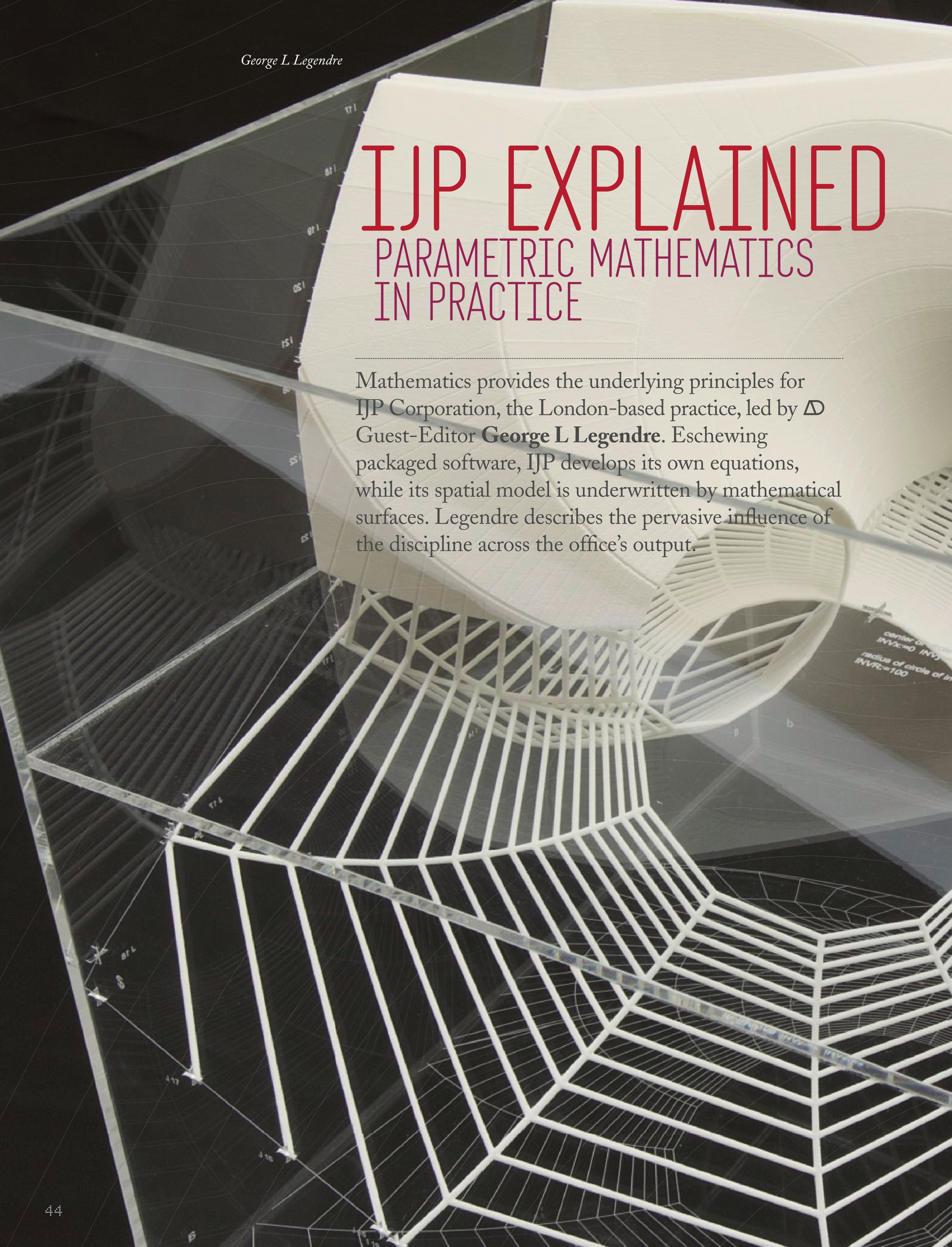
George L Legendre



IJP EXPLAINED

PARAMETRIC MATHEMATICS IN PRACTICE

Mathematics provides the underlying principles for IJP Corporation, the London-based practice, led by AD Guest-Editor **George L Legendre**. Eschewing packaged software, IJP develops its own equations, while its spatial model is underwritten by mathematical surfaces. Legendre describes the pervasive influence of the discipline across the office's output.

Figure 1. IJP with John Pickering, F01(b), London, 2009
In this project, IJP explored the parametric deployment of a simple homothetical (scaling) transformation known as inversion, to which the Wolverhampton-based artist John Pickering has devoted several decades of work. IJP determined the analytic equations of the transformation and used them to invert, under Pickering's guidance, a pair of ordinary cylinders into an intricate aggregation of (partial) cycloid surfaces.

