Model Solutions, Exam 2020-12-11

I The argument incorrectly refers to distributional effects of ordinary market interactions as externalities. The decisions to cooperate between firms, workers and customers are voluntary, with or without robots. The mere fact that some form of voluntary interaction—in this case an employment relationship—does not take place, does not give rise to an externality (any more than me favoring one cafe over another would).

A common reason for points deductions was the use of multiple irrelevant arguments, in addition to the correct one, that were either patently wrong, or unnecessary and relying on additional assumptions. Points were neither lost or gained for discussing how goals other than efficiency might or might not justify the tax.

II (a) This is the payoff matrix in part IIa.

		Mr Montana		
	\$100k	A	B	C
Tax authority	A	8,0	-2,10	-2,10
	B	-2,10	8,0	-2,10
	C	-2,10	-2,10	8,0
	None	0,10	0,10	0,10

Note: Mr Montana's payoffs can be defined either as gains relative to losing the money (as above) or as losses relative to keeping his money.

(b) There can be no pure strategy Nash equilibria in this game, because the authority wants to match Mr Montana's choice while Mr Montana wants to avoid the authority's choice. The best a player can do in equilibrium is to keep the other player guessing and indifferent between their actions. Thus the Nash equilibrium will be in mixed strategies.

Notice that the game is symmetric with respect to the mansions for both players. Mr Montana will therefore use each of the three mansions with equal probability, 1/3. No matter which mansion the tax authority raids its expected value is $(1/3)8 + (2/3)(-2) = 4/3$. This is better than the zero from not raiding any mansion, which is therefore indeed a dominated strategy and will not be part of the mixed strategy. The authority will also mix between all mansions with equal probability 1/3.

Additional comment. As always, the mixing probabilities could also be solved from the players' indifference conditions. The tax authority is indifferent between the mansions if Mr Montana selects them with probabilities $\{p_A, p_B, 1-p_A-p_B\}$ *such that* $V_T = 8p_A - 2p_B - 2(1-p_A-p_B)$, $V_T = -2p_A + 8p_B - 2(1-p_A-p_B)$, *and* $V_T = -2p_A - 2p_B + 8(1-p_A-p_B)$. *This is a system of three linear equations with three unknowns, which is straightforward to solve for* $p_A = 1/3, p_B = 1/3, V_T = 4/3$. *The expected value from choosing not to raid is* $0 < V_T$ *and thus indeed dominated.*

Similarly, Mr Montana is indifferent when the tax authority selects mansions with probabilities $\{q_A, q_B, 1 - q_A - q_B\}$ *such that* $V_M = 10q_B + 10(1 - q_A - q_B)$, $V_M = 10q_A + 10(1 - q_A - q_B)$, *and* $V_M = 10q_A + 10q_B$, *which holds when* $q_A = 1/3, q_B = 1/3, V_M = 20/3$.

III (a) Kärry's profit is maximized when its marginal cost equals its marginal revenue. Its marginal cost is the wholesale price P_W plus the additional marginal cost it incurs, i.e., $\mathrm{MC}_K = P_W + 5$. Its total revenue is $\mathrm{TR}_K = P^D(Q)Q = 200Q - (1/4)Q^2$, where P^D is the inverse of demand for $Q^D(p) = 800 - 4p$. Marginal revenue is then $\mathrm{MR}_K(Q) = 200 - Q/2$. The manufacturer knows that Kärry will select Q to maximize its own profits, which requires $\mathrm{MR}_K(Q) = \mathrm{MC}_K \implies 200 - Q/2 = P_W + 5 \implies P_W = 195 - Q/2$, which is in effect the demand faced by the manufacturer.

The manufacturer's total revenue from Lintukoto is then $\mathrm{TR}_M(Q) = (195 - Q/2)Q = 195Q - (1/2)Q^2$, so its marginal revenue is $\mathrm{MR}_M(Q) = 195 - Q$. Its profits are maximized when $\mathrm{MC}_M = \mathrm{MR}_M(Q) \implies \mathrm{MC}_M = 195 - Q$. Kärry is currently dealing 160 vehicles, so the MC_M that is consistent with the manufacturer currently charging a profit-maximizing wholesale price is $\mathrm{MC}_M = 195 - 160 = 35$.

