c) Consumer surplus is higher the more is produced, or equivalently, the lower the price. There is no need to calculate anything new: it suffices to compare total output with and without pre-empting by Alpha. Total output was $2 \times 8 = 16$ without pre-empting (part IIIa) and 12 with pre-empting (part IIIb), so consumers are better off if Alpha cannot pre-empt.

[2] Alternatively, maximize Alpha's profits while plugging in Beta's BR in its profit function, resulting in $Q_A^* = 12$. Beta's implied choice $\mathrm{BR}(12) = 6$ would give it negative profits, so it will not enter. However, here answers that treated Beta's fixed costs as sunk were also accepted, in which case $Q_B^* = 6$, pre-empting is worth it for Alpha if $x < 1$, and consumers benefit from pre-empting in IIIc.

IV Only sellers observe the quality level that buyers care about, so adverse selection could be a problem. Let's summarize the setup:

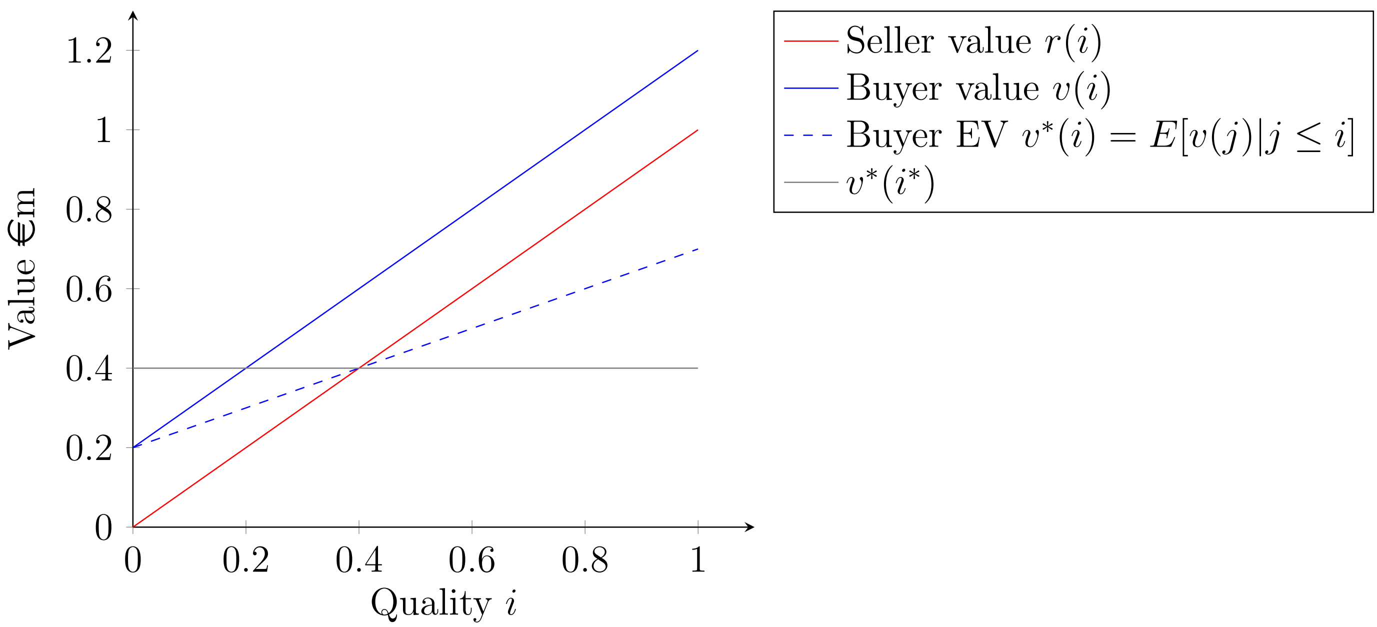
1. Seller valuations r uniform in $[0, 1]$, so $r(i) = i$ for seller at quantile i
2. Buyers value cottages by €0.2m more than sellers: $v(i) = v(r(i)) = 0.2 + i$
3. When highest quality sold is at the ith quantile then buyer EV is $v^*(i) = E[v(j)|j \leq i]$

(a) All cottages are valued higher by non-owners than by owners, so it would be efficient for all cottages to be traded. Then buyer EV would be $E[v(i)] = 0.2 + E[i] = 0.7$, which is below the valuation of the highest quality owner, $r(1) = 1$, so adverse selection is indeed a problem. Highest qualities will not be traded.

In equilibrium, the sellers of the highest traded quality obtain exactly their reservation value, which must equal the buyer EV for all qualities being traded. If i is the highest quality traded, then traded qualities are uniformly distributed in $[0, i]$ so their EV is $E[j|j \leq i] = 0.5i$. Then buyer EV is $v^*(i) = E[v(j)|j \leq i] = 0.2 + E[j|j \leq i] = 0.2 + 0.5i$. Equilibrium share of traded cottages satisfies $r(i) = v^*(i)$, so

$$i = 0.2 + 0.5i \Rightarrow i^* = 0.4.$$

In equilibrium, 40% of cottages are traded at the market price of €0.4 million.



Adverse selection in the cottage market. At the equilibrium point buyer EV equals seller value.

(b) If it were possible to verify the quality of cottages then 100% of cottages would be sold. Every trade increases welfare by €0.2 million. We saw in part IVa that under asymmetric information 60% of cottages are not traded. Thus the resulting increase in average welfare per cottage would be $0.6 \times 0.2 = 0.12$ €m.