A.03 *Midterm Proposal*

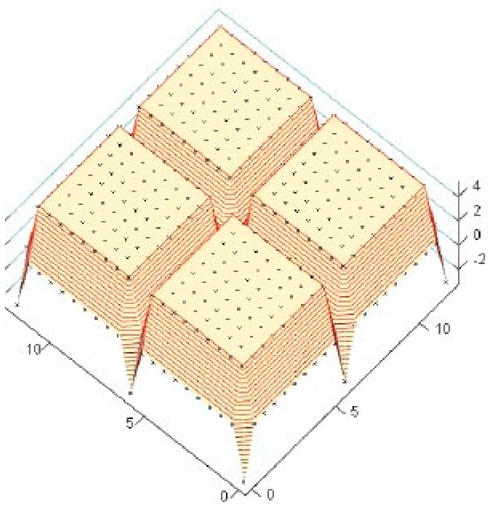
GSD2404 LEGENDRE

LIBERATORE Cara

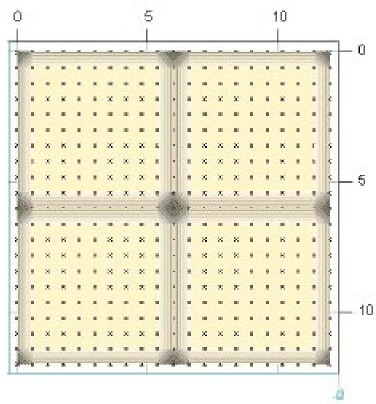
_ranges $n1 := 0, 1 .. 20$ $m1 := 0, 1 .. 20$

_equations $iT(u,v) := 6 \cdot \frac{u}{\pi}$ $jT(u,v) := 6 \cdot \frac{v}{\pi}$ $sh(u,v) := \left(\cos(u)^{2000} + \cos(v)^{2000}\right) \cdot -4 + 4 + 1$

$\Pi_{m1,n1} := iT\left(\frac{m1}{20} 2\pi, \frac{n1}{20} 2\pi\right)$ $\Theta_{m1,n1} := jT\left(\frac{m1}{20} \cdot 2\pi, \frac{n1}{20} 2\pi\right)$ $K_{m1,n1} := sh\left(\frac{m1}{20} \cdot 2\pi, \frac{n1}{20} \cdot 2\pi\right)$



(Θ, Π, K)



(Θ, Π, K)

Figure 2. IJP, Asymptotic Box, Parametric equations, 2004–10
The Asymptotic Box© is an implicit 3-D surface derived by analytic means. A blob trying to pass itself off as a box, this curious surface is produced by raising a periodic product to a (very) high exponent in order to deform an ordinary pliant surface into a 'near box'. There are algebraic limits to this kind of game, as infinite tangents are inadmissible. Hence the term 'Asymptotic', whereby the box tends towards orthogonality – without reaching it.

Since its inception in 2004, the London-based practice IJP Corporation has been using a mathematical knowledge model as a blueprint for the design of novel structures. Proposals that are drastically different in size and scope follow the same instrumental premises. This is evident in F01(b) (2009), a collaboration with the artist John Pickering; in the Art Fund Pavilion (2009); Yeosu 2012 Thematic Pavilion (2009); the Shenzhen Museum of Contemporary Art and Planning (2007), with architect Max Kahlen; and the Henderson Waves Bridge (2004–9) with RSP Architects, Planners and Engineers. Bypassing the conveniences of modelling software in favour of elemental mathematics, these projects share a common basis of analytic geometry. Rather than simply consuming software, IJP produces the very material software is made of – raw equations – usually taken for granted under the hood, and hence maintains a far greater amount of control over what it designs and manufactures. In this sense, the office turns its back on a dazzling technological design agenda, preferring to work at an infra-technological level, where a symbolic language common to all computational design processes exists.

IJP's model of spatial cognition is based on the notion of mathematical surface. In analytic terms, all mathematical parametric surfaces form and deform in direct response to numerical relations they hold in their midst. To the observer who knows what to look for, the conformation of a parametric surface exhibits the traces of these internal motions as markedly as the plump figure of midlife will subsume the sharper inflections of a youthful physique. Parametric surfaces surge upwards because of a genetic antecedent of linearity, a pattern of linear growth exhibited to some degree by one of the dependent relations they quite literally incorporate. They undulate by peaks and depressions because of a periodic internal makeup, pointing to the presence of cyclical behaviours in one of their three respective antecedents. Finally, they spiral up and down under the confluence of linearity, periodicity and transposition. Each term follows a distinct pattern and determines the overall form of the surface in tandem with the other two, through a composite process yielding the most complex results.