(b) The combined profit is maximized when total marginal revenue equals total marginal cost. Note that payments between the manufacturer and the retailer cancel out—they are costs for one, but revenue for the other—so total marginal revenue is the same as faced by Kärry in IIIa. The total marginal cost is the sum of the real marginal incurred by the retailer and the manufacturer, i.e., $35 + 5 = 40$. The profit-maximizing condition is therefore $200 - Q/2 = 40 \implies Q = 320$, which plugged into the inverse demand yields the retail price $p^m = P^D(320) = 120$.

(c) Currently, Kärry's profit is $\mathrm{TR}_K - Q \times \mathrm{MC}_K = 160 \times 160 - 160 \times 120 = 6400$ and the manufacturer's profit is $\mathrm{TR}_M - Q \times \mathrm{MC}_M = 115 \times 160 - 160 \times 35 = 12800$. We know from IIIb that combined profits are maximized when $P^D = 120$, yielding combined profits of $\Pi_C(P^D) = 960 \times 120 - 4 \times 120^2 - 32000 = 25600$. Hence the increase in combined profits is $25600 - (6400 + 13600) = 25600 - 6400 - 12800 = 6400$. This can be achieved with a two-part tariff, which optimally involves the manufacturer selling vehicles to Kärry at its marginal cost €35k and only making profits from the license fee. This gets rid of the problem of *double marginalization* seen in part IIIa, where both the retailer and the manufacturer used simple pricing in succession, raising the consumer price to a level that is inefficiently high from the point of view of combined profits.

For the resulting increase in combined profits to be shared equally between the parties, Kärry must pay a license fee equal to the manufacturer's original profit plus half of the increase in profits, $12800 + 6400/2 = 16000$, i.e., €16m per year.

IV (a) Total welfare is $W(n) = 2nY(n) - 100n = 300n - n^2$. This is maximized when $W'(n) = 300 - 2n - 100 = 0 \implies n^* = 150$.

With unrestricted entry, boats will enter until the per-boat revenue falls below the cost of operating a boat, i.e., until $2Y(n) = 100 \implies 400 - n = 100 \implies n^0 = 300$.

(b) Denote by n_D and n_R the number of boats sent by Dunwich and Rungholt respectively, so that $n_D + n_R = n$. The profit for Dunwich is then $\Pi(n_D, n_R) = 2n_D Y(n_D + n_R) - 100n_D$. Maximizing this with its own decision variable yields the best response function for Dunwich in terms of Rungholt's decision: $\frac{\partial}{\partial n_D}(300n_D - n_D^2 - n_D n_R) = 0 \implies 300 - 2n_D - n_R = 0 \implies \text{BR}(n_R) = 150 - n_R/2$. Since the game is symmetric, Rungholt's best response takes the same form. In equilibrium both towns $i \in \{D, R\}$ send n_i boats such that $\text{BR}(n_i) = n_i$. Hence $150 - n_i/2 = n_i \implies n_i = 100$. Both towns send 100 boats, for a total of 200 boats in equilibrium.

(c) An equilibrium yielding half of the maximal profits for both towns would have both towns sending in $n^*/2 = 75$ boats every year, yielding profits of $\Pi(75, 75) = 2 \times 75 \times Y(150) - 100 \times 75 = 11250$ per year for both. Suppose the towns would try to uphold this situation with a grim trigger strategy, where both would punish any deviation from this by reverting to a Nash equilibrium. The most a town could gain by "cheating" is by sending in their best response number of boats $\text{BR}(75) \approx 113$. This would give a profit of $\Pi(113, 75) = 2 \times 113 \times (200 - 0.5 \times (113 + 75)) - 100 \times 113 = 23956 - 11300 = 12656$.

If cooperation fails then both towns earn the Nash equilibrium profits of $\Pi(100, 100) = (2 \times 200 \times (200 - 0.5 \times 200) - 100 \times 200)/2 = 10000$ for ever after.

With a 10% discount rate, the present value of cooperating forever is $11250 + 11250/0.1 = 112656$ while the present value from cheating followed by Nash equilibrium forever is $12656 + 10000/0.1 \approx 111505$. The present value from cooperation is higher, so {Grim,Grim} is indeed an equilibrium in the repeated game where both towns get half of the maximal profits every year.