TIØ 1: Financial Engineering in Energy Markets

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COURSE OUTLINE

- Introduction (Chs 1–2)
- Mathematical Background (Chs 3–4)
- Investment and Operational Timing (Chs 5–6)
- Entry, Exit, Lay-Up, and Scrapping (Ch 7)
- Recent Theoretical Work I: Capacity Sizing
- Recent Theoretical Work II: Risk Aversion and Multiple Risk Factors
- Applications to the Energy Sector I: Capacity Sizing, Timing, and Operational Flexibility
- Applications to the Energy Sector II: Modularity and Technology Choice

LECTURE OUTLINE

★ Review of the now-or-never NPV approach

★ Options and irreversibility

★ Simple examples with uncertainty

TRADITIONAL APPROACH TO INVESTMENT

- The neoclassical view of investment under uncertainty is that a project should be accepted if its expected NPV is positive
  - How to estimate cash flows?
  - Which discount rate to use?
  - Can also express the decision rule in terms of the marginal benefit and cost of the incremental unit
- Jorgenson (1963) compares the marginal value of capital with its periodic rental cost
- Tobin (1969) assesses the ratio q of the imputed value of capital to its purchase price
- This approach ignores irreversibility, uncertainty, and discretion over timing, however

OPTION APPROACH

- The option approach accounts for:
  - Irreversibility in terms of sunk costs
  - Uncertainty in the cash flows
  - Timing over the investment decision, which includes the possibility to wait for more information about the cash flows
- Consequently, the NPV decision rule must be modified since the value of the project must exceed the investment cost by an amount equal to the opportunity cost of killing the option
- In practice, firms do not invest until the output price, for example, exceeds the long-run average cost substantially
- Investment under uncertainty is, thus, analogous to the exercise of financial options

IRREVERSIBILITY AND DEFERRAL

- Why are investment costs sunk?
  - Industry-specific capital is not recoverable because the circumstances that lead a firm to exit the industry will also make its assets less attractive to other firms
  - Equipment that is not industry specific will suffer from the lemons problem: buyers will lack information and pay only the average cost, while owners of an above-average machine will be hesitant to sell (Akerlof (1970))
  - Government regulation, such as capital controls, may also lead to irreversibility
- Deferral is valuable because the additional information gained by waiting often outweighs the cost, e.g., in terms of lost revenue
  - Invest only if the output price of the asset increases (just like with a call option)
  - With threat of entry by other firms, the value of waiting may be lower

NON-ECONOMIC APPLICATIONS

- Marriage and suicide
  - Waiting for a better match has option value, especially if the costs (in terms of courtship or divorce) are high
  - Societies that make divorce difficult would observe longer waiting before deciding
  - However, such societies possibly internalise this and have better matchmaking facilities
  - In terms of suicide, perhaps most aggrieved people would ignore the option value of staying alive and exercise the option too quickly
  - Again, societal taboos raise the perceived cost of the act
- Legal reform
  - Politicians may act too soon based on current public opinion to change laws and ignore the option value of waiting before opinion is sufficiently well entrenched
  - Constitutional framers often impose high costs for changing the law, which corrects for the myopia of ignoring the option value

TWO-PERIOD EXAMPLE

- Suppose that we can invest in a widget factory that will last forever, i.e., no irreversibility or suspension
  - Factory may be built instantaneously at cost $I = 1600$ and produce one unit per year at zero operating cost
  - Current widget price is $P_0 = 200$ and will rise (fall) to $P_1 = 300$ ($P_1 = 100$) with probability $q = \frac{1}{2}$ ($1 - q = \frac{1}{2}$) next year
- After next year, there will be no more price changes
  - NPV of investing now (with a future expected price of \$200) given the risk-free interest rate $r = 0.1$ is $NPV = -I + \sum_{t=0}^{\infty} \frac{P_0}{(1+r)^t} = -1600 + 200 + \frac{200}{0.1} = 600 \Rightarrow$ invest now
- What if we wait a year and invest only if the price increases?
  - $NPV = \frac{1}{2}\left[-\frac{1600}{1.1} + \sum_{t=1}^{\infty} \frac{300}{1.1^t}\right] = \frac{1}{2}\left[-1455 + 3000\right] = 773$
  - Now, it seems better to wait a year before investing
- The option to delay and irreversibility introduce the opportunity cost to investing now