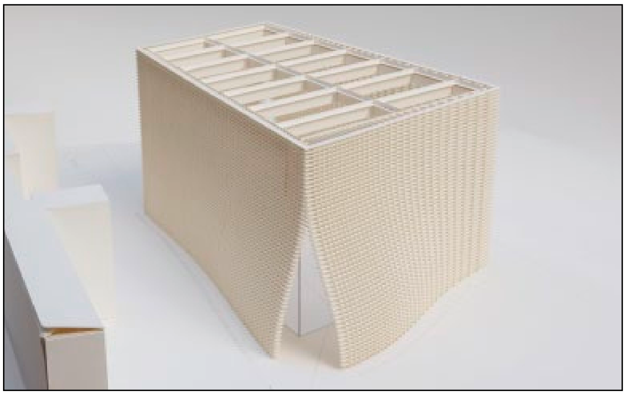


Figure 3. IJP, The Art Fund Pavilion, Woking, Surrey, 2009
This lightweight structure is clad with a prefabricated timber lattice of highly variable, side-specific density. A single room enclosed by four rigid panels of extruded acrylic, the Art Fund Pavilion may be endlessly reconfigured into a temporary support space for the neighbouring Light Box contemporary art museum for which it was commissioned. The pavilion is designed to be assembled and taken apart in 72 hours by a team of just two. View of laser-cut model.

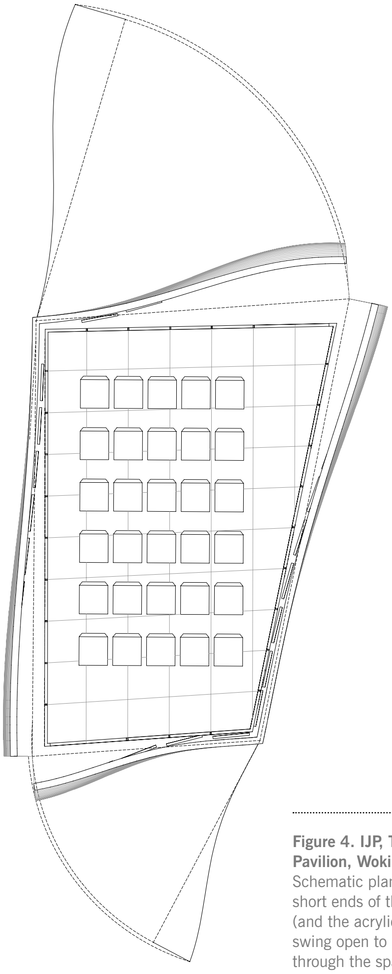


Figure 4. IJP, The Art Fund Pavilion, Woking, Surrey, 2009
Schematic plan. On busy nights, the short ends of the timber honeycomb (and the acrylic panels behind) swing open to let visitors overflow through the space.

Variable Densities

A commitment to mathematics is not consistent with a bias towards a given style or Gestalt. For IJP, variable curvature is not a matter of architectural vocabulary, but a heuristic device, an operative tool to conceptualise space. The office develops plain boxes, for instance, while remaining equally committed to meeting the challenge of continuity, which gets surprisingly tricky where mundane conditions of orthogonality (or proper tectonic corners) are concerned. Hence the tendency is to avoid discontinuous, piecemeal assemblies where each side of a notionally continuous envelope is dealt with separately, considering instead alternative holistic options such as plotting circles or ellipses with only four points, raising periodic expressions to a high exponent (IJP's trademark Asymptotic Box©), or substituting Fourier summations for ordinary periodic functions, all of which involve bona fide blobs (successfully) trying to pass themselves off as a box.

For reasons related to the programmatic requirements spelled out by the organising body of the open international competition for a new temporary art space sited by the Light Box Art Museum in Surrey, UK, the Art Fund Pavilion (2009) is the exception to the rule. Here the overriding issue is the differential filtering of natural daylight, so the emphasis is kept on each individual side, and the practical possibility of redistributing its indexical threads without altering the form

Geometry and algebra, the study of figures and that of symbols, separated more than 400 years ago; as noted in the introduction to this issue, this separation lies at the root of mathematical modernity, and reminds us that unlike art history, or even technology, progress in mathematics is extremely fast paced.

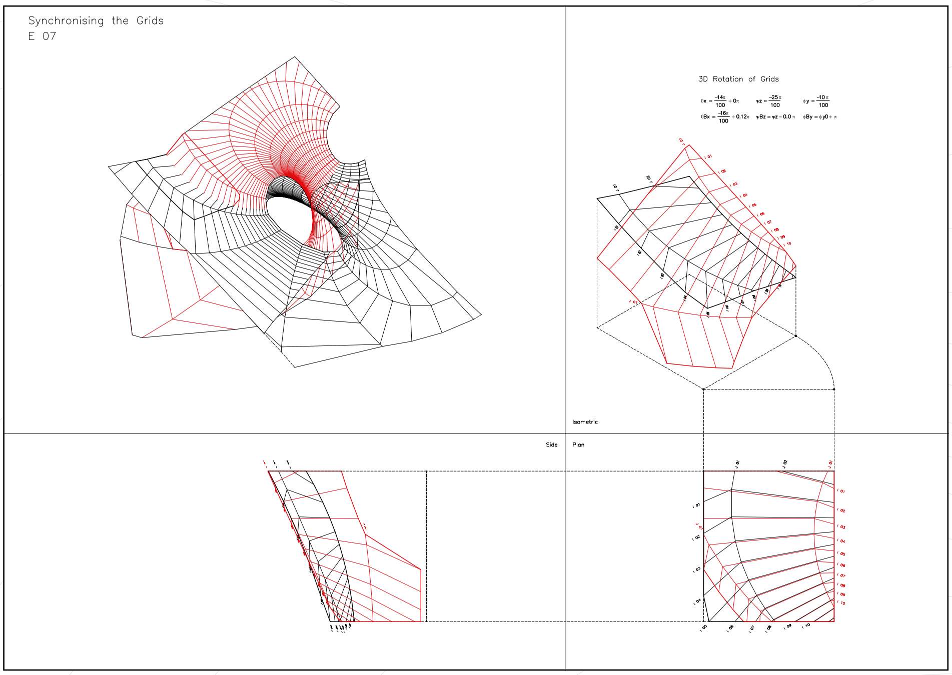


Figure 5. IJP with John Pickering, F01(b), London, 2009
Study diagram of intersection between the solid and reticular portions of the model. The multiple intersections of surface threads establish a material continuity between discrete figures in space and enhance the cohesiveness – and stability – of the final piece.

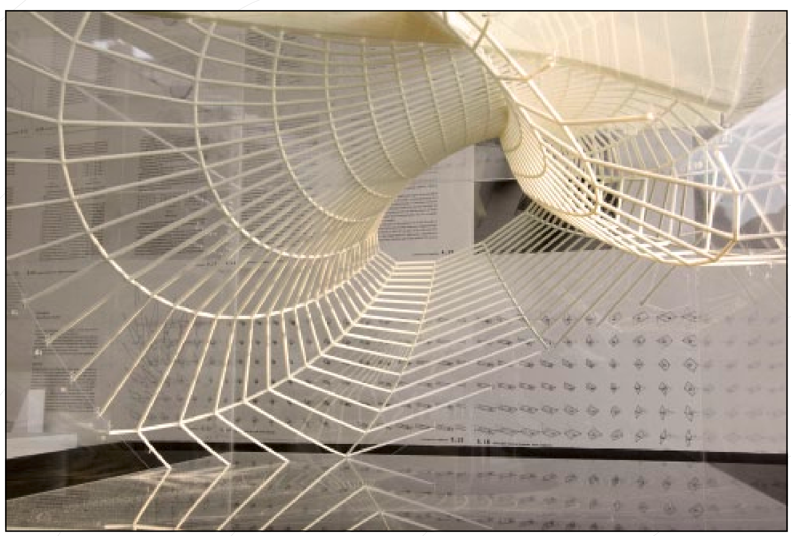


Figure 6. IJP with John Pickering, F01(b), London, 2009
Close-up view. The figure is encased in a translucent box that crops the infinite surfaces produced by the transformation. The rapid-prototyped silicate and engraved acrylic give the finished model a light and reflective quality.

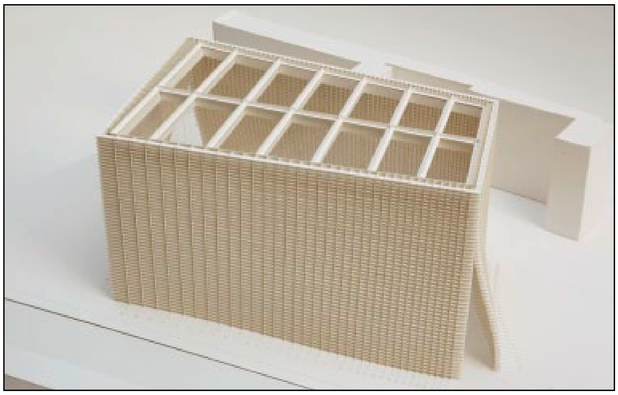


Figure 7. IJP, The Art Fund Pavilion, Woking, Surrey, 2009
Model of northwest corner, with zip-up entrance.

itself. The proposal consists of a single room with four walls and a roof. This temporary structure is meant to support its host institution by variously housing cultural seminars, temporary exhibitions or fundraising events. It is orientated like a traditional artist's studio for a maximum intake of northern daylight, and its envelope is made of pliant parametric surfaces extruded into a honeycomb, behind which a lightweight thermal barrier of acrylic panels marks the boundary of the exhibition space proper.

Each of the pavilion's walls is designed by altering the sequence of a numerical range through the deployment of equations. The rhythm of the south-facing studs follows a periodic distribution that aggregates proximate threads to within an inch of one another, closing the honeycomb and blocking direct sunlight. On the north side, the distribution is precisely inversed. 'Stitching' overlapping edges secures the water-tightness of contiguous walls, whereas pulling the edges apart produces zipper-shaped entrances and exits at the opposite corners of the room. The materiality of the honeycomb is based on a lattice of flat, CNC-cut profiles of shallow radii meeting at a right angle, and the profiles are notched to maximise adherence without chemical bonding. The completed honeycomb is dense, stiff and so redundant structurally that other forms of support, including steel posts or panel frames, are unnecessary. In this project it is the curtains that carry the 3.6-metre (12-foot) high wall, frame and roofing material, and not the other way around. High ranges are always used to solve parametric surface equations offering the opportunity of experimenting with the sturdiness of redundant lattices at drastically different scales, from a single room like the Art Fund Pavilion, to a 2.4-metre (8-foot) wide, 152.4-metre (500-foot) tall high-rise building.

Curvature as Fragmented Flatness

From small installations to extra-large infrastructure, this mathematical knowledge model migrates seamlessly across scales and contexts. Working, for instance, with the brand of geometry pioneered by the British sculptor John Pickering has uncovered unexpected forms of tectonic efficiency. Based on a projective transformation known as inversion (or scaling relative to a fixed point, but with a variable coefficient), Pickering's art is an exercise in absolute reduction. IJP has worked on three short projects in collaboration with the artist, resulting initially in the production of licensed similes of his original artwork with state-of-the art software and numerically controlled fabrication, with a view to soliciting a grant from the Arts Council in order to rebuild them as large-scale installations in a gallery. The first two pieces re-created at his behest closely follow his working blueprint, canning Pickering's manual approach into a scripting routine and exporting the result to a rapid-prototyping device. As such, the resulting pieces are little more than copies (or forgeries, depending on one's point of view) of the original.

The third instalment of the collaboration, on the other hand, an installation titled F01(b) commissioned by Pickering for his own collection, disregards the manual two-dimensional strategy favoured by the artist, using instead the parametric equations of a relatively simple homothetical transformation in space. Developing F01(b) through equations rather than drawings was not only a matter of re-evaluating the Modernist sensibility of the previous installations, but of taking a profoundly different approach that involved a shift from figures to symbols, in line with an argument more at home in the 17th century than in the 21st. Geometry and algebra, the study of figures and that of symbols, separated more than 400 years ago; as noted in the introduction to this issue, this separation lies at the root of mathematical modernity, and reminds us that unlike art history, or even technology, progress in mathematics is extremely fast paced. Hence, rather than recasting the static geometry favoured by the artist in a new technical idiom, deploying the analytic equations of inversion in F01(b) opens up the possibility of a contemporary re-evaluation of his modus operandi itself.

F01(b) has two parts, both made of overlapping cones inversed relative to the same centre. For purely formal reasons, the top part is solid and the bottom reticular. The resulting figure is encased in a translucent box that crops the infinite surfaces produced by inversion, and the choice of materials (rapid prototyping and engraved acrylic) give the finished model a light and reflective quality. Critically F01(b) secretes an exciting discovery. Inversion is a transformation: it does not create anything new, it just alters what is already there. In this context circles invert (mostly) into circles, and spheres (mostly) into spheres. Planes invert into spheres too. However, unlike the standard polar variety (which must be rationalised through triangulation if it is to be built), these spheres are made of flat quadrilaterals. When the right conditions are met, standard primitives such as planes, cones or cylinders invert into non-standard surfaces, such as spheres, cycloids or cross-caps, without the need to rationalise, triangulate or develop the result.

Inverse Configurations

An operative shift from geometry to algebra – and the surprising tectonic properties uncovered in F01(b) – are wont to accelerate the cycle of design experimentation, making it possible to deploy the principle of inversion to full-blown building scale. As the point of convergence of the Yeosu 2012 World Expo, the form of IJP's Thematic Pavilion is simple, yet memorable, involving a simple spherical form

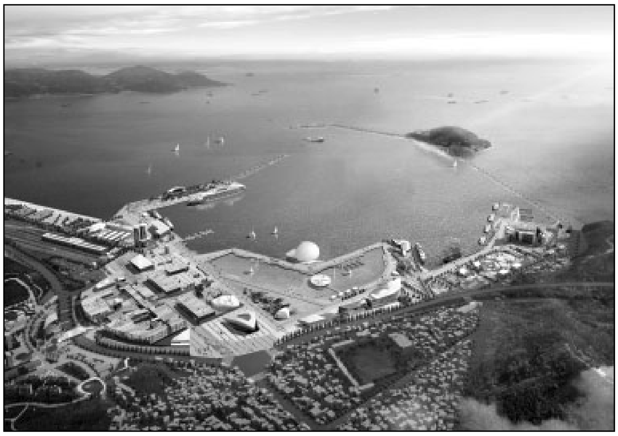


Figure 8. IJP, Yeosu 2012 Thematic Pavilion, Yeosu, South Korea, 2009
Masterplan. IJP's Thematic Pavilion features an elementary spherical form sitting on the edge of the sea, and topped with a large circular opening. It is located on the geographic centre of the boundary defined by the six public access points of the Yeosu 2012 World Fair.