TWO-PERIOD EXAMPLE: Figure 2.1

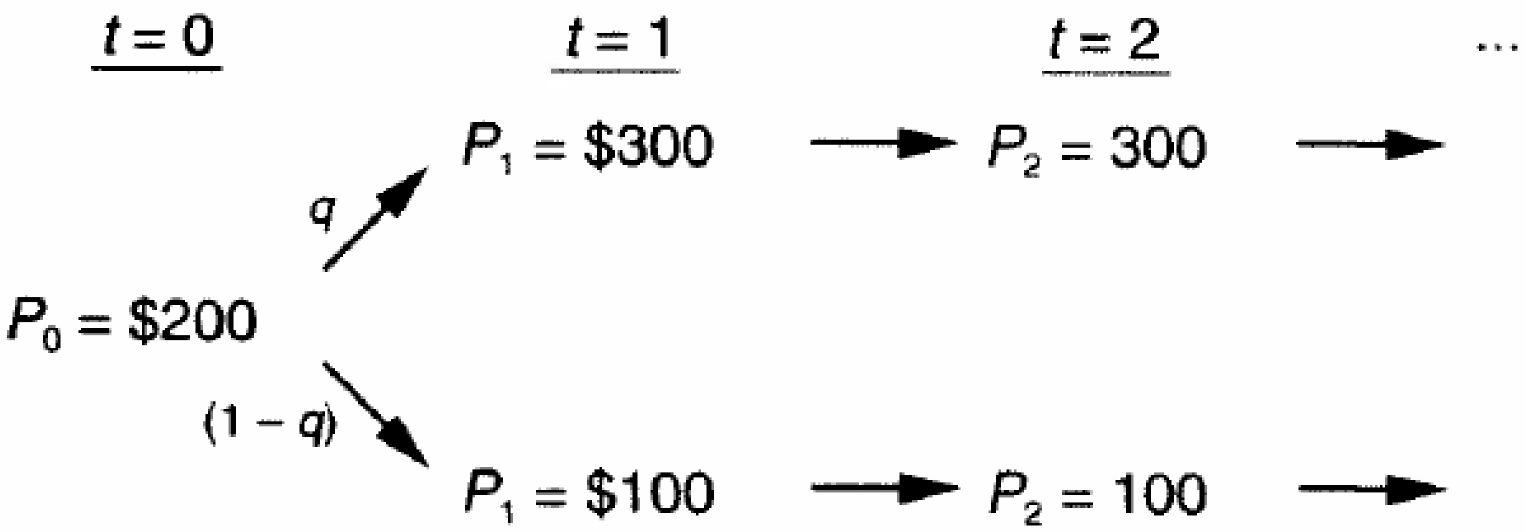

Figure 2.1. Price of Widgets

VALUE OF FLEXIBILITY

- Having the option to delay the investment is worth \$773-\$600=\$173
- Another way to frame it: how high should I be in the flexible case before it is no more valuable than the inflexible case?
  - Solve the following equation for $\bar{I}$: $\frac{1}{2}\left[-\frac{\bar{I}}{1.1}+\sum_{t=1}^{\infty}\frac{300}{1.1^t}\right]=600 \Rightarrow \bar{I}=1980>I$
  - Building a factory only now at a cost of \$1600 has the same value as the opportunity to build one either now or next year at a cost of \$1980
- No gain from hedging exposure to widget price
  - Sell futures contract for delivery of 11 units in one year at a price of \$200: if the price increases to \$300 (decreases to \$100), then lose \$1100 (gain \$1100) on derivatives
  - Since the project is worth \$3300 (\$1100) in case of a price increase (decrease), the PV of cash flows is \$2200

ANALOGY TO FINANCIAL OPTIONS

★ Use standard option pricing methods:

- Let F_0 (F_1) be the value of the investment opportunity today (next year)
- First, determine F_1: it is worth $\sum_{t=0}^{\infty} \frac{300}{1.1^t} - 1600 = 1700$ next year if the price is \$300 and zero otherwise
- Determine F_0 by constructing a risk-free portfolio, Φ_0, that consists of one unit of the option and is short n units of the underlying
- Portfolio today is worth $\Phi_0 = F_0 - nP_0 = F_0 - 200n$
- Next year, it is worth $\Phi_1 = F_1 - nP_1$, which is equal to $1700 - 300n$ if $P_1 = 300$ and $-100n$ otherwise
- In order for the portfolio to be risk free, we must have the same payoff regardless of the state of nature: $1700 - 300n = -100n \Rightarrow n = 8.5, \Phi_1 = -850$
- No-arbitrage condition: return on portfolio must equal the risk-free rate earned on its initial value
- $\Phi_1 - \Phi_0 - rnP_0 = \Phi_1 - F_0 + nP_0 - rnP_0 = 680 - F_0$
- Since $r\Phi_0 = 0.1(F_0 - 1700)$, we obtain $F_0 = 773$

SENSITIVITY ANALYSIS: Cost of Investment

- In case of arbitrary investment cost, I, we have $\Phi_1 = 3300 - I - 300n$ if $P_1 = 300$ and $\Phi_1 = -100n$ otherwise
  - Equating the two implies $n = 16.5 - 0.005I$ and $\Phi_1 = 0.5I - 1650$
- Since $\Phi_0 = F_0 - nP_0 = F_0 - 3300 + I$, the expected portfolio appreciation is $\Phi_1 - \Phi_0 - rnP_0 = 1320 - F_0 - 0.4I$
- Equating this to the instantaneous risk-free return, $r\Phi_0 = 0.1F_0 - 330 + 0.1I$, yields $F_0 = 1500 - 0.455I$
- Invest today if $V_0 > F_0 + I \Rightarrow 2200 > 1500 + 0.545I \Rightarrow I < 1283.33$ (Figure 2.2)
- Never invest if $F_0 < 0 \Rightarrow I > 3300$

SENSITIVITY ANALYSIS : Figure 2.2

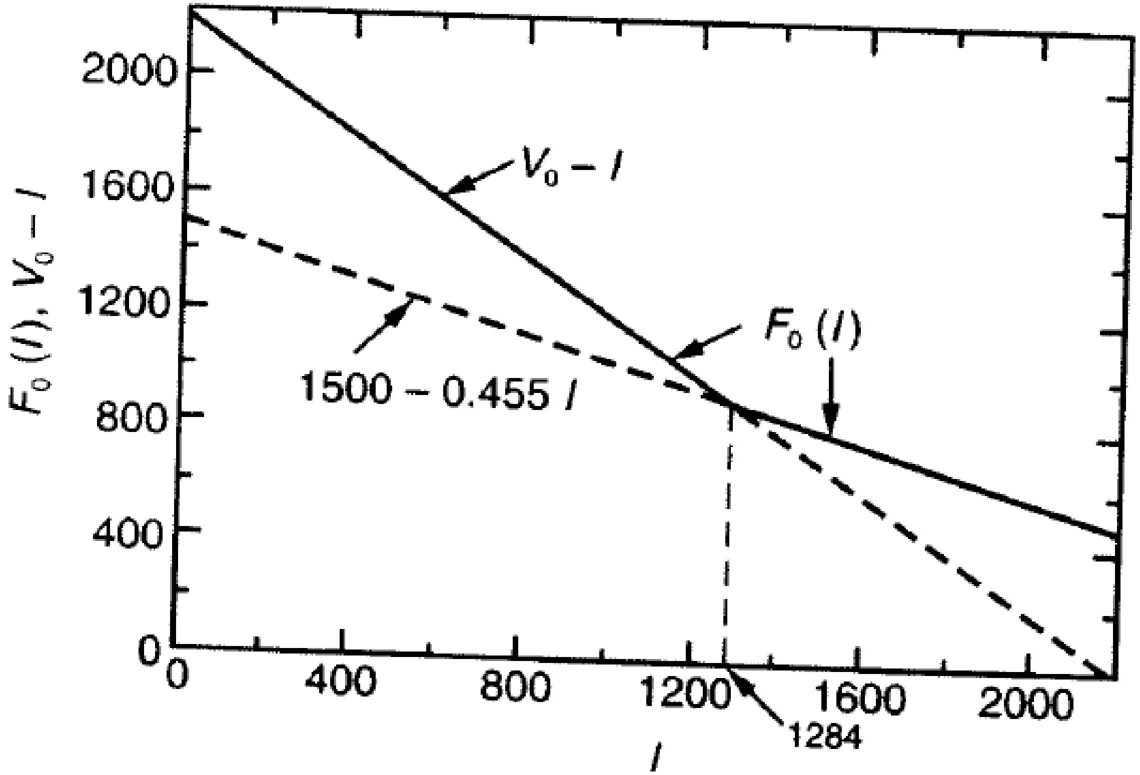

Figure 2.2. *Option to Invest in Widget Factory*

SENSITIVITY ANALYSIS: Initial Price

- ★ Have an arbitrary P_0 so that $P_1 = 1.5P_0$ or $P_1 = 0.5P_0$
  - ▶ This implies that $F_1 = \max(0, 11P_1 - 1600)$
  - ▶ Consequently, $\Phi_1 = 11\ \mathrm{P}_1 - 1600 - nP_1 = 16.5P_0 - 1600 - 1.5nP_0$ if $P_1 = 1.5P_0$ and $\Phi_1 = -0.5nP_0$ otherwise
  - ▶ Equating the two yields $n = 16.5 - \frac{1600}{P_0}$ and $\Phi_1 = -8.25P_0 + 800$
- ★ In year 0, $\Phi_0 = F_0 - nP_0 = F_0 - 16.5P_0 + 1600$
- ★ Expected net appreciation is $\Phi_1 - \Phi_0 - rnP_0 = 6.6P_0 - F_0 - 640$
- ★ Equate this to the rate of return on the portfolio, $r\Phi_0 = 0.1F_0 - 1.65P_0 + 160$, to yield $F_0 = 7.5P_0 - 727$
- ★ Never invest if $F_0 < 0 \Rightarrow P_0 < 97$
- ★ Invest now if $F_0 + I < V_0 \Rightarrow 7.5P_0 - 727 + 1600 < 11P_0 \Rightarrow P_0 > 249$ (Figure 2.4)

SENSITIVITY ANALYSIS: Figure 2.4

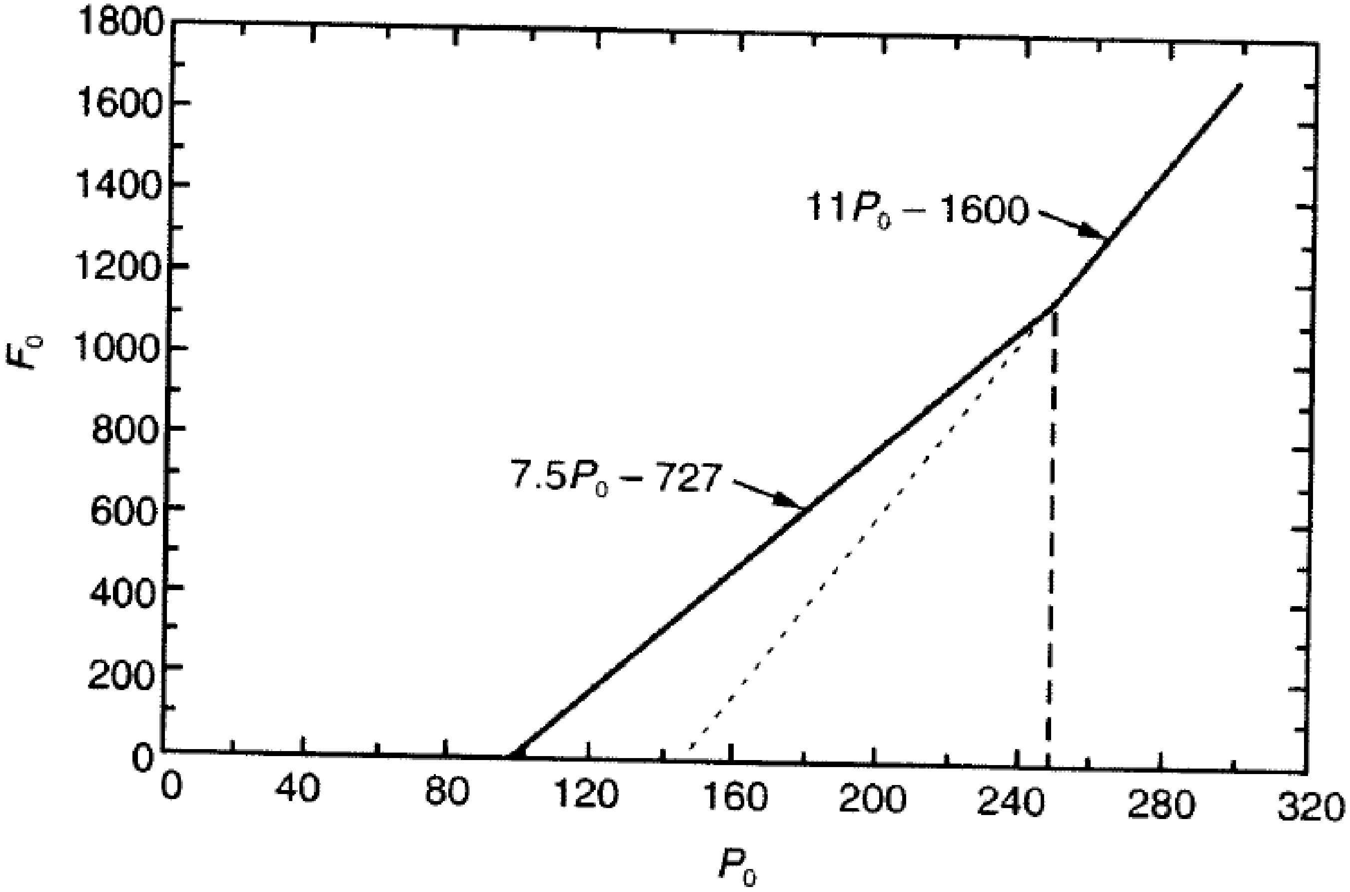

Figure 2.4. Value of Option to Invest as a Function of Initial Price

SENSITIVITY ANALYSIS: Price Uncertainty

- Either $P_1 = 1.75P_0$ or $P_1 = 0.25P_0$
  - This implies $F_1 = \max(0, 11P_1 - 1600)$ so that $\Phi_1 = 19.25P_0 - 1600 - 1.75nP_0$ if $P_1 = 1.75P_0$ and $\Phi_1 = -0.25nP_0$ otherwise
  - Equating the two yields $n = 12.83 - \frac{1066.67}{P_0}$
  - Thus, $\Phi_1 = -3.2075P_0 + 266.67$ and $\Phi_0 = F_0 - 12.83P_0 + 1066.67$
- Expected net appreciation is $\Phi_1 - \Phi_0 - rnP_0 = 8.3395P_0 - F_0 - 693.33$
- Risk-free return is $r\Phi_0 = 0.1F_0 - 1.283P_0 + 106.67$
- Equating the two yields $F_0 = 8.747P_0 - 727$, which is more sensitive to P_0 than before
- Never invest if $F_0 < 0 \Rightarrow P_0 < 83.15$
- Invest now if $F_0 + I < V_0 \Rightarrow P_0 > 388$

BAD NEWS PRINCIPLE

- Investment threshold price depends on the size of the downward price movement, d (Bernanke (1983))
  - Ability to avoid consequences of bad news that leads us to wait
- Suppose that $P_1 = (1+u)P_0$ with probability q and $P_1 = (1-d)P_0$ otherwise
- Invest now: $NPV_0 = -I + P_0 + q\sum_{t=1}^{\infty} \frac{(1+u)P_0}{1.1^t} + (1-q)\sum_{t=1}^{\infty} \frac{(1-d)P_0}{1.1^t} = -I + 10P_0[1.1 + q(u+d) - d]$
- Invest next year: $NPV_1 = \frac{1}{1.1}\left[q\max(0, -I + 11(1+u)P_0\right]$
  - Indifference point: $NPV_0 = NPV_1 \Rightarrow P_0^* = \frac{I}{11}\frac{[0.1+(1-q)]}{[0.1+(1-q)(1-d)]}$
- Only magnitude of the downward move affects the investment threshold

THREE-PERIOD EXAMPLE

- Now, assume that the price can also change in period 2, i.e., P_2 can be either $2.25P_0$, $0.75P_0$, or $0.25P_0$
- There are now five possible strategies
  - Never invest
  - Invest only in period 2 if $P_2 = 2.25P_0$
  - Invest in period 1 if $P_1 = 1.5P_0$ but never invest if $P_1 = 0.5P_0$
  - Invest in period 1 if $P_1 = 1.5P_0$ and wait otherwise
  - Invest in period 0
- Solve this problem by starting in period 1 assuming no investment and working backwards
  - If $P_1 = 0.5P_0$, then invest next year only if price rises
  - Repeat for $P_1 = 1.5P_0$

THREE-PERIOD EXAMPLE: t=1

★ If $P_1 = 0.5P_0$, then invest at $t = 2$ if $P_2 = 0.75P_0$ and do nothing if $P_2 = 0.25P_0$

- For $P_2 = 0.75P_0$, $F_2 = \sum_{t=0}^{\infty} \frac{0.75P_0}{1.1^t} - 1600 = 8.25P_0 - 1600$ and $\Phi_2 = 8.25P_0 - 1600 - 0.75n_1P_0$
- Otherwise, for $P_2 = 0.25P_0$, $F_2 = 0$ and $\Phi_2 = -0.25n_1P_0$
- Thus, $n_1 = 16.5 - \frac{3200}{P_0}$ and $\Phi_2 = 800 - 4.125P_0$
- Portfolio return: $\Phi_2 - \Phi_1 - rn_1P_1 = 3.3P_0 - F_1 - 640$
- Risk-free return: $r\Phi_1 = 0.1F_1 - 0.825P_0 + 160$
- No-arbitrage condition: $F_1 = 3.75P_0 - 727.3$
- Do not invest if $F_1 < 0 \Rightarrow P_0 < 193.94$

THREE-PERIOD EXAMPLE: t=1

- If $P_1 = 1.5P_0$, then invest at $t = 2$ if $P_2 = 2.25P_0$ and do nothing if $P_2 = 0.75P_0$
  - For $P_2 = 2.25P_0$, $F_2 = \sum_{t=0}^{\infty} \frac{2.25P_0}{1.1^t} - 1600 = 24.75P_0 - 1600$ and $\Phi_2 = 24.75P_0 - 1600 - 2.25n_1P_0$
  - Otherwise, for $P_2 = 0.75P_0$, $F_2 = 0$ and $\Phi_2 = -0.75n_1P_0$
  - Thus, $n_1 = 16.5 - \frac{1067}{P_0}$ and $\Phi_2 = 800 - 12.375P_0$
  - Portfolio return: $\Phi_2 - \Phi_1 - rn_1P_1 = 9.9P_0 - F_1 - 640$
  - Risk-free return: $r\Phi_1 = 0.1F_1 - 2.475P_0 + 160$
  - No-arbitrage condition: $F_1 = 11.25P_0 - 727.3$
  - Do not invest if $F_1 < 0 \Rightarrow P_0 \leq 64.65$
  - Invest immediately if $F_1 + I < V_1 \Rightarrow P_0 > 166.23$
  - Otherwise, if $64.65 < P_0 \leq 166.23$, then wait for period 2

THREE-PERIOD EXAMPLE: t=0

- The five possible strategies are:
  - $P_0 \leq 64.65$: never invest, i.e., $F_0 = 0$
  - $64.65 < P_0 \leq 166.23$: invest only in period 2 if $P_2 = 2.25P_0$
  - $166.23 < P_0 \leq 193.94$: invest in period 1 if $P_1 = 1.5P_0$ but never invest otherwise
  - $193.94 < P_0 \leq P_I$: invest in period 1 if $P_1 = 1.5P_0$ and wait otherwise
  - $P_0 > P_I$: invest immediately in period 0
- $64.65 < P_0 \leq 166.23$
  - Invest only in $t = 2$ if the price increases both times
  - $P_1 = 1.5P_0$: $F_1 = 11.25P_0 - 727.3$ and $\Phi_1 = 11.25P_0 - 727.3 - 1.5n_0P_0$
  - $P_1 = 0.5P_0$: $F_1 = 0$ and $\Phi_1 = -0.5n_0P_0$
  - Thus, $n_0 = 11.25 - \frac{727.3}{P_0}$ and $\Phi_1 = 363.65 - 5.625P_0$
  - Portfolio return: $\Phi_1 - \Phi_0 - rn_0P_0 = 4.5P_0 - F_0 - 290.92$
  - Risk-free return: $r\Phi_0 = 0.1F_0 - 1.125P_0 + 72.73$
  - No-arbitrage condition: $F_0 = 5.11P_0 - 330.6$

THREE-PERIOD EXAMPLE: t=0

- $166.23 < P_0 \leq 193.94$
  - Invest in $t = 1$ if the price increases and never invest otherwise
  - $P_1 = 1.5P_0$: $F_1 = V_1 - I = 16.5P_0 - 1600$ and $\Phi_1 = 16.5P_0 - 1600 - 1.5n_0P_0$
  - $P_1 = 0.5P_0$: $F_1 = 0$ and $\Phi_1 = -0.5n_0P_0$
  - Thus, $n_0 = 16.5 - \frac{1600}{P_0}$ and $\Phi_1 = 800 - 8.25P_0$
  - Portfolio return: $\Phi_1 - \Phi_0 - rn_0P_0 = 6.6P_0 - F_0 - 640$
  - Risk-free return: $r\Phi_0 = 0.1F_0 - 1.65P_0 + 160$
  - No-arbitrage condition: $F_0 = 7.5P_0 - 727.3$
- $193.94 < P_0 \leq P_I$
  - Invest in $t = 1$ if the price increases and wait otherwise
  - $P_1 = 1.5P_0$: $F_1 = 16.5P_0 - 1600$ and $\Phi_1 = 16.5P_0 - 1600 - 1.5n_0P_0$
  - $P_1 = 0.5P_0$: $F_1 = 3.75P_0 - 727.3$ and $\Phi_1 = 3.75P_0 - 727.3 - 0.5n_0P_0$
  - Thus, $n_0 = 12.75 - \frac{872.7}{P_0}$ and $\Phi_1 = -290.95 - 2.625P_0$
  - Portfolio return: $\Phi_1 - \Phi_0 - rn_0P_0 = 8.85P_0 - F_0 - 1076.38$
  - Risk-free return: $r\Phi_0 = 0.1F_0 - 1.275P_0 + 87.27$
  - No-arbitrage condition: $F_0 = 9.2P_0 - 1057.9$

THREE-PERIOD EXAMPLE: t=0

- $P_I < P_0$
  - Invest immediately at $t = 0$
  - $V_0 - I = 11P_0 - 1600$
  - Find P_I by equating F_0 and $V_0 - I$
  - Thus, $11P_I = 1600 + 9.2P_I - 1057.9$, which implies $P_I = 301.2$
- Value of option to invest is still a piecewise linear function of the price, but now has five parts (Figure 2.6)
- Extension of the problem to more periods will result in more kinks in the function and later on explore continuous fluctuation of the payoff

THREE-PERIOD EXAMPLE: Figure 2.6

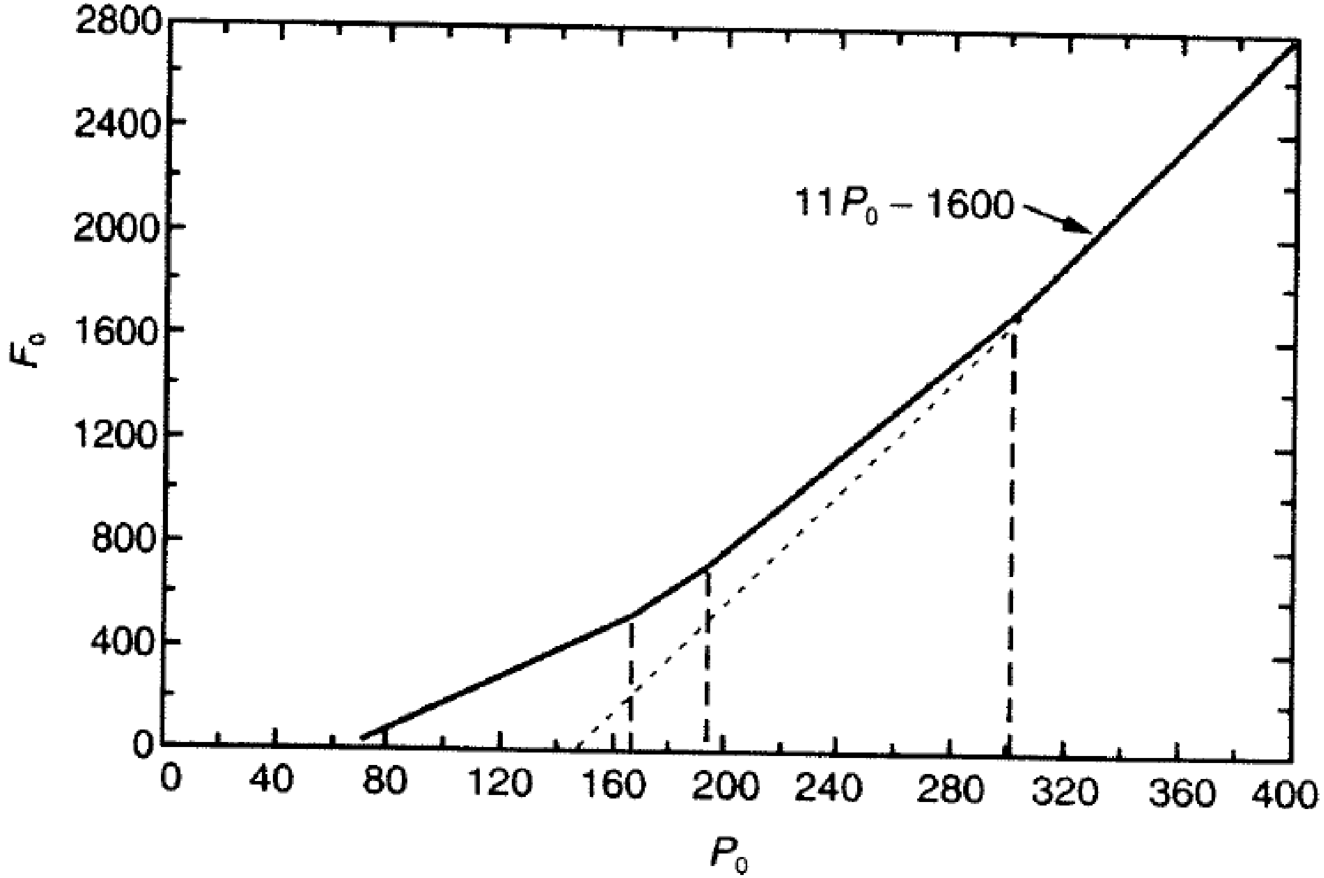

Figure 2.6. *Value of Option to Invest as a Function of Initial Price*

COST UNCERTAINTY

- Suppose $P_0 = 200$ is fixed, but that I can vary
  - $I_0 = 1600$ and either $I_1 = 2400$ or $I_1 = 800$ each with probability $\frac{1}{2}$
- If invest today, then $NPV_0 = 2200 - 1600 = 600$
- Otherwise, if we wait one year, then $NPV_0' = \frac{1}{2}\left[\sum_{t=1}^{\infty} \frac{200}{1.1^t} - \frac{800}{1.1}\right] = \frac{1}{2}\left[2000 - \frac{800}{1.1}\right] = 636$
- Therefore, it is better to wait when there is cost uncertainty as well
- In case of technical uncertainty, it may be better to invest in the first phase in order to obtain more information about the cost

INTEREST RATE UNCERTAINTY

- Have $r_0 = 0.10$ and either $r_1 = 0.15$ or $r_1 = 0.05$ each with probability $\frac{1}{2}$
  - Since the perpetuity function is convex, i.e., $\frac{1}{r}$, Jensen's Inequality implies that $\mathcal{E}\left[\frac{1}{r}\right] = 13.33 > \frac{1}{\mathcal{E}[r]} = 10$
- If $P_0 = 200$ and $I = 2000$ are fixed, then the PV of cash flows next year will be $V_1 = \sum_{t=0}^{\infty} \frac{200}{1.1^t} = 2200$ if $r = 0.1$
- But with uncertain r, either $V_1 = 1533$ or $V_1 = 4200$ with equal probability; thus, $\mathcal{E}[\tilde{V}_1] = 2867 > V_1$
  - Without uncertainty: $NPV_0 = 2200 - 2000 = 200$ if invest today and $NPV_0 = 181.82$ if we invest next year
  - With uncertainty: $\mathcal{E}[NPV_0] = -2000 + 200 + \frac{\mathcal{E}[\tilde{V}_1]}{1.1} = 806$ if invest today and $\mathcal{E}[NPV_0] = \frac{1}{2}\left[-\frac{2000}{1.1} + \frac{1}{1.1}\sum_{t=0}^{\infty} \frac{200}{1.05^t}\right] = 1000$
- Mean-preserving volatility increases option value along with the incentive to wait

SCALE VERSUS FLEXIBILITY

- Economies of scale are often counterbalanced by flexibility
  - Power company faces demand growth of 100 MW per annum
  - Plan A: 200 MW coal-fired power plant, which will cost \$180 million to build and \$19 million per annum per 100 MW to operate forever
  - Plan B: 100 MW oil-fired power plant, which will cost \$100 million to build and \$20 million per annum per 100 MW to operate
  - If no uncertainty and $r = 0.1$, then select coal
- Oil price is relatively more uncertain, so suppose that it can either rise to \$30 million or fall to \$10 million after a year and 100 MW of capacity is built in the first two years
  - $PV_A = 180 + \sum_{t=0}^{\infty} \frac{19}{1.1^t} + \sum_{t=1}^{\infty} \frac{19}{1.1^t} = 579$ (coal today)
  - $PV_B = 100 + \frac{100}{1.1} + \sum_{t=0}^{\infty} \frac{20}{1.1^t} + \sum_{t=1}^{\infty} \frac{20}{1.1^t} = 611$ (oil today)
  - $PV_B^{'} = 100 + \sum_{t=0}^{\infty} \frac{20}{1.1^t} + \frac{1}{2}\left[\frac{100}{1.1} + \sum_{t=1}^{\infty} \frac{10}{1.1^t}\right] + \frac{1}{2}\left[\frac{180}{1.1} - \frac{90}{1.1^2} + \sum_{t=1}^{\infty} \frac{19}{1.1^t}\right] = 555$ (oil today and cheaper of the two next year)

QUESTIONS