topped with a large circular opening; in turns sea creature, beached monster or fishing basket, the form brings to mind disparate but related associations. The pavilion sits near the geographic centre of the area defined by the six public access points of the fair. The programme is housed on circular floor plates distributed around a large atrium, and the interiors are naturally lit through windows and internal openings. The large circular oculus pours ambient daylight levels throughout the building (perceived at the end of the public exhibition route, this gigantic opening looms over the visitor like a rising orb). On the ocean side, the Floating Island extends the pavilion to create an artificial shoreline, further extended underwater by a man-made Marine Life reef.

The mathematical subtleties of inversion help resolve the complementary demands of the brief by providing a common spatial blueprint that unites the two main features of the proposal – the spherical exhibitions volume and the fanning surface dedicated to marine life. The two figures are derived by inverting two primitives relative to the same centre, but with a different coefficient. The inverse figures intertwine in a seemingly continuous way about a single focal point, coinciding with a public observatory. The fanning outline of the Marine Reef illustrates how a closed primitive positioned next to the centre of the transformation will invert into an open superficial expanse for the same geometric reason that a circle passing though the centre of inversion will map onto an (infinite) straight line. The principle of continuity between solid and reticulated form established in F01(b) carries over seamlessly to this project, where the very same superficial geometry is given alternate material expressions above and below the variable flood line.

Material Efficiencies

Ultimately, the key question raised by the deployment of analytic mathematics in design is whether it produces material efficiencies. The surface of discrete analytic mathematics does not actually exist: what the parametric formulas produce is only a discrete array of indexical threads grouped in two sets, notated I and J (after which IJP is named). This conceptual model ensures a stable transition to materiality: if the threads are two-dimensional, they are used to define centre lines for laser-cut material profiles. If the indexical threads are not two-dimensional, they are used to print double-curved members in depth (the threads are effectively laminated from the ground up with rapid prototyping, stereo-lithography or casting). Eventually all morphogenetic results can be traced back to fundamental issues of algebraic modulation; there are various machinery-consistent equations, in other words some better suited to sheet-cutting, others to lamination.

IJP's first built project (in collaboration with RSP), the Henderson Waves Bridge is a project commissioned by the Urban Redevelopment Authority (URA), Singapore, following an open international competition. It involves an infrastructural intervention with a complex structure and a simple brief: to link two ridge summits with a continuous plane on the southern coast of the island of Singapore. The equation that was used here offers a direct application of IJP's research in periodicity[1] where, like the rhetorical algebra of medieval Arabia, it is narrated without symbols. The 'parametric pillow' is the product of three space-relations: the first may be diagrammed as an oblique plane; the second is more complex and produces a flowing periodic oscillation; and the third (and most intricate) represents a product of periodic out-of-phase

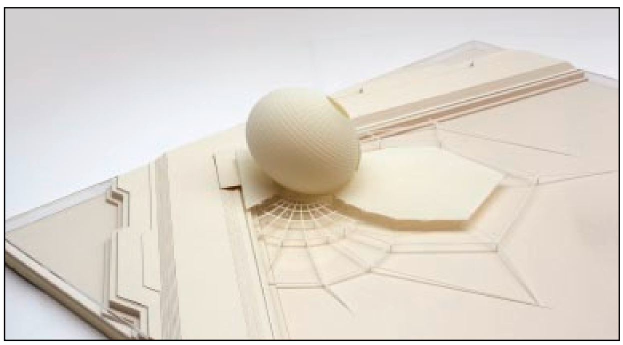


Figure 9. IJP, Yeosu 2012 Thematic Pavilion, Yeosu, South Korea, 2009
Model seen from the northwest. Side view of Floating Island and Marine Life reef.

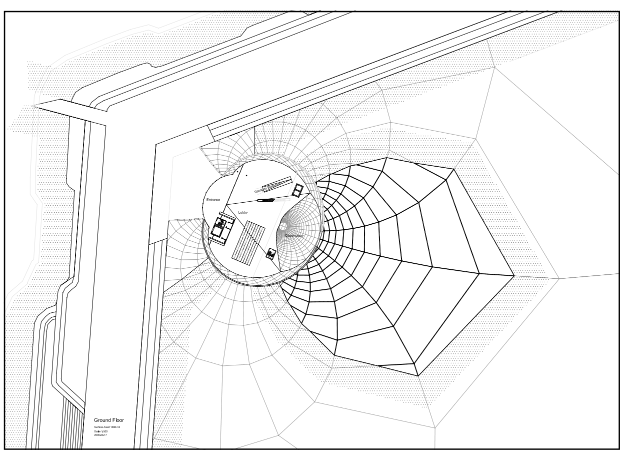


Figure 10. IJP, Yeosu 2012 Thematic Pavilion, Yeosu, South Korea, 2009
Ground-floor plan of the pavilion showing the entrance lobby, the Floating Island and the Marine Life reef that surrounds it.

F 02_06.6.8 STUDY INVERSE SURFACE

_matrices

```
TTx := | i ← last_in + 1          TTy := | i ← last_in + 1          TTz := | i ← last_in + 1
       | j ← last_jn + 1                 | j ← last_jn + 1                 | j ← last_jn + 1
       | a ← matrix(i, j, FinalX)        | a ← matrix(i, j, FinalY)        | a ← matrix(i, j, FinalZ)
       | a                               | a                               | a
```

_INVERSION CONE B   _modulo of point/inverse

$$MOD\_B_{in,\,jn} := \frac{invR^2}{\sqrt{\left(COM\_Y\_B_{in,\,jn} - INVy\right)^2 + \left(COM\_X\_B_{in,\,jn} - INVx\right)^2 + \left(COM\_Z\_B_{in,\,jn} - INVz\right)^2}}$$

_equations

$$FinalX\_B(x,y) := \left(COM\_Y\_B_{x,\,y}\right) \cdot MOD\_B_{x,\,y} - INVy \cdot \left(MOD\_B_{x,\,y} - 1\right)$$

$$FinalY\_B(x,y) := \left(COM\_X\_B_{x,\,y}\right) \cdot MOD\_B_{x,\,y} - INVx \cdot \left(MOD\_B_{x,\,y} - 1\right)$$

$$FinalZ\_B(x,y) := COM\_Z\_B_{x,\,y} \cdot MOD\_B_{x,\,y} - INVz \cdot \left(MOD\_B_{x,\,y} - 1\right)$$

_matrices

```
TTBx := | i ← last_in + 1          TTBy := | i ← last_in + 1          TTBz := | i ← last_in + 1
        | j ← last_jn + 1                  | j ← last_jn + 1                  | j ← last_jn + 1
        | a ← matrix(i, j, FinalX_B)       | a ← matrix(i, j, FinalY_B)       | a ← matrix(i, j, FinalZ_B)
        | a                                | a                                | a
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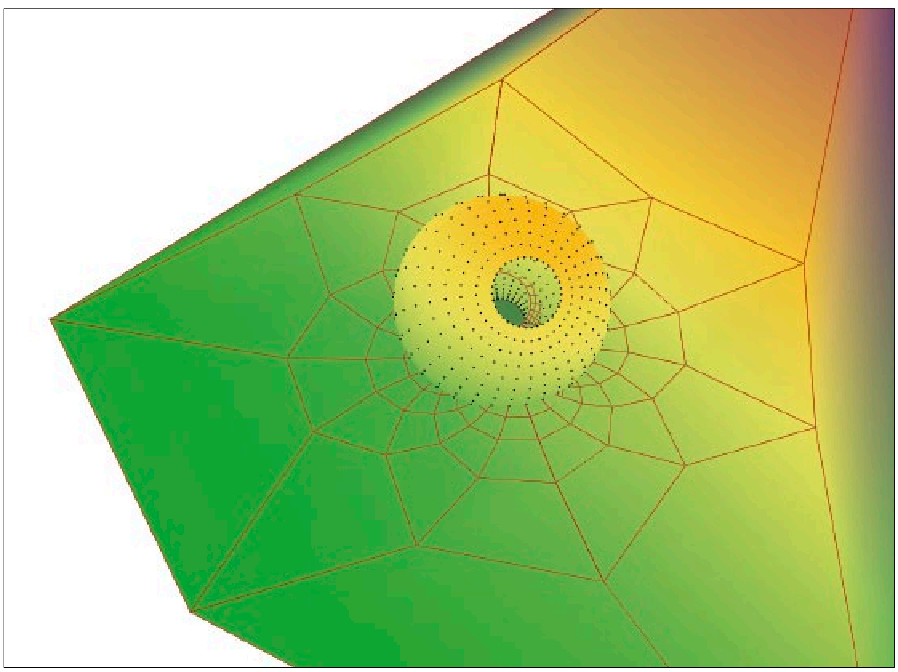
inversion



(TTy, TTx, TTz), (TTBy, TTBx, TTBz)

Figure 11. IJP, Yeosu 2012 Thematic Pavilion, Yeosu, South Korea, 2009
Partial mathematical formulation. Such worksheets (greatly simplified here for purposes of publication) lie at the heart of the office's methodology. The upper half of the sheet features the parametric surface calculations that are the true engine of the process; the lower, a read-only illustration of the result. For designers skilled in this methodology, visualising the act of 'writing form' is not strictly necessary, but it is useful in helping to alleviate the abstraction of the process.

Figure 12. IJP and RSP Architects Planners and Engineers, Henderson Waves, Singapore, 2008
Location plan of the bridge and timber end-works by IJP (competition stage). The project begins with a timber pathway linking the springing point of the bridge to the busy vehicular loop on Mount Faber –shown in the lower-right corner of the plan. The project continues with the bridge itself and concludes with a ramp that connects the bridge to another existing circular path, winding its way around the summit of Telok Blangah Hill (in the upper-left corner of the plan). The entrance pathway, bridge deck and connecting ramp are given the same steel and timber treatment and can hardly be distinguished.

Figure 13. IJP and RSP Architects Planners and Engineers, Henderson Waves, Singapore, 2008
View of the central span and completed deck in May 2008.

Figure 14. IJP and RSP Architects Planners and Engineers, Henderson Waves, Singapore, 2008
Timber deck of the main span under construction. This central arch spans 57 metres (187 feet), and rises to more than 6 metres (19.6 feet) at its apex. Too large to be prefabricated off-site and craned into position, it was assembled on a makeshift platform directly above Henderson Road, then raised into position by a battery of hydraulic jacks. The centre-lines of the steel members shown (central arch, edge member, mid-height member and curved ribs) are determined by a single set of parametric equations.

oscillations spreading in perpendicular dimensions. In space, the pillow resembles an egg-crate-like arrangement of peaks and depressions. As the ranges vary, it divides like a cell into two, three, or N swelling bulges, as if held in place by knots.

The structural considerations driving this morphogenetic process determined a comparatively rational scheme that leverages the structural integrity of the surface's indexical threads, a scheme formulated as a succession of arches and catenaries, behaving like a dual, differentiated beam. In terms of structural expression, the bridge systematises and amplifies the problem of converting selected indexical threads into centre lines of material members with structural roles. Named after the ubiquitous index I (reappearing here as a steel beam), the iThreads provide the physical edge and lateral stability of the surface form. Along the other dimension, the J threads fulfil the gravitational demands of the structure, and the piles sit at both ends of each span where the surface self-intersects and the section of the structure is reduced to a single beam.

The timber deck presents a subtler challenge. Largely in evidence in its completed form, the recurrence of kinks along its surface indicates that it is discontinuous. To accommodate wheelchair access, the deck is bent, sheared, oblique and punctuated with thresholds that break its continuity at regular intervals. Every flight and threshold is computed by a specific variant of the equation, and the numbers collected from 160 different formulas in a spreadsheet. At competition stage, this super-surface was envisioned as a thin timber veneer stretched over the steel members, peeling off the structure to provide seating and playing areas. Eventually the form of the veneer was given by the same overall equation, with minor adjustments for the seating areas, which required their own custom calculations. The 1,500-square-metre (16,145-square-foot) expanse of tropical hardwood is the fulcrum of Henderson Waves. Its double-curved areas form a tapestry of 5,000 modular boards, each varying by a single degree every few metres and all tapered to measure.

The Relational Body

IJP's methodology is no different from a traditional approach based on precedent, otherwise known as 'type', but with the proviso that its types are abstract rather than figurative, and invisible rather than conspicuous. Its commitment to typological reduction goes hand in hand with a rejection of traditional modes of composition, as the traditional understanding of form begins and ends with what is, from the standpoint of its mathematical model, an unnecessary premise: the separation of the whole into parts. Parametric surfaces are naturally inured to this mode of thought because their constitutive parts are not fragments, in the sense that a cornice would constitute a fragment of an elevation, but relationships.

Relationships act as parts only in a loose, strictly functional sense, inasmuch as they can be manipulated independently to alter a whole. Their role is neither 'pure' nor distinct, and the combined impact they visit on the body muddles their respective areas of influence. A form shaped by modulation has no discrete limbs; one cannot chop it off into pieces nor indulge in the permutation and scaling of parts to which parametric 'invention' is often reduced. Consider, for instance, the parametric seed of the pillow that in time hatched into Henderson Waves. What exactly is it made of?

This common surface is obtained by composing one linear transformation with two periodic ones. The three relations determine the motions that shape it in breadth, width and depth, and clarify – if their interplay can be unravelled – why it looks the way it does. The symmetries of the pillow can be traced back to periodic cycles with identical beginnings and ends; its upright stance will be traced back to a linear range increase. And the pillow's subsequent cell-like division into two, three or more swellings reflects the number of phases fed to a periodic function.

None of these surfaces looks like the pillow itself. None resembles the form, yet all jointly determine it. Had they not been there, the form could not have been produced because it cannot be modelled simply by deforming other, ready-made surfaces, however sophisticated the software may be. Had they not been identified as antecedents, it could not have been retrospectively read. With their dependent functions, variable parametric surfaces are both a means to complexity and the way out of its mystifying embrace. They are, in other words, the ultimate objects of knowledge. ⌂

Notes

1. See George Liaropoulos-Legendre, *IJP: The Book of Surfaces*, AA Publications (London), 2003, pp 2, 